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V 1

A TEXTBOOK

ON

STEAM ENGINEERING

INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

ARITHMETIC
MENSURATION AND USE OF LETTERS IN FORMULAS
PRINCIPLES OF MECHANICS
MACHINE ELEMENTS
MECHANICS OF FLUIDS
STRENGTH OF MATERIALS

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ARITHMETIC.

(PART 1.)

DEFINITIONS.

1. **Arithmetic** is the art of reckoning, or the study of numbers.

2. A **unit** is *one*, or a single thing, as *one*, *one* bolt, *one* pulley, *one* dozen.

3. A **number** is a unit, or a collection of units, as *one*, *three* engines, *five* boilers.

4. The **unit of a number** is one of the collection of units which constitutes the number. Thus, the unit of *twelve* is *one*, of *twenty* dollars is *one* dollar, of *one hundred* bolts is *one* bolt.

5. A **concrete number** is a number applied to some particular kind of object or quantity, as three *grate bars*, five *dollars*, ten *pounds*.

6. An **abstract number** is a number that is not applied to any object or quantity, as *three*, *five*, *ten*.

7. **Like numbers** are numbers which express units of the *same kind*, as six *days* and ten *days*, two *feet* and five *feet*.

8. **Unlike numbers** are numbers which express units of *different kinds*, as ten *months* and eight *miles*, seven *wrenches* and five *bolts*.

NOTATION AND NUMERATION.

9. Numbers are expressed in three ways: (1) by words; (2) by figures; (3) by letters.

10. **Notation** is the art of expressing numbers by figures or letters.

11. **Numeration** is the art of reading the numbers which have been expressed by figures or letters.

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12. The **Arabic notation** is the method of expressing numbers by figures. This method employs ten different **figures** to represent numbers, viz.:

Figures	0	1	2	3	4	5	6	7	8	9
Names	*naught, cipher, or zero.*	*one*	*two*	*three*	*four*	*five*	*six*	*seven*	*eight*	*nine*

The first character (0) is called **naught, cipher,** or **zero,** and when standing alone has no value.

The other nine figures are called **digits,** and each has a value of its own.

Any whole number is called an **integer.**

13. As there are only ten *figures* used in expressing numbers, each figure must have a different *value* at different times.

14. The value of a figure depends upon its *position* in relation to other figures.

15. Figures have **simple** values, and **local,** or **relative,** values.

16. The **simple** value of a figure is the value it expresses when standing alone.

17. The **local,** or **relative,** value of a figure is the *increased* value it expresses by having other figures placed on its right.

For instance, if we see the figure 6 standing alone, thus 6

we consider it as *six units*, or simply **six.**

Place another 6 to the *left* of it; thus 66

The original figure is still *six units*, but the second figure is *ten times* 6, or 6 **tens.**

If a third 6 be now placed still one place further to the *left*, it is increased in value *ten times* more, thus making it 6 **hundreds** 666

A fourth 6 would be 6 **thousands** 6666

A fifth 6 would be 6 **tens of thousands,** or **sixty thousand** 66666

A sixth 6 would be 6 **hundreds of thousands** . . 666666

A seventh 6 would be 6 **millions** 6666666

The entire line of seven figures is read *six millions six hundred sixty-six thousands six hundred sixty-six.*

18. The **increased value** of each of these figures is its *local*, or *relative*, value. Each figure is *ten times* greater in value than the one immediately on its *right.*

19. The **cipher** (0) has no value in itself, but it is useful in determining the place of other figures. To represent the number *four hundred five*, two digits only are necessary, one to represent *four hundred* and the other to represent *five units;* but if these two digits are placed together, as 45, the 4 (being in the second place) will mean 4 *tens.* To mean 4 *hundreds*, the 4 should have two figures on its right, and a *cipher* is therefore inserted in the place usually given to *tens*, to show that the number is composed of *hundreds* and *units* only, and that there are no *tens. Four hundred five* is therefore expressed as 405. If the number were *four thousand and five*, two ciphers would be inserted; thus, 4005. If it were *four hundred fifty*, it would have the *cipher* at the right-hand side to show that there were no *units*, and only *hundreds and tens;* thus, 450. *Four thousand and fifty* would be expressed 4050.

20. In *reading* figures, it is usual to *point off* the number into groups of three figures each, beginning at the right-hand, or **units,** column, a comma (,) being used to point off these groups.

Billions.			*Millions.*			*Thousands.*			*Units.*		
Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds of Units.	Tens of Units.	Units.
4	3	2 ,	1	9	8 ,	7	6	5 ,	4	3	2

In *pointing off* these figures, begin at the right-hand figure and count—*units, tens of units, hundreds of units;* the next

group of three figures is *thousands;* therefore, we insert a comma (,) before beginning with them. Beginning at the figure 5, we say *thousands, tens of thousands, hundreds of thousands*, and insert another comma. We next read *millions, tens of millions, hundreds of millions* (insert another comma), *billions, tens of billions, hundreds of billions.*

The entire line of figures would be read: *Four hundred thirty-two billions one hundred ninety-eight millions seven hundred sixty-five thousands four hundred thirty-two.* When we thus *read* a line of figures, it is called *numeration*, and if the numeration be changed back to *figures*, it is called *notation.*

For instance, the writing of the following figures,

72,584,623,

would be the notation, and the numeration would be *seventy-two millions five hundred eighty-four thousands six hundred twenty-three.*

21. NOTE.—It is customary to leave the "s" off the words millions, thousands, etc. in cases like the above, both in speaking and writing; hence, the above would usually be expressed seventy-two million five hundred eighty-four thousand six hundred twenty-three.

22. The four fundamental processes of arithmetic are addition, subtraction, multiplication, and division. They are called fundamental processes because all operations in arithmetic are based upon them.

ADDITION.

23. **Addition** is the *process of finding the sum of two or more numbers.* The sign of addition is +. It is read **plus,** and means *more.* Thus, 5 + 6 is read *5 plus 6*, and means that 5 and 6 are to be added.

24. The sign of equality is =. It is read **equals** or **is equal to.** Thus, 5 + 6 = 11 may be read *5 plus 6 equals 11*, or *5 plus 6 is equal to 11.*

25. *Like numbers* can be added; *unlike numbers* cannot be added. 6 dollars can be added to 7 dollars, and the sum will be 13 dollars; but 6 dollars cannot be added to 7 feet.

26. The following table gives the sum of any two numbers from 1 to 12; it should be carefully committed to memory:

1 and 1 is 2	2 and 1 is 3	3 and 1 is 4	4 and 1 is 5
1 and 2 is 3	2 and 2 is 4	3 and 2 is 5	4 and 2 is 6
1 and 3 is 4	2 and 3 is 5	3 and 3 is 6	4 and 3 is 7
1 and 4 is 5	2 and 4 is 6	3 and 4 is 7	4 and 4 is 8
1 and 5 is 6	2 and 5 is 7	3 and 5 is 8	4 and 5 is 9
1 and 6 is 7	2 and 6 is 8	3 and 6 is 9	4 and 6 is 10
1 and 7 is 8	2 and 7 is 9	3 and 7 is 10	4 and 7 is 11
1 and 8 is 9	2 and 8 is 10	3 and 8 is 11	4 and 8 is 12
1 and 9 is 10	2 and 9 is 11	3 and 9 is 12	4 and 9 is 13
1 and 10 is 11	2 and 10 is 12	3 and 10 is 13	4 and 10 is 14
1 and 11 is 12	2 and 11 is 13	3 and 11 is 14	4 and 11 is 15
1 and 12 is 13	2 and 12 is 14	3 and 12 is 15	4 and 12 is 16
5 and 1 is 6	6 and 1 is 7	7 and 1 is 8	8 and 1 is 9
5 and 2 is 7	6 and 2 is 8	7 and 2 is 9	8 and 2 is 10
5 and 3 is 8	6 and 3 is 9	7 and 3 is 10	8 and 3 is 11
5 and 4 is 9	6 and 4 is 10	7 and 4 is 11	8 and 4 is 12
5 and 5 is 10	6 and 5 is 11	7 and 5 is 12	8 and 5 is 13
5 and 6 is 11	6 and 6 is 12	7 and 6 is 13	8 and 6 is 14
5 and 7 is 12	6 and 7 is 13	7 and 7 is 14	8 and 7 is 15
5 and 8 is 13	6 and 8 is 14	7 and 8 is 15	8 and 8 is 16
5 and 9 is 14	6 and 9 is 15	7 and 9 is 16	8 and 9 is 17
5 and 10 is 15	6 and 10 is 16	7 and 10 is 17	8 and 10 is 18
5 and 11 is 16	6 and 11 is 17	7 and 11 is 18	8 and 11 is 19
5 and 12 is 17	6 and 12 is 18	7 and 12 is 19	8 and 12 is 20
9 and 1 is 10	10 and 1 is 11	11 and 1 is 12	12 and 1 is 13
9 and 2 is 11	10 and 2 is 12	11 and 2 is 13	12 and 2 is 14
9 and 3 is 12	10 and 3 is 13	11 and 3 is 14	12 and 3 is 15
9 and 4 is 13	10 and 4 is 14	11 and 4 is 15	12 and 4 is 16
9 and 5 is 14	10 and 5 is 15	11 and 5 is 16	12 and 5 is 17
9 and 6 is 15	10 and 6 is 16	11 and 6 is 17	12 and 6 is 18
9 and 7 is 16	10 and 7 is 17	11 and 7 is 18	12 and 7 is 19
9 and 8 is 17	10 and 8 is 18	11 and 8 is 19	12 and 8 is 20
9 and 9 is 18	10 and 9 is 19	11 and 9 is 20	12 and 9 is 21
9 and 10 is 19	10 and 10 is 20	11 and 10 is 21	12 and 10 is 22
9 and 11 is 20	10 and 11 is 21	11 and 11 is 22	12 and 11 is 23
9 and 12 is 21	10 and 12 is 22	11 and 12 is 23	12 and 12 is 24

27. For *addition*, place the numbers to be added directly under one another, taking care to place *units* under *units*, *tens* under *tens*, *hundreds* under *hundreds*, and so on.

When the numbers are thus written, the *right-hand figure of one number is placed directly under the right-hand figure of the one above it, thus bringing units under units, tens under tens, etc.* Proceed as in the following examples:

28. EXAMPLE.—What is the sum of 131, 222, 21, 2, and 413?

SOLUTION.—

$$\begin{array}{r} 131 \\ 222 \\ 21 \\ 2 \\ 413 \\ \hline \textit{sum}\ 789 \end{array}$$ Ans.

EXPLANATION.—After placing the numbers in proper order, begin at the bottom of the right-hand, or units, column and add; thus, $3 + 2 + 1 + 2 + 1 = 9$, the sum of the numbers in units column. Place the 9 directly beneath as the first, or units, figure in the sum.

The sum of the numbers in the next, or tens, column equals 8 tens, which is the second, or tens, figure in the sum.

The sum of the numbers in the next, or hundreds, column equals 7 hundreds, which is the third, or hundreds, figure in the sum.

The sum, or answer, is 789.

29. EXAMPLE.—What is the sum of 425, 36, 9,215, 4, and 907?

SOLUTION.—

$$\begin{array}{r} 425 \\ 36 \\ 9215 \\ 4 \\ 907 \\ \hline 27 \\ 60 \\ 1500 \\ 9000 \\ \hline \textit{sum}\ 10587 \end{array}$$ Ans.

EXPLANATION.—The sum of the numbers in the first, or units, column is 27 units; i. e., 2 tens and 7 units. Write 27 as shown. The sum of the numbers in the second, or tens, column is 6 tens, or 60. Write 60 underneath 27 as shown. The sum of the numbers in the third, or hundreds, column is 15 hundreds, or 1,500. Write 1,500 under the two preceding results as shown. There is only one number in the fourth, or thousands, column, 9, which represents 9,000.

Write 9,000 under the three preceding results. Adding these four results, the sum is 10,587, which is the sum of 425, 36, 9,215, 4, and 907.

30. The addition may also be performed as follows:

425
36
9215
4
907
sum 10587 Ans.

EXPLANATION.—The sum of the numbers in units column = 27 units, or 2 tens and 7 units. Write the 7 units as the first, or right-hand, figure in the sum. Reserve the 2 tens and add them to the figures in tens column. The sum of the figures in the tens column plus the 2 tens reserved and carried from the units column is 8, which is written down as the second figure in the sum. There is nothing to carry to the next column, because 8 is less than 10. The sum of the numbers in the next column is 15 hundreds, or 1 thousand and 5 hundreds. Write down the 5 as the third, or hundreds, figure in the sum and carry the 1 to the next column. $1 + 9 = 10$, which is written down at the left of the other figures.

The second method saves space and figures, but the first is to be preferred when adding a long column.

31. EXAMPLE.—Add the numbers in the column below:

SOLUTION.—

890
82
90
393
281
80
770
83
492
80
383
84
191
sum 3899 Ans.

EXPLANATION.—The sum of the digits in the first column equals 19 units, or 1 ten and 9 units. Write the 9 and carry 1 to the next column. The sum of the digits in the second column + the 1 carried from the first column is 109 tens, or 10 hundreds and 9 tens. Write down the 9 and carry the 10 to the next column. The sum of the digits in this column plus the 10 carried from the second column is 38.

The entire sum is 3,899.

32. Rule.—I. *Begin at the right, add each column separately, and write the sum, if it be only one figure, under the column added.*

II. *If the sum of any column consists of two or more figures, put the right-hand figure of the sum under that column and add the remaining figure or figures to the next column.*

33. Proof.—*To prove addition, add each column from top to bottom. If you obtain the same result as by adding from bottom to top, the work is probably correct.*

EXAMPLES FOR PRACTICE.

34. Find the sum of:

(*a*)	104 + 203 + 613 + 214.	Ans.	(*a*) 1,134.
(*b*)	1,875 + 3,143 + 5,826 + 10,832.		(*b*) 21,676.
(*c*)	4,865 + 2,145 + 8,173 + 40,084.		(*c*) 55,267.
(*d*)	14,204 + 8,173 + 1,065 + 10,042.		(*d*) 33,484.
(*e*)	10,832 + 4,145 + 3,133 + 5,872.		(*e*) 23,982.
(*f*)	214 + 1,231 + 141 + 5,000.		(*f*) 6,586.
(*g*)	123 + 104 + 425 + 126 + 327.		(*g*) 1,105.
(*h*)	6,354 + 2,145 + 2,042 + 1,111 + 3,333.		(*h*) 14,985.

1. A week's record of coal burned in an engine room is as follows: Monday, 1,800 pounds; Tuesday, 1,655 pounds; Wednesday, 1,725 pounds; Thursday, 1,690 pounds; Friday, 1,648 pounds; Saturday, 1,020 pounds. How much coal was burned during the week?

Ans. 9,538 lb.

2. A steam pump, in one hour, pumps out of a cistern 4,200 gallons; in the next hour, 5,420 gallons; and in 45 minutes more an additional 3,600 gallons, when the cistern becomes empty. How many gallons were in the cistern at first? Ans. 13,220 gal.

3. What is the total cost of a steam plant, the several items of expense being as follows: steam engine, $900; boiler, $775; fittings and connections, $225; erecting the plant, $125; engine house, $650?

Ans. $2,675.

SUBTRACTION.

35. In arithmetic, **subtraction** is the process of finding how much greater one number is than another.

The greater of the two numbers is called the **minuend.**

The smaller of the two numbers is called the **subtrahend.**

The number left after subtracting the subtrahend from the minuend is called the **difference,** or **remainder.**

36. The sign of subtraction is —. It is read **minus,** and means *less.* Thus, 12 — 7 is read *12 minus 7,* and means that 7 is to be taken from 12.

37. EXAMPLE.—From 7,568 take 3,425.

SOLUTION.—

minuend	7 5 6 8	
subtrahend	3 4 2 5	
remainder	4 1 4 3	Ans.

EXPLANATION.—Begin at the right-hand, or units, column and subtract in succession each figure in the subtrahend from the one directly above it in the minuend, and write the remainders below the line. The result is the entire remainder.

38. When there are more figures in the minuend than in the subtrahend, and when some figures in the minuend are *less* than the figures directly under them in the subtrahend, proceed as in the following example:

EXAMPLE.—From 8,453 take 844.

SOLUTION.—

minuend	8 4 5 3	
subtrahend	8 4 4	
remainder	7 6 0 9	Ans.

EXPLANATION.—Begin at the right-hand, or units, column to subtract. We cannot take 4 from 3, and must,

therefore, borrow 1 from 5 in tens column and annex it to the 3 in units column. The 1 ten = 10 units, which added to the 3 in units column = 13 units. 4 from 13 = 9, the first, or units, figure in the remainder.

Since we borrowed 1 from the 5, only 4 remains; 4 from 4 = 0, the second, or tens, figure. We cannot take 8 from 4, so borrow 1 thousand, or 10 hundreds, from 8; 10 hundreds + 4 hundreds = 14 hundreds, and 8 from 14 = 6, the third, or hundreds, figure in the remainder.

Since we borrowed 1 from 8, only 7 remains, from which there is nothing to subtract; therefore, 7 is the next figure in the remainder, or answer.

The operation of borrowing is placing 1 before the figure following the one from which it is borrowed. In the above example the 1 borrowed from 5 is placed before 3, making it 13, from which we subtract 4. The 1 borrowed from 8 is placed before 4, making 14, from which 8 is taken.

39. EXAMPLE.—Find the difference between 10,000 and 8,763.

SOLUTION.—

minuend	10000
subtrahend	8763
remainder	1237 Ans.

EXPLANATION.—In the above example we borrow 1 from the second column and place it before 0, making 10; 3 from 10 = 7. In the same way we borrow 1 and place it before the next cipher, making 10; but as we have borrowed 1 from this column and have taken it to the units column, only 9 remains from which to subtract 6; 6 from 9 = 3. For the same reason we subtract 7 from 9 and 8 from 9 for the next two figures, and obtain a total remainder of 1,237.

40. Rule.—*Place the subtrahend (or smaller) number under the minuend (or larger) number, in the same manner as for addition, and proceed as in Arts.* **37, 38,** *and* **39.**

41. Proof.—*To prove an example in subtraction, add the subtrahend and the remainder. The sum should equal the minuend. If it does not, a mistake has been made, and the work should be done over.*

Proof of the above example:

subtrahend	8763
remainder	1237
minuend	10000

EXAMPLES FOR PRACTICE.

42. From:

(*a*)	94,278 take 62,574.	Ans. (*a*)	31,704.
(*b*)	53,714 take 25,824.	(*b*)	27,890.
(*c*)	71,832 take 58,109.	(*c*)	13,723.
(*d*)	20,804 take 10,408.	(*d*)	10,396.
(*e*)	310,465 take 102,141.	(*e*)	208,324.
(*f*)	(81,043 + 1,041) take 14,831.	(*f*)	67,253.
(*g*)	(20,482 + 18,216) take 21,214.	(*g*)	17,484.
(*h*)	(2,040 + 1,213 + 542) take 3,791.	(*h*)	4.

1. A cistern is fed by two pipes which supply 1,200 and 2,250 gallons per hour, respectively, and is being emptied by a pump which delivers 5,800 gallons per hour. Starting with 8,000 gallons in the cistern, how much water does it contain at the end of an hour? Ans. 5,650 gal.

2. A train in running from New York to Buffalo travels 38 miles the first hour, 42 the second, 39 the third, 56 the fourth, 52 the fifth, and 48 the sixth hour. How many miles remain to be traveled at the end of the sixth hour, the distance between the two places being 410 miles?

Ans. 135 mi.

3. On Monday morning a bank had on hand $2,862. During the day $1,831 were deposited and $2,172 drawn out; on Tuesday, $3,126 were deposited and $1,954 drawn out. How many dollars were on hand Wednesday morning? Ans. $3,693.

MULTIPLICATION.

43. To **multiply** a number is to *add* it to itself a certain number of times.

44. **Multiplication** is the process of multiplying one number by another.

The number thus added to itself, or the number to be multiplied, is called the **multiplicand.**

The number which shows how many times the *multiplicand* is to be taken, or the number by which we *multiply*, is called the **multiplier.**

The result obtained by multiplying is called the **product.**

45. In the following table, the product of any two numbers (neither of which exceeds 12) may be found:

1 times 1 is 1	2 times 1 is 2	3 times 1 is 3
1 times 2 is 2	2 times 2 is 4	3 times 2 is 6
1 times 3 is 3	2 times 3 is 6	3 times 3 is 9
1 times 4 is 4	2 times 4 is 8	3 times 4 is 12
1 times 5 is 5	2 times 5 is 10	3 times 5 is 15
1 times 6 is 6	2 times 6 is 12	3 times 6 is 18
1 times 7 is 7	2 times 7 is 14	3 times 7 is 21
1 times 8 is 8	2 times 8 is 16	3 times 8 is 24
1 times 9 is 9	2 times 9 is 18	3 times 9 is 27
1 times 10 is 10	2 times 10 is 20	3 times 10 is 30
1 times 11 is 11	2 times 11 is 22	3 times 11 is 33
1 times 12 is 12	2 times 12 is 24	3 times 12 is 36
4 times 1 is 4	5 times 1 is 5	6 times 1 is 6
4 times 2 is 8	5 times 2 is 10	6 times 2 is 12
4 times 3 is 12	5 times 3 is 15	6 times 3 is 18
4 times 4 is 16	5 times 4 is 20	6 times 4 is 24
4 times 5 is 20	5 times 5 is 25	6 times 5 is 30
4 times 6 is 24	5 times 6 is 30	6 times 6 is 36
4 times 7 is 28	5 times 7 is 35	6 times 7 is 42
4 times 8 is 32	5 times 8 is 40	6 times 8 is 48
4 times 9 is 36	5 times 9 is 45	6 times 9 is 54
4 times 10 is 40	5 times 10 is 50	6 times 10 is 60
4 times 11 is 44	5 times 11 is 55	6 times 11 is 66
4 times 12 is 48	5 times 12 is 60	6 times 12 is 72
7 times 1 is 7	8 times 1 is 8	9 times 1 is 9
7 times 2 is 14	8 times 2 is 16	9 times 2 is 18
7 times 3 is 21	8 times 3 is 24	9 times 3 is 27
7 times 4 is 28	8 times 4 is 32	9 times 4 is 36
7 times 5 is 35	8 times 5 is 40	9 times 5 is 45
7 times 6 is 42	8 times 6 is 48	9 times 6 is 54
7 times 7 is 49	8 times 7 is 56	9 times 7 is 63
7 times 8 is 56	8 times 8 is 64	9 times 8 is 72
7 times 9 is 63	8 times 9 is 72	9 times 9 is 81
7 times 10 is 70	8 times 10 is 80	9 times 10 is 90
7 times 11 is 77	8 times 11 is 88	9 times 11 is 99
7 times 12 is 84	8 times 12 is 96	9 times 12 is 108
10 times 1 is 10	11 times 1 is 11	12 times 1 is 12
10 times 2 is 20	11 times 2 is 22	12 times 2 is 24
10 times 3 is 30	11 times 3 is 33	12 times 3 is 36
10 times 4 is 40	11 times 4 is 44	12 times 4 is 48
10 times 5 is 50	11 times 5 is 55	12 times 5 is 60
10 times 6 is 60	11 times 6 is 66	12 times 6 is 72
10 times 7 is 70	11 times 7 is 77	12 times 7 is 84
10 times 8 is 80	11 times 8 is 88	12 times 8 is 96
10 times 9 is 90	11 times 9 is 99	12 times 9 is 108
10 times 10 is 100	11 times 10 is 110	12 times 10 is 120
10 times 11 is 110	11 times 11 is 121	12 times 11 is 132
10 times 12 is 120	11 times 12 is 132	12 times 12 is 144

This table should be carefully committed to memory.
Since 0 has no value, the product of 0 and any number is 0.

46. The sign of multiplication is $\times$. It is read **times** or **multiplied by.** Thus, 9×6 is read *9 times 6*, or *9 multiplied by 6.*

47. It matters not in what order the numbers to be multiplied together are placed. Thus, 6×9 is the same as 9×6.

48. To multiply a number by one figure only:

EXAMPLE.—Multiply 425 by 5.

SOLUTION.—

multiplicand	4 2 5
multiplier	5
product	2 1 2 5 Ans.

EXPLANATION.—For convenience, the multiplier is generally written under the right-hand figure of the multiplicand. On looking in the multiplication table, we see that 5×5 is 25. Multiplying the first figure at the right of the multiplicand, or 5, by the multiplier 5, it is seen that 5×5 units is 25 units, or 2 tens and 5 units. Write the 5 units in units place in the product and reserve the 2 tens to add to the product of tens. Looking in the multiplication table again, we see that 5×2 is 10. Multiplying the second figure of the multiplicand by the multiplier 5, we see that 5 times 2 tens is 10 tens, which, plus the 2 tens reserved, is 12 tens, or 1 hundred and 2 tens. Write the 2 tens in tens place and reserve the 1 hundred to add to the product of hundreds. Again, we see by the multiplication table that 5×4 is 20. Multiplying the third, or last, figure of the multiplicand by the multiplier 5, we see that 5 times 4 hundreds is 20 hundreds, which, plus the 1 hundred reserved, is 21 hundreds, or 2 thousands and 1 hundred, which we write in thousands and hundreds places, respectively.

Hence, the product is 2,125.

This result is the same as adding 425 five times. Thus,

	4 2 5
	4 2 5
	4 2 5
	4 2 5
	4 2 5
sum	2 1 2 5 Ans.

EXAMPLES FOR PRACTICE.

49. Find the product of:

(*a*)	61,483 × 6.	Ans. (*a*)	368,898.
(*b*)	12,375 × 5.	(*b*)	61,875.
(*c*)	10,426 × 7.	(*c*)	72,982.
(*d*)	10,835 × 3.	(*d*)	32,505.
(*e*)	98,376 × 4.	(*e*)	393,504.
(*f*)	10,873 × 8.	(*f*)	86,984.
(*g*)	71,543 × 9.	(*g*)	643,887.
(*h*)	218,734 × 2.	(*h*)	437,468.

1. A stationary engine makes 5,520 revolutions per hour. Running 9 hours a day, 5 days in the week, and 5 hours on Saturday, how many revolutions would it make in 4 weeks? Ans. 1,104,000 rev.

2. An engineer earns $650 a year and his average expenses are $548. How much could he save in 8 years at that rate? Ans. $816.

3. The connection between an engine and boiler is made up of 5 lengths of pipe, three of which are 12 feet long, one 2 feet 6 inches long, and one 8 feet 6 inches long. If the pipe weighs 9 pounds per foot, what is the total weight of the pipe used? Ans. 423 lb.

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50. To multiply a number by two or more figures:

EXAMPLE.—Multiply 475 by 234.

SOLUTION.—

multiplicand	475
multiplier	234
	1900
	1425
	950
product	111150 Ans.

EXPLANATION.—For convenience, the multiplier is generally written under the multiplicand, placing units under units, tens under tens, etc.

We cannot multiply by 234 at one operation; we must, therefore, multiply by the *parts* and then *add* the **partial products.**

The parts by which we are to multiply are 4 units, 3 tens, and 2 hundreds. 4 times 475 = 1,900, the *first partial*

product; 3 times 475 = 1,425, the *second partial product, the right-hand figure of which is written directly under the figure multiplied by*, or 3; 2 times 475 = 950, the *third partial product*, the right-hand figure of which is written directly under the figure multiplied by, or 2.

The sum of these three partial products is 111,150, which is the *entire product.*

51. **Rule.—I.** *Write the multiplier under the multiplicand, so that units are under units, tens under tens, etc.*

II. *Begin at the right and multiply each figure of the multiplicand by each successive figure of the multiplier, placing the right-hand figure of each partial product directly under the figure used as a multiplier.*

III. *The sum of the partial products will equal the required product.*

52. **Proof.**—*Review the work carefully, or multiply the multiplier by the multiplicand; if the results agree, the work is correct.*

53. When there is a *cipher* in the *multiplier*, multiply by it the same as with the other figures; since the result will be zero, place a cipher under the cipher in the multiplier and write the next partial product to the left of it, as shown below:

(*a*)	(*b*)	(*c*)	(*d*)
0	2	15	708
× 0	× 0	× 0	× 0
0 Ans.	0 Ans.	0 Ans.	0 Ans.

(*e*)	(*f*)	(*g*)
3114	4008	31264
203	305	1002
9342	20040	62528
62280	120240	3126400
632142 Ans.	1222440 Ans.	31326528 Ans.

EXAMPLES FOR PRACTICE.

54. Find the product of:

(*a*) 3,842 × 26.	Ans.	(*a*) 99,892.
(*b*) 3,716 × 45.		(*b*) 167,220.
(*c*) 1,817 × 124.		(*c*) 225,308.
(*d*) 675 × 38.		(*d*) 25,650.
(*e*) 1,875 × 33.		(*e*) 61,875.
(*f*) 4,836 × 47.		(*f*) 227,292.
(*g*) 5,682 × 543.		(*g*) 3,085,326.
(*h*) 3,257 × 246.		(*h*) 801,222.
(*i*) 2,875 × 302.		(*i*) 868,250.
(*j*) 17,819 × 1,004.		(*j*) 17,890,276.
(*k*) 38,674 × 205.		(*k*) 7,928,170.
(*l*) 18,304 × 100.		(*l*) 1,830,400.
(*m*) 7,834 × 10.		(*m*) 78,340.
(*n*) 87,543 × 1,000.		(*n*) 87,543,000.
(*o*) 48,763 × 100.		(*o*) 4,876,300.

1. If the area of a steam-engine piston is 113 square inches, what is the total pressure upon it when the steam pressure is 85 pounds per square inch? Ans. 9,605 lb.

2. A steam engine which indicated 164 horsepower was found to consume 4 pounds of coal per horsepower per hour. Being replaced by a new engine of the same horsepower, another test was made, which showed a consumption of 3 pounds per horsepower per hour. What was the saving of coal for a year of 309 days, if the engine's average run was 14 hours a day? Ans. 709,464 lb.

3. Two steamers are 7,846 miles apart and are sailing towards each other, one at the rate of 18 miles an hour and the other at the rate of 15 miles an hour. How far apart will they be at the end of 205 hours? Ans. 1,081 mi.

DIVISION.

55. **Division** is the process of finding how many times one number is contained in another of the same kind.

The number to be *divided* is called the **dividend.**

The number by which we *divide* is called the **divisor.**

The number which shows how many times the divisor is contained in the dividend is called the **quotient.**

56. The sign of division is ÷. It is read **divided by.** 54 ÷ 9 is read *54 divided by 9.* Another way to write 54

divided by 9 is $\frac{54}{9}$. Thus, $54 \div 9 = 6$, or $\frac{54}{9} = 6$. The expression $\frac{54}{9}$ may be read either 54 divided by 9 or 54 *over* 9, the word "over" implying that there is a line between the two numbers indicating that the upper number is to be divided by the lower one. In both of these cases 54 is the *dividend* and 9 is the *divisor*.

Division is the reverse of multiplication.

57. To divide when the divisor consists of but one figure, proceed as in the following example:

EXAMPLE.—What is the quotient of $875 \div 7$?

SOLUTION.—

divisor dividend quotient

7) 8 7 5 (1 2 5 Ans.
7
1 7
1 4
3 5
3 5
remainder 0

EXPLANATION.— 7 is contained in 8 hundreds, 1 hundred times. Place the 1 as the first, or left-hand, figure of the quotient. Multiply the divisor, 7, by the 1 hundred of the quotient, and place the product, 7 hundreds, under the 8 hundreds in the dividend, and subtract. Beside the remainder, 1, bring down the 7 tens, making 17 tens; 17 divided by 7 = 2 tens. Write the 2 as the second figure of the quotient. Multiply the divisor, 7, by the 2, and subtract the product from 17. Beside the remainder, 3, bring down the 5 units of the dividend, making 35 units. 7 is contained in 35, 5 times, and 5 is placed in the quotient. 5 times 7 = 35, which, subtracted from 35, under which it is placed, leaves 0. Therefore, the quotient is 125. This method is called **long division.**

58. In **short division,** only the divisor, dividend, and quotient are written, the operations being performed mentally.

dividend
divisor 7) 8¹7³5
quotient 1 2 5 Ans.

The mental operation is as follows: 7 is contained in 8, 1 time and 1 remainder; 1 placed before 7 makes 17; 7 is contained in 17, 2 times and 3 over; the 3 placed before 5 makes 35; 7 is contained in 35, 5 times. These partial quotients, placed in order as they are found, make the entire quotient, 125.

59. If the divisor consists of *two or more* figures, proceed as in the following example:

EXAMPLE.—Divide 2,702,826 by 63.

divisor *dividend* *quotient*
SOLUTION.— 63) 2702826 (42902 Ans.
252
182
126
568
567
126
126

EXPLANATION.—As 63 is not contained in the first two figures, 27, we must use the first three figures, 270. Now, by trial we must find how many times 63 is contained in 270. 6 is contained in the first two figures of 270, 4 times. Place the 4 as the first, or left-hand, figure in the quotient. Multiply the divisor, 63, by 4, and subtract the product, 252, from 270. The remainder is 18, beside which we write the next figure of the dividend, 2, making 182. Now, 6 is contained in the first two figures of 182, 3 times, but on multiplying 63 by 3, we see that the product, 189, is too great, so we try 2 as the second figure of the quotient. Multiplying the divisor, 63, by 2 and subtracting the product, 126, from 182, the remainder is 56, beside which we bring down the next figure of the dividend, making 568. 6 is contained in 56 about 9 times. Multiply the divisor, 63, by 9 and subtract the product, 567, from 568. The remainder is 1, and

bringing down the next figure of the dividend, 2, gives 12. As 12 is smaller than 63, we write 0 in the quotient and bring down the next figure, 6, making 126. 63 is contained in 126, 2 times, without a remainder. Therefore, 42,902· is the quotient.

60. Rule.—I. *Write the divisor at the left of the dividend, with a line between them.*

II. *Find how many times the divisor is contained in the lowest number of the left-hand figures of the dividend that will contain it, and write the result at the right for the first figure of the quotient.*

III. *Multiply the divisor by this quotient; write the product under the partial dividend used, and subtract, and to the remainder annex the next figure of the dividend. Divide as before, and thus continue until all the figures of the dividend have been used.*

IV. *If any partial dividend will not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend, and proceed as before.*

V. *If there be at last a remainder, write it after the quotient, with the divisor underneath.*

61. Proof.—*Multiply the quotient by the divisor and add the remainder, if there be any, to the product. The result will be the dividend.* Thus,

divisor dividend quotient

63) 4235 (67$\frac{14}{63}$ Ans.

378

455

441

remainder 14

PROOF. *quotient* 67

divisor 63

201

402

4221

remainder 14

dividend 4235

EXAMPLES FOR PRACTICE.

62. Divide the following:

(*a*)	126,498 by 58.	Ans. (*a*)	2,181.
(*b*)	3,207,594 by 767.	(*b*)	4,182.
(*c*)	11,408,202 by 234.	(*c*)	48,753.
(*d*)	2,100,315 by 581.	(*d*)	3,615.
(*e*)	969,936 by 4,008.	(*e*)	242.
(*f*)	7,481,888 by 1,021.	(*f*)	7,328.
(*g*)	1,525,915 by 5,003.	(*g*)	305.
(*h*)	1,646,301 by 381.	(*h*)	4,321.

1. In a mile there are 5,280 feet. How many rails would it take to lay a double row 1 mile long, each rail being 30 feet long?

Ans. 352 rails.

2. How many rivets will be required for the longitudinal seams of a cylindrical boiler 20 feet long, the joint being double-riveted, and the rivets being spaced 4 inches apart? Ans. 120 rivets.

NOTE.—First find the length of the boiler in inches.

3. It requires 7,020,000 bricks to build a large foundry. How many teams will it require to draw the bricks in 60 days, if each team draws 6 loads per day and 1,500 bricks at a load? Ans. 13 teams.

NOTE.—Find how many loads 7,020,000 bricks make; then, how many days it will take one team to draw the bricks.

CANCELATION.

63. **Cancelation** is the process of shortening operations in division by casting out equal factors from both dividend and divisor.

64. The **factors** of a number are those numbers which, when multiplied together, will *equal that number*. Thus, 5 and 3 are the factors of 15, since $5 \times 3 = 15$. Likewise, 8 and 7 are the factors of 56, since $8 \times 7 = 56$.

65. A **prime number** is one which cannot be divided by any number except itself and 1. Thus, 2, 3, 11, 29, etc. are prime numbers.

66. A **prime factor** is any factor that is a prime number.

Any number that is not a prime is called a **composite** number, and may be produced by multiplying together its

prime factors. Thus, 60 is a composite number, and is equal to the product of its prime factors, $2 \times 2 \times 3 \times 5$.

Numbers are said to be **prime to each other** when no two of them can be divided by any number except 1; the numbers themselves *may* be either prime or composite. Thus, the numbers 3, 5, and 11 are prime to one another; so also are 22, 25, and 21—all three of which are composite numbers.

67. *Canceling equal factors from both dividend and divisor does not change the quotient.*

Canceling a factor in both dividend and divisor is the same as *dividing them both by the same number*, which, by the principle of division, *does not change the quotient.*

Write the numbers forming the dividend above the line, and those forming the divisor below it.

68. EXAMPLE.—Divide $4 \times 45 \times 60$ by 9×24.

SOLUTION.—Placing the dividend over the divisor, and canceling,

$$\frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{6}{\cancel{24}}} = \frac{50}{1} = 50. \quad \text{Ans.}$$

EXPLANATION.—The 4 in the dividend and the 24 in the divisor are both divisible by 4, since 4 divided by 4 equals 1, and 24 divided by 4 equals 6. Cross off the 4 and write the 1 over it; also cross off the 24 and write the 6 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times 45 \times 60}{9 \times \underset{6}{\cancel{24}}} =$$

60 in the dividend and 6 in the divisor are divisible by 6, since 60 divided by 6 equals 10, and 6 divided by 6 equals 1. Cross off the 60 and write 10 over it; also cross off the 6 and write 1 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times 45 \times \overset{10}{\cancel{60}}}{9 \times \underset{1}{\cancel{24}}} =$$

Again, 45 in the dividend and 9 in the divisor are divisible by 9, since 45 divided by 9 equals 5, and 9 divided by 9

equals 1. Cross off the 45 and write the 5 over it; also cross off the 9 and write the 1 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{1}{\cancel{24}}} = \underset{1}{\cancel{6}}$$

Since there are no two remaining numbers (one in the dividend and one in the divisor) divisible by any number except 1, without a remainder, it is impossible to cancel further.

Multiply all the uncanceled numbers in the dividend together and divide their product by the product of all the uncanceled numbers in the divisor. The result will be the quotient. The product of all the uncanceled numbers in the dividend equals $5 \times 1 \times 10 = 50$; the product of all the uncanceled numbers in the divisor equals $1 \times 1 = 1$.

Hence, $\dfrac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{1}{\cancel{24}}} = \underset{1}{\cancel{6}} = \dfrac{1 \times 5 \times 10}{1 \times 1} = 50.$ Ans.

It is usual to omit the 1's when canceling, instead of writing them as above.

69. Rule.—I. *Cancel the common factors from both the dividend and the divisor.*

II. *Then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

EXAMPLES FOR PRACTICE.

70. Divide:

(*a*) $14 \times 18 \times 16 \times 40$ by $7 \times 8 \times 6 \times 5 \times 3$.
(*b*) $3 \times 65 \times 50 \times 100 \times 60$ by $30 \times 60 \times 13 \times 10$.
(*c*) $8 \times 4 \times 3 \times 9 \times 11$ by $11 \times 9 \times 4 \times 3 \times 8$.
(*d*) $164 \times 321 \times 6 \times 7 \times 4$ by $82 \times 321 \times 7$.
(*e*) $50 \times 100 \times 200 \times 72$ by $1{,}000 \times 144 \times 100$.
(*f*) $48 \times 63 \times 55 \times 49$ by $7 \times 21 \times 11 \times 48$.
(*g*) $110 \times 150 \times 84 \times 32$ by $11 \times 15 \times 100 \times 64$.
(*h*) $115 \times 120 \times 400 \times 1{,}000$ by $23 \times 1{,}000 \times 60 \times 800$.

Ans. (*a*) 32. (*b*) 250. (*c*) 1. (*d*) 48. (*e*) 5. (*f*) 105. (*g*) 42. (*h*) 5.

ARITHMETIC.

(PART 2.)

FRACTIONS.

REMARK.—If a stick of wood is divided into, say, 12 equal parts, one of these parts is called a twelfth. If we take away 5 of these equal parts, we shall have left 7 parts or 7-twelfths. Since it would be very inconvenient to spell out the names of the number of parts into which an object has been (or is supposed to have been) divided, mathematicians invented, long ago, a kind of a shorthand method of expressing 7-twelfths, 25-forty-fifths, etc., viz.: they wrote the number of the equal parts taken or considered above a horizontal line, and called this number the *numerator;* then they wrote below the horizontal line the number which denoted the number of equal parts into which the object was supposed to be divided, and called it the *denominator.* Hence, instead of writing 7-twelfths, 25-forty-fifths, etc., they wrote $\frac{7}{12}$, $\frac{25}{45}$, etc.; but they read them the same as if they had been written the other way.

1. A **fraction** is *one or more equal parts of a unit. One-half, two-thirds, seven-fifths* are fractions.

2. *Two* numbers are required to express a fraction; one is called the **numerator** and the other the **denominator.**

3. The numerator is placed above the denominator with a line between them, as $\frac{2}{3}$. Here 3 is the denominator, and shows into how many *equal parts* the *unit*, or *one*, is divided. The numerator 2 shows how many of these *equal parts* are

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taken or considered. The denominator also indicates the names of the parts.

$\frac{1}{2}$ is read one-half.
$\frac{3}{4}$ is read three-fourths.
$\frac{3}{8}$ is read three-eighths.
$\frac{5}{16}$ is read five-sixteenths.

In the expression "$\frac{3}{4}$ of an apple," the denominator, 4, shows that the apple is cut into 4 *equal parts*, and the numerator, 3, shows that *three of these parts*, or *fourths*, are taken or considered.

If each of the parts, or fourths, of the apple were cut into *two equal pieces*, one of these pieces would be $\frac{1}{2}$ of $\frac{1}{4}$, or $\frac{1}{8}$ of the whole apple, and three of the pieces would be $\frac{3}{8}$ of the apple. Thus, it is evident that the larger the *denominators*, the smaller the parts into which anything is divided and the less the value of the fraction, the numerators being the same. Thus, $\frac{3}{8}$ is less than $\frac{3}{4}$.

4. The **value** of a fraction is the numerator divided by the denominator, as $\frac{4}{2} = 2$, $\frac{6}{2} = 3$.

5. The line between the numerator and the denominator means *divided by*, or $\div$.

$\frac{3}{4}$ is equivalent to $3 \div 4$.
$\frac{5}{8}$ is equivalent to $5 \div 8$.

6. The numerator and the denominator of a fraction are called the **terms** of a fraction.

7. The *value* of a fraction whose numerator and denominator are equal is 1.

$\frac{4}{4}$, or four-fourths $= 1$.
$\frac{8}{8}$, or eight-eighths $= 1$.
$\frac{64}{64}$, or sixty-four sixty-fourths $= 1$.

8. A **proper fraction** is a fraction whose numerator is *less* than its denominator. Its value is *less* than 1, as $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{16}$.

9. An **improper fraction** is a fraction whose numerator is *equal* to or is *greater* than the denominator. Its value is 1 or *more than* 1, as $\frac{4}{4}$, $\frac{9}{8}$, $\frac{42}{32}$.

10. A **mixed number** is a whole number and a fraction united. $4\frac{2}{3}$ is a mixed number and is equivalent to $4 + \frac{2}{3}$. It is read *four and two-thirds.*

REDUCTION OF FRACTIONS.

11. **Reduction of fractions** is the process of changing their forms without changing their *value.*

12. *A fraction is reduced to higher terms by multiplying both terms of the fraction by the same number.* Thus, $\frac{3}{4}$ is reduced to $\frac{6}{8}$ by multiplying both terms by 2.

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8}.$$

The *value* is not changed, for $\frac{3}{4} = \frac{6}{8}$.

13. *A fraction is reduced to lower terms by dividing both terms by the same number.* Thus, $\frac{8}{10}$ is reduced to $\frac{4}{5}$ by dividing both terms by 2.

$$\frac{8 \div 2}{10 \div 2} = \frac{4}{5}.$$

14. *A fraction is reduced to lowest terms when its numerator and denominator cannot be divided by the same number,* as $\frac{3}{4}$, $\frac{2}{3}$, $\frac{11}{17}$.

15. **To reduce a whole number or a mixed number to an improper fraction:**

EXAMPLE 1.—How many fourths in 5?

SOLUTION.—Since there are 4 fourths in 1 ($\frac{4}{4} = 1$), in 5 there will be 5×4 fourths, or 20 fourths; i. e., $5 \times \frac{4}{4} = \frac{20}{4}$. Ans.

EXAMPLE 2.—Reduce $8\frac{3}{4}$ to an improper fraction.

SOLUTION.— $8 \times \frac{4}{4} = \frac{32}{4}$. $\frac{32}{4} + \frac{3}{4} = \frac{35}{4}$. Ans.

16. **Rule.**—*Multiply the whole number by the denominator of the fraction, add the numerator to the product, and place the denominator under the result.*

EXAMPLES FOR PRACTICE.

17. Reduce to improper fractions:

(*a*) $4\frac{1}{5}$.
(*b*) $5\frac{1}{9}$.
(*c*) $10\frac{2}{10}$.
(*d*) $37\frac{3}{4}$.
(*e*) $50\frac{1}{8}$.

Ans. (*a*) $\frac{21}{5}$. (*b*) $\frac{46}{9}$. (*c*) $\frac{102}{10}$. (*d*) $\frac{151}{4}$. (*e*) $\frac{201}{4}$.

18. To reduce an improper fraction to a whole or a mixed number:

EXAMPLE.—Reduce $\frac{21}{4}$ to a mixed number.

SOLUTION.— 4 is contained in 21, 5 times and 1 remaining; as this is also divided by 4, its value is $\frac{1}{4}$. Therefore, $5 + \frac{1}{4}$, or $5\frac{1}{4}$, is the number. Ans.

19. Rule.—*Divide the numerator by the denominator, the quotient will be the whole number; the remainder, if there be any, will be the numerator of the fractional part, of which the denominator is the same as the denominator of the improper fraction.*

EXAMPLES FOR PRACTICE.

20. Reduce to whole or mixed numbers:

(*a*) $\frac{145}{6}$.
(*b*) $\frac{185}{8}$.
(*c*) $\frac{701}{6}$.
(*d*) $\frac{149}{3}$.
(*e*) $\frac{76}{15}$.
(*f*) $\frac{125}{25}$.

Ans. (*a*) $24\frac{1}{6}$. (*b*) $61\frac{3}{8}$. (*c*) $116\frac{5}{6}$. (*d*) $49\frac{2}{3}$. (*e*) 4. (*f*) 5.

21. A **common denominator** of two or more fractions is a number which will contain all the denominators of the fractions without a remainder. The **least common denominator** is the least number that will contain all the denominators of the fractions without a remainder.

22. To find the least common denominator:

EXAMPLE.—Find the least common denominator of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{16}$.

SOLUTION.—We first place the denominators in a row, separated by commas.

2) 4, 3, 9, 16
2) 2, 3, 9, 8
3) 1, 3, 9, 4
3) 1, 1, 3, 4
4) 1, 1, 1, 4
1, 1, 1, 1

$2 \times 2 \times 3 \times 3 \times 4 = 144$, the least common denominator. **Ans.**

EXPLANATION.—Divide each of them by some prime number which will divide at least two of them without a remainder (if possible), bringing down those denominators to the

row below which will not contain the divisor without a remainder. Dividing each of the numbers by 2, the second row becomes 2, 3, 9, 8, since 2 will not divide 3 and 9 without a remainder. Dividing again by 2, the result is 1, 3, 9, 4. Dividing the third row by 3, the result is 1, 1, 3, 4. So continue until the last row contains only 1's. The product of all the divisors, or $2 \times 2 \times 3 \times 3 \times 4 = 144$, is the least common denominator.

23. EXAMPLE.—Find the least common denominator of $\frac{1}{3}$, $\frac{5}{12}$, $\frac{7}{18}$.

SOLUTION.—

$$\begin{array}{r} 3\,)\,9,\ 12,\ 18 \\ 3\,)\,3,\ 4,\ 6 \\ 2\,)\,1,\ 4,\ 2 \\ 2\,)\,1,\ 2,\ 1 \\ 1,\ 1,\ 1 \end{array}$$

$3 \times 3 \times 2 \times 2 = 36$. Ans.

24. To reduce two or more fractions to fractions having a common denominator:

EXAMPLE.—Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ to fractions having a common denominator.

SOLUTION.—The common denominator is a number which will contain 3, 4, and 2. The least common denominator is 12, because it is the smallest number which can be divided by 3, 4, and 2 without a remainder. $\frac{2}{3} = \frac{8}{12}$, $\frac{3}{4} = \frac{9}{12}$, $\frac{1}{2} = \frac{6}{12}$.

Reducing $\frac{2}{3}$, 3 is contained in 12, 4 times. By multiplying both numerator and denominator of $\frac{2}{3}$ by 4, we find

$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$. In the same way we find $\frac{3}{4} = \frac{9}{12}$ and $\frac{1}{2} = \frac{6}{12}$.

25. Rule.—*Divide the common denominator by the denominator of the given fraction and multiply both terms of the fraction by the quotient.*

EXAMPLES FOR PRACTICE.

26. Reduce to fractions having a common denominator:

(*a*) $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$.
(*b*) $\frac{3}{16}$, $\frac{3}{4}$, $\frac{7}{32}$.
(*c*) $\frac{7}{8}$, $\frac{7}{88}$, $\frac{10}{11}$.
(*d*) $\frac{3}{8}$, $\frac{5}{8}$, $\frac{11}{40}$.
(*e*) $\frac{4}{10}$, $\frac{6}{40}$, $\frac{9}{80}$.
(*f*) $\frac{7}{15}$, $\frac{17}{30}$, $\frac{21}{30}$.

Ans.
(*a*) $\frac{6}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.
(*b*) $\frac{6}{32}$, $\frac{24}{32}$, $\frac{7}{32}$.
(*c*) $\frac{77}{88}$, $\frac{7}{88}$, $\frac{80}{88}$.
(*d*) $\frac{24}{40}$, $\frac{25}{40}$, $\frac{11}{40}$.
(*e*) $\frac{16}{40}$, $\frac{6}{40}$, $\frac{18}{40}$.
(*f*) $\frac{14}{30}$, $\frac{17}{30}$, $\frac{21}{30}$.

ADDITION OF FRACTIONS.

27. *Fractions cannot be added unless they have a common denominator.* We cannot add $\frac{3}{4}$ to $\frac{7}{8}$ as they now stand, since the denominators represent parts of different sizes. Fourths cannot be added to eighths.

The fractions should be reduced to the *least* common denominator, or the *least* number which will contain all the denominators.

28. EXAMPLE.—Find the sum of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$.

SOLUTION.—The *least common denominator*, or the *least number* which will contain all the denominators, is 8.

$$\tfrac{1}{2} = \tfrac{4}{8},\ \tfrac{3}{4} = \tfrac{6}{8},\ \text{and}\ \tfrac{5}{8} = \tfrac{5}{8}.$$

EXPLANATION.—As the denominator tells or indicates the *names* of the parts, the numerators only are added to obtain the total number of parts indicated by the denominator.

$$\tfrac{4}{8} + \tfrac{6}{8} + \tfrac{5}{8} = \frac{4+6+5}{8} = \frac{15}{8} = 1\tfrac{7}{8}. \quad \text{Ans.}$$

29. EXAMPLE 1.—What is the sum of $12\frac{3}{4}$, $14\frac{5}{8}$, and $7\frac{5}{16}$?

SOLUTION.—The least common denominator in this case is 16.

$$\begin{aligned} 12\tfrac{3}{4} &= 12\tfrac{12}{16} \\ 14\tfrac{5}{8} &= 14\tfrac{10}{16} \\ 7\tfrac{5}{16} &= 7\tfrac{5}{16} \\ \hline \textit{sum}\ \ 33 + \tfrac{27}{16} &= 33 + 1\tfrac{11}{16} = 34\tfrac{11}{16}. \quad \text{Ans.} \end{aligned}$$

The sum of the fractions $= \frac{27}{16}$ or $1\frac{11}{16}$, which added to the sum of the whole numbers $= 34\frac{11}{16}$.

EXAMPLE 2.—What is the sum of 17, $13\frac{3}{16}$, $\frac{9}{32}$, and $3\frac{1}{2}$?

SOLUTION.—The least common denominator is 32. $13\frac{3}{16} = 13\frac{6}{32}$, $3\frac{1}{2} = 3\frac{8}{32}$.

$$\begin{aligned} &17 \\ &13\tfrac{6}{32} \\ &\tfrac{9}{32} \\ &3\tfrac{8}{32} \\ \hline \textit{sum}\ \ &33\tfrac{23}{32} \quad \text{Ans.} \end{aligned}$$

30. Rule.—I. *Reduce the given fractions to fractions having the least common denominator and write the sum of the numerators over the common denominator.*

II. *When there are mixed numbers and whole numbers, add the fractions first, and if their sum is an improper fraction, reduce it to a mixed number and add the whole number with the other whole numbers.*

EXAMPLES FOR PRACTICE.

31. Find the sum of:

(*a*) $\frac{4}{9}$, $\frac{7}{24}$, $\frac{5}{8}$.
(*b*) $\frac{2}{3}$, $\frac{5}{16}$, $\frac{24}{48}$.
(*c*) $\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{16}$.
(*d*) $\frac{5}{6}$, $1\frac{1}{12}$, $\frac{13}{18}$.
(*e*) $\frac{10}{11}$, $\frac{6}{33}$, $\frac{23}{66}$.
(*f*) $\frac{23}{45}$, $\frac{11}{15}$, $\frac{14}{45}$.
(*g*) $\frac{4}{11}$, $\frac{7}{22}$, $\frac{14}{22}$.
(*h*) $\frac{3}{7}$, $\frac{14}{49}$, $\frac{2}{7}$.

Ans. (*a*) $1\frac{7}{72}$. (*b*) $1\frac{3}{16}$. (*c*) $1\frac{3}{16}$. (*d*) $1\frac{5}{12}$. (*e*) $1\frac{23}{66}$. (*f*) $1\frac{5}{9}$. (*g*) $1\frac{7}{22}$. (*h*) 1.

1. The weights of a number of castings were $412\frac{3}{4}$ pounds, $270\frac{1}{2}$ pounds, 1,020 pounds, $75\frac{1}{4}$ pounds, and $68\frac{1}{2}$ pounds. What was their total weight? Ans. 1,847 lb.

2. Four bolts are required, $2\frac{3}{4}$, $1\frac{7}{8}$, $2\frac{5}{16}$, and $1\frac{13}{32}$ inches long. How long a piece of iron will be required to cut them from, allowing $\frac{3}{4}$ of an inch altogether for cutting off and finishing the ends? Ans. $9\frac{3}{32}$ in.

SUBTRACTION OF FRACTIONS.

32. *Fractions cannot be subtracted without first reducing them to a common denominator.*

EXAMPLE.—Subtract $\frac{3}{8}$ from $\frac{13}{16}$.

SOLUTION.—The common denominator is 16.

$$\frac{3}{8} = \frac{6}{16}. \quad \frac{13}{16} - \frac{6}{16} = \frac{13 - 6}{16} = \frac{7}{16}. \quad \text{Ans.}$$

33. EXAMPLE.—From 7 take $\frac{5}{8}$.

SOLUTION.— $1 = \frac{8}{8}$; therefore, $7 = 6 + \frac{8}{8} = 6\frac{8}{8}$; $6\frac{8}{8} - \frac{5}{8} = 6\frac{3}{8}$. Ans.

34. EXAMPLE.—What is the difference between $17\frac{9}{16}$ and $9\frac{15}{32}$?

SOLUTION.—The common denominator of the fractions is 32. $17\frac{9}{16} = 17\frac{18}{32}$.

minuend	$17\frac{18}{32}$
subtrahend	$9\frac{15}{32}$
difference	$8\frac{3}{32}$ Ans.

35. EXAMPLE.—From $9\frac{1}{4}$ take $4\frac{7}{16}$.

SOLUTION.—The common denominator of the fractions is 16. $9\frac{1}{4} = 9\frac{4}{16}$.

minuend	$9\frac{4}{16}$ or $8\frac{20}{16}$	
subtrahend	$4\frac{7}{16}$	$4\frac{7}{16}$
difference	$4\frac{13}{16}$	$4\frac{13}{16}$ Ans.

EXPLANATION. — As the fraction in the subtrahend is greater than the fraction in the minuend, it cannot be subtracted; therefore, *borrow* 1, or $\frac{16}{16}$, from the 9 in the minuend and add it to the $\frac{4}{16}$; $\frac{4}{16} + \frac{16}{16} = \frac{20}{16}$. $\frac{7}{16}$ from $\frac{20}{16} = \frac{13}{16}$. Since 1 was borrowed from 9, 8 remains; 4 from $8 = 4$; $4 + \frac{13}{16} = 4\frac{13}{16}$.

36. EXAMPLE.—From 9 take $8\frac{3}{16}$.

SOLUTION.—		
minuend	9	or $8\frac{16}{16}$
subtrahend	$8\frac{3}{16}$	$8\frac{3}{16}$
difference	$\frac{13}{16}$	$\frac{13}{16}$ Ans.

EXPLANATION.—As there is no fraction in the minuend from which to take the fraction in the subtrahend, borrow 1, or $\frac{16}{16}$, from 9. $\frac{3}{16}$ from $\frac{16}{16} = \frac{13}{16}$. Since 1 was borrowed from 9, only 8 is left. 8 from $8 = 0$.

37. **Rule.—I.** *Reduce the fractions to fractions having a common denominator. Subtract one numerator from the other and place the remainder over the common denominator.*

II. *When there are mixed numbers, subtract the fractions and whole numbers separately, and place the remainders side by side.*

III. *When the fraction in the subtrahend is greater than the fraction in the minuend, borrow 1 from the whole number in the minuend and add it to the fraction in the minuend, from which subtract the fraction in the subtrahend.*

IV. *When the minuend is a whole number, borrow 1; reduce it to a fraction whose denominator is the same as the denominator of the fraction in the subtrahend, and place it over that fraction for subtraction.*

EXAMPLES FOR PRACTICE.

38. Subtract:

(*a*) $\frac{10}{24}$ from $\frac{11}{12}$.
(*b*) $\frac{7}{14}$ from $\frac{17}{36}$.
(*c*) $\frac{4}{30}$ from $\frac{5}{10}$.
(*d*) $\frac{15}{35}$ from $\frac{45}{70}$.
(*e*) $\frac{15}{18}$ from $\frac{57}{48}$.
(*f*) $13\frac{1}{4}$ from $30\frac{1}{2}$.
(*g*) $12\frac{1}{3}$ from 27.
(*h*) $5\frac{1}{4}$ from 30.

Ans. (*a*) $\frac{1}{4}$. (*b*) $\frac{3}{36}$. (*c*) $\frac{11}{30}$. (*d*) $\frac{3}{14}$. (*e*) $\frac{1}{2}$. (*f*) $17\frac{1}{2}$. (*g*) $14\frac{2}{3}$. (*h*) $24\frac{3}{4}$.

1. An engineer found that he had on hand $48\frac{1}{2}$ gallons of cylinder oil. During the following week he used $\frac{3}{4}$ of a gallon each day for three days, $\frac{7}{8}$ of a gallon on the fourth day, $\frac{15}{16}$ of a gallon on the fifth day, and $\frac{1}{2}$ of a gallon on the sixth day. How much oil remained at the end of the week? Ans. $43\frac{15}{16}$ gal.

2. The main line shaft of a manufacturing plant is run by an engine and waterwheel. A test of the plant showed that the engine was capable of developing $251\frac{3}{4}$ H. P. (horsepower), and the waterwheel, under full gate, $67\frac{1}{4}$ H. P. It was also found that the machinery consumed $210\frac{11}{16}$ H. P., and the friction of the shafting and belting was $32\frac{1}{4}$ H. P. How much power remained unused? Ans. $76\frac{233}{560}$ H. P.

MULTIPLICATION OF FRACTIONS.

39. *In multiplication of fractions it is not necessary to reduce the fractions to fractions having a common denominator.*

Multiplying the numerator or dividing the denominator multiplies the fraction.

EXAMPLE.—Multiply $\frac{3}{4}$ by 4.

SOLUTION.— $\frac{3}{4} \times 4 = \frac{3 \times 4}{4} = \frac{12}{4} = 3$. Ans.

Or, $\frac{3}{4} \times 4 = \frac{3}{4 \div 4} = \frac{3}{1} = 3$. Ans.

40. The word "of" in multiplication of fractions means the same as $\times$, or times. Thus,

$$\tfrac{3}{4} \text{ of } 4 = \tfrac{3}{4} \times 4 = 3.$$

$$\tfrac{1}{8} \text{ of } \tfrac{5}{16} = \tfrac{1}{8} \times \tfrac{5}{16} = \tfrac{5}{128}.$$

EXAMPLE.—Multiply $\frac{3}{8}$ by 2.

SOLUTION.— $2 \times \frac{3}{8} = \frac{3 \times 2}{8} = \frac{6}{8} = \frac{3}{4}$. Ans.

Or, $2 \times \frac{3}{8} = \frac{3}{8 \div 2} = \frac{3}{4}$. Ans.

41. EXAMPLE.—What is the product of $\frac{4}{16}$ and $\frac{7}{8}$?

SOLUTION.— $\frac{4}{16} \times \frac{7}{8} = \frac{4 \times 7}{16 \times 8} = \frac{28}{128} = \frac{7}{32}$. **Ans.**

Or, by cancelation, $\frac{\cancel{4} \times 7}{\cancel{16} \times 8} = \frac{7}{4 \times 8} = \frac{7}{32}$. **Ans.**

42. EXAMPLE.—What is $\frac{4}{8}$ of $\frac{3}{4}$ of $\frac{16}{32}$?

SOLUTION.— $\frac{\cancel{4} \times 3 \times \cancel{16}}{8 \times \cancel{4} \times \cancel{32}} = \frac{3}{8 \times 2} = \frac{3}{16}$. **Ans.**

43. EXAMPLE.—What is the product of $9\frac{3}{4}$ and $5\frac{5}{8}$?

SOLUTION.— $9\frac{3}{4} = \frac{39}{4}$; $5\frac{5}{8} = \frac{45}{8}$.

$$\frac{39}{4} \times \frac{45}{8} = \frac{39}{4} \times \frac{45}{8} = \frac{1755}{32} = 54\frac{27}{32}. \quad \textbf{Ans.}$$

44. EXAMPLE.—Multiply $15\frac{7}{8}$ by 3.

SOLUTION.—

$$\begin{array}{rcl} 15\frac{7}{8} & & 15\frac{7}{8} \\ 3 & \text{or} & 3 \\ \hline 47\frac{5}{8} & & 45 + \frac{21}{8} = 45 + 2\frac{5}{8} = 47\frac{5}{8}. \end{array}$$ **Ans.**

45. **Rule.—I.** *Divide the product of the numerators by the product of the denominators. All factors common to the numerators and denominators should first be cast out by cancelation.*

II. *To multiply one mixed number by another, reduce them both to improper fractions.*

III. *To multiply a mixed number by a whole number, first multiply the fractional part by the multiplier, and if the product is an improper fraction, reduce it to a mixed number and add the whole number part to the product of the multiplier and the whole number.*

EXAMPLES FOR PRACTICE.

46. Find the product of:

(*a*) $7 \times \frac{3}{19}$.
(*b*) $14 \times \frac{5}{16}$.
(*c*) $\frac{21}{32} \times \frac{5}{14}$.
(*d*) $\frac{16}{27} \times 4$.
(*e*) $\frac{19}{18} \times 7$.
(*f*) $17\frac{18}{21} \times 7$.
(*g*) $\frac{105}{224} \times 32$.
(*h*) $\frac{15}{28} \times 14$.

Ans. (*a*) $1\frac{3}{19}$. (*b*) $4\frac{3}{8}$. (*c*) $\frac{15}{64}$. (*d*) $2\frac{10}{27}$. (*e*) $7\frac{7}{18}$. (*f*) 125. (*g*) 15. (*h*) $7\frac{1}{2}$.

1. A single belt can transmit $107\frac{3}{8}$ horsepower, but as it is desired to use more power, a double belt of the same width is substituted for it. Supposing the double belt to be capable of transmitting $\frac{10}{7}$ as much power as the single belt, how many horsepower can be used after the change? Ans. $153\frac{17}{28}$ H. P.

2. What is the weight of $2\frac{3}{8}$ miles of copper wire weighing $5\frac{1}{4}$ pounds per 100 feet? There are 5,280 feet in a mile. Ans. $796\frac{13}{20}$ lb.

3. The grate of a steam boiler contains $20\frac{1}{2}$ square feet. If the boiler burns $8\frac{8}{10}$ pounds of coal an hour per square foot of grate area and can evaporate $7\frac{1}{2}$ pounds of water an hour per pound of coal burned, how many pounds of water are evaporated by the boiler in 1 hour? Ans. $1,276\frac{1}{2}$ lb.

DIVISION OF FRACTIONS.

47. *In division of fractions it is not necessary to reduce the fractions to fractions having a common denominator.*

48. *Dividing the numerator or multiplying the denominator divides the fraction.*

EXAMPLE 1.—Divide $\frac{6}{8}$ by 3.

SOLUTION.—When *dividing the numerator*, we have

$$\tfrac{6}{8} \div 3 = \frac{6 \div 3}{8} = \tfrac{2}{8} = \tfrac{1}{4}. \quad \text{Ans.}$$

When *multiplying the denominator*, we have

$$\tfrac{6}{8} \div 3 = \frac{6}{8 \times 3} = \tfrac{6}{24} = \tfrac{1}{4}. \quad \text{Ans.}$$

EXAMPLE 2.—Divide $\frac{3}{16}$ by 2.

SOLUTION.— $\tfrac{3}{16} \div 2 = \dfrac{3}{16 \times 2} = \tfrac{3}{32}$. Ans.

EXAMPLE 3.—Divide $\frac{14}{32}$ by 7.

SOLUTION.— $\tfrac{14}{32} \div 7 = \dfrac{14 \div 7}{32} = \tfrac{2}{32} = \tfrac{1}{16}$. Ans.

49. To **invert** a fraction is to *turn it upside down;* that is, make the numerator and denominator *change places.*

Invert $\frac{3}{4}$ and it becomes $\frac{4}{3}$.

50. EXAMPLE.—Divide $\frac{9}{16}$ by $\frac{3}{16}$.

SOLUTION.—1. The fraction $\frac{3}{16}$ is contained in $\frac{9}{16}$, 3 times, for the denominators are the same, and one numerator is contained in the other 3 times. 2. If we now invert the divisor $\frac{3}{16}$ and multiply, the solution is

$$\frac{9}{16} \times \frac{16}{3} = \frac{\overset{3}{\cancel{9}} \times \cancel{16}}{\cancel{16} \times \underset{1}{\cancel{3}}} = 3. \quad \text{Ans.}$$

This brings the same quotient as in the first case.

51. EXAMPLE.—Divide $\frac{3}{8}$ by $\frac{1}{4}$.

SOLUTION.—We cannot divide $\frac{3}{8}$ by $\frac{1}{4}$, as in the first case above, for the denominators are not the same; therefore, we must solve as in the second case.

$$\frac{3}{8} \div \frac{1}{4} = \frac{3}{8} \times \frac{4}{1} = \frac{3 \times \overset{1}{\cancel{4}}}{\underset{2}{\cancel{8}} \times 1} = \frac{3}{2} = 1\tfrac{1}{2}. \quad \text{Ans.}$$

52. EXAMPLE 1.—Divide 5 by $\frac{10}{16}$.

SOLUTION.— $\frac{10}{16}$ inverted becomes $\frac{16}{10}$.

$$5 \times \frac{16}{10} = \frac{\overset{1}{\cancel{5}} \times \overset{8}{\cancel{16}}}{\underset{1}{\cancel{10}}} = 8. \quad \text{Ans.}$$

EXAMPLE 2.—How many times is $3\frac{1}{4}$ contained in $7\frac{7}{16}$?

SOLUTION.— $3\frac{1}{4} = \frac{13}{4}$; $7\frac{7}{16} = \frac{119}{16}$.

$\frac{13}{4}$ inverted equals $\frac{4}{13}$.

$$\frac{119}{16} \times \frac{4}{15} = \frac{119 \times \overset{1}{\cancel{4}}}{\underset{4}{\cancel{16}} \times 15} = \frac{119}{60} = 1\tfrac{59}{60}. \quad \text{Ans.}$$

53. Rule.—*Invert the divisor and proceed as in multiplication.*

54. We have learned that a line placed between two numbers indicates that the number above the line is to be divided by the number below it. Thus, $\frac{18}{3}$ shows that 18 is to be divided by 3. This is also true if a fraction or a fractional expression be placed above or below a line.

$\frac{9}{\frac{3}{8}}$ means that 9 is to be divided by $\frac{3}{8}$; $\dfrac{3 \times 7}{\frac{8+4}{16}}$ means that 3×7 is to be divided by the value of $\dfrac{8+4}{16}$.

$\frac{\frac{1}{4}}{\frac{3}{8}}$ is the same as $\frac{1}{4} \div \frac{3}{8}$.

It will be noticed that there is a heavy line between the 9 and the $\frac{3}{8}$. This is necessary, since otherwise there would be nothing to show as to whether 9 was to be divided by $\frac{3}{8}$ or $\frac{9}{3}$ was to be divided by 8. Whenever a heavy line is used, as shown here, it indicates that *all above the line is to be divided by all below it.*

55. Whenever an expression like one of the three following ones is obtained, it may always be simplified by transposing the denominator from *above* to *below* the line, or from *below* to *above*, as the case may be, taking care, however, to indicate that the denominator when so transferred is a multiplier.

1. $\frac{\frac{3}{4}}{9} = \frac{3}{9 \times 4} = \frac{3}{36} = \frac{1}{12}$; for, regarding the fraction above the heavy line as the numerator of a fraction whose denominator is 9, $\frac{\frac{3}{4} \times 4}{9 \times 4} = \frac{3}{9 \times 4}$, as before.

2. $\frac{9}{\frac{3}{4}} = \frac{9 \times 4}{3} = 12$. The proof is the same as in the first case.

3. $\frac{\frac{5}{9}}{\frac{3}{4}} = \frac{5 \times 4}{3 \times 9} = \frac{20}{27}$; for, regarding $\frac{5}{9}$ as the numerator of a fraction whose denominator is $\frac{3}{4}$, $\frac{\frac{5}{9} \times 9}{\frac{3}{4} \times 9} = \frac{5}{\frac{3 \times 9}{4}}$; and $\frac{\frac{5}{3 \times 9}}{\frac{4}{4}} \times 4 = \frac{5 \times 4}{3 \times 9} = \frac{20}{27}$, as above.

This principle may be used to great advantage in cases like $\frac{\frac{1}{4} \times 310 \times \frac{27}{12} \times 72}{40 \times 4\frac{1}{2} \times 5\frac{1}{6}}$. Reducing the mixed numbers to fractions, the expression becomes $\frac{\frac{1}{4} \times 310 \times \frac{27}{12} \times 72}{40 \times \frac{9}{2} \times \frac{31}{6}}$. Now transferring the denominators of the fractions and canceling,

$$\frac{1 \times 310 \times 27 \times 72 \times 2 \times 6}{40 \times 9 \times 31 \times 4 \times 12} = \frac{1 \times \overset{10}{\cancel{310}} \times \overset{3}{\cancel{27}} \times \overset{3}{\cancel{72}} \times \cancel{2} \times \overset{3}{\cancel{6}}}{\underset{4}{\cancel{40}} \times \cancel{9} \times \cancel{31} \times \underset{2}{\cancel{4}} \times \cancel{12}}$$

$= \frac{27}{2} = 13\frac{1}{2}$.

Greater exactness in results can usually be obtained by using this principle than can be obtained by reducing the fractions to decimals. The principle, however, should not be employed *if a sign of addition or subtraction occurs either above or below the dividing line.*

EXAMPLES FOR PRACTICE.

56. Divide:

(*a*) 15 by $6\frac{3}{7}$.		(*a*) $2\frac{1}{3}$.
(*b*) 30 by $\frac{3}{5}$.		(*b*) 40.
(*c*) 172 by $\frac{4}{5}$.		(*c*) 215.
(*d*) $\frac{14}{15}$ by $1\frac{7}{15}$.	Ans.	(*d*) $\frac{113}{207}$.
(*e*) $\frac{102}{5}$ by $14\frac{1}{2}$.		(*e*) $1\frac{15}{58}$.
(*f*) $\frac{142}{27}$ by $17\frac{1}{9}$.		(*f*) $\frac{71}{231}$.
(*g*) $\frac{14}{15}$ by $\frac{145}{72}$.		(*g*) $\frac{56}{145}$.
(*h*) $\frac{138}{15}$ by $72\frac{1}{3}$.		(*h*) $\frac{644}{651}$.

1. A $\frac{3}{8}$-inch boiler plate containing 24 square feet of surface weighs $362\frac{4}{10}$ pounds. What is its weight per square foot? Ans. $15\frac{1}{10}$ lb.

2. A certain boiler has $927\frac{1}{2}$ square feet of heating surface, which is equal to 35 times the area of the grate. What is the area of the grate in square feet? Ans. $26\frac{1}{2}$ sq. ft.

3. If the distance around the rim of a locomotive driving wheel is $13\frac{1}{12}$ feet, how many revolutions will the wheel make in traveling 682 feet? Ans. $52\frac{20}{157}$ rev.

ARITHMETIC.

(PART 3.)

DECIMALS.

1. **Decimals** are *tenth* fractions; that is, the parts of a unit are expressed on the scale of ten, as *tenths*, *hundredths*, *thousandths*, etc.

2. The *denominator*, which is always 10, 100, 1,000, etc., is not expressed, as it would be in fractions, by writing it under the *numerator* with a line between them, as $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, but is expressed by placing a *period* (.), which is called a **decimal point**, to the *left* of the *figures of the numerator*, to indicate that the figures on the right form the numerator of a fraction whose denominator is *ten*, *one hundred*, *one thousand*, etc.

3. The *reading* of a decimal number depends upon the number of decimal places in it, i. e., the number of figures to the right of the decimal point.

One decimal place expresses *tenths*.
Two decimal places express *hundredths*.
Three decimal places express *thousandths*.
Four decimal places express *ten-thousandths*.
Five decimal places express *hundred-thousandths*.
Six decimal places express *millionths*.

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Thus:

.3 = $\frac{3}{10}$ = 3 tenths.
.03 = $\frac{3}{100}$ = 3 hundredths.
.003 = $\frac{3}{1000}$ = 3 thousandths.
.0003 = $\frac{3}{10000}$ = 3 ten-thousandths.
.00003 = $\frac{3}{100000}$ = 3 hundred-thousandths.
.000003 = $\frac{3}{1000000}$ = 3 millionths.

We see in the above that the *number of decimal places in a decimal equals the number of ciphers to the right of the figure 1 in the denominator of its equivalent fraction.* This fact kept in mind will be of much assistance in reading and writing decimals.

Whatever may be written to the *left* of a decimal point is a whole number. *The decimal point affects only the figures to its right.*

When a whole number and decimal are written together, the expression is a *mixed number.* Thus, 8.12 and 17.25 are mixed numbers.

The relation of decimals and whole numbers to each other is clearly shown by the following table :

hundreds of millions.	tens of millions.	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.	decimal point.	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	ten-millionths.	hundred-millionths.
9	8	7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9

The figures to the *left* of the decimal point represent *whole numbers ;* those to the *right* are *decimals.*

In both decimals and whole numbers, the ***units*** place is made the starting point of notation and numeration. The decimals *decrease* on the scale of *ten* to the ***right***, and the whole numbers *increase* on the scale of *ten* to the ***left***. The *first* figure to the *left* of units is *tens*, and the ***first*** figure to the ***right*** of units is *tenths.* The *second* figure to the *left* of

units is *hundreds*, and the *second* figure to the *right* is *hundredths*. The *third* figure to the *left* is *thousands*, and the *third* to the *right* is *thousandths*, and so on, the *whole* numbers on the *left* and the *decimals* on the *right*. The figures equally distant from units place correspond in name. The *decimals* have the ending *ths*, which distinguishes them from *whole* numbers. The following is the numeration of the number in the above table: nine hundred eighty-seven million six hundred fifty-four thousand three hundred twenty-one and twenty-three million four hundred fifty-six thousand seven hundred eighty-nine hundred-millionths.

The decimals increase to the *left* on the scale of *ten*, the same as whole numbers; for, beginning at, say, 4-*thousandths* in the table, the next figure to the left is *hundredths*, which is ten times as great, and the next *tenths*, or ten times the *hundredths*, and so on through both decimals and whole numbers.

4. *Annexing or taking away a cipher at the right of a decimal does not affect its value.*

.5 is $\frac{5}{10}$; .50 is $\frac{50}{100}$, but $\frac{5}{10} = \frac{50}{100}$; therefore, $.5 = .50$.

5. *Inserting a cipher between a decimal and the decimal point divides the decimal by 10.*

$$.5 = \tfrac{5}{10}\ ;\ \tfrac{5}{10} \div 10 = \tfrac{5}{100} = .05.$$

6. *Taking away a cipher from the left of a decimal multiplies the decimal by 10.*

$$.05 = \tfrac{5}{100}\ ;\ \tfrac{5}{100} \times 10 = \tfrac{5}{10} = .5.$$

ADDITION OF DECIMALS.

7. The only respect in which addition of decimals differs from addition of whole numbers is in the placing of the numbers to be added.

Whole numbers begin at units and increase on the scale of 10, to the left. Decimals decrease on the scale of 10, to the right. Whole numbers are to the left of the decimal point, and decimals are to the right of it. In whole numbers the *right-hand* side of a column of figures to be added must be in line,

and in decimals the *left-hand* side must be in line, which brings the decimal points directly under one another.

whole numbers	*decimals*	*mixed numbers*
342	.342	342.032
4234	.4234	4234.5
26	.26	26.6782
3	.03	3.06
sum 4605 Ans.	*sum* 1.0554 Ans.	*sum* 4606.2702 Ans.

8. EXAMPLE.—What is the sum of 242, .36, 118.725, 1.005, 6, and 100.1?

SOLUTION.—

	242.
	.36
	118.725
	1.005
	6.
	100.1
sum	468.190 Ans.

9. Rule.—*Place the numbers to be added so that the decimal points will be directly under one another. Add as in whole numbers, and place the decimal point in the sum directly under the decimal points above.*

EXAMPLES FOR PRACTICE.

10. Find the sum of:

(*a*)	.2143, .105, 2.3042, and 1.1417.	(*a*)	3.7652.
(*b*)	783.5, 21.473, .2101, and .7816.	(*b*)	805.9647.
(*c*)	21.781, 138.72, 41.8738, .72, and 1.413.	(*c*)	204.5078.
(*d*)	.3724, 104.15, 21.417, and 100.042.	(*d*)	225.9814.
(*e*)	200.172, 14.105, 12.1465, .705, and 7.2.	(*e*)	234.3285.
(*f*)	1,427.16, .244, .32, .032, and 10.0041.	(*f*)	1,437.7601.
(*g*)	2,473.1, 41.65, .7243, 104.067, and 21.073.	(*g*)	2,640.6143.
(*h*)	4,107.2, .00375, 21.716, 410.072, and .0345.	(*h*)	4,539.02625.

Ans.

1. The estimated weights of the parts of a return-tubular boiler were as follows: shell, 3,626 pounds; tubes, 3,564.5 pounds; manhole cover, ring, and yoke, 270.34 pounds; stays, etc., 1,089.4 pounds; steam nozzles, 236.07 pounds; handhole covers and yokes, 120.25 pounds; feedpipe, 34.75 pounds; boiler supports, 350.6 pounds. What was the total estimated weight of the boiler? Ans. 9,291.91 lb.

2. A bill for engine-room supplies had the following items: 1 waste can, $8.30; 20 feet of 4-inch belting, $11.20; 1 pipe wrench, $1.65; 12 pounds of waste, $0.84; 5 gallons of cylinder oil, $8.75; 20 gallons of machine oil, $24. How much did the bill amount to? Ans. $54.74.

SUBTRACTION OF DECIMALS.

11. For the same reason as in addition of decimals, the left-hand figures of decimal numbers are placed in line and the decimal points under each other.

EXAMPLE.—Subtract .132 from .3063.

SOLUTION.—

minuend	.3 0 6 3
subtrahend	.1 3 2
difference	.1 7 4 3 Ans.

12. EXAMPLE.—What is the difference between 7.895 and .725?

SOLUTION.—

minuend	7.8 9 5
subtrahend	.7 2 5
difference	7.1 7 0, or 7.17 Ans.

13. EXAMPLE.—Subtract .625 from 11.

SOLUTION.—

minuend	1 1.0 0 0
subtrahend	.6 2 5
difference	1 0.3 7 5 Ans.

14. Rule.—*Place the subtrahend under the minuend, so that the decimal points will be directly under each other. Subtract as in whole numbers, and place the decimal point in the remainder directly under the decimal points above.*

When the figures in the decimal part of the subtrahend extend beyond those in the minuend, place ciphers in the minuend above them and subtract as before.

EXAMPLES FOR PRACTICE.

15. From:

(*a*)	407.385 take 235.0004.	(*a*)	172.3846.
(*b*)	22.718 take 1.7042.	(*b*)	21.0138.
(*c*)	1,368.17 take 13.6817.	(*c*)	1,354.4883.
(*d*)	70.00017 take 7.000017.	(*d*)	63.000153.
(*e*)	630.630 take .6304.	(*e*)	629.9996.
(*f*)	421.73 take 217.162.	(*f*)	204.568.
(*g*)	1.000014 take .00001.	(*g*)	1.000004.
(*h*)	.783652 take .542314.	(*h*)	.241338.

Ans.

1. If the temperature of steam at 5 pounds pressure is 227.964 degrees and at 100 pounds pressure is 337.874 degrees, how many degrees hotter is the steam at the higher pressure?

Ans. 109.91 degrees.

2. The outside diameter of 2½-inch wrought-iron pipe is 2.87 inches and the inside diameter is 2.46 inches. How thick is the pipe?

Ans. .41 ÷ 2 = .205 in.

3. In a cistern that will hold 326.5 barrels of water there are 178.625 barrels? How much does it lack of being full?

Ans. 147.875 bbl.

4. A wrought-iron rod is 2.53 inches in diameter. What must be the thickness of metal turned off, so that the rod will be 2.495 inches in diameter? Ans. .035 ÷ 2 = .0175 in.

MULTIPLICATION OF DECIMALS.

16. In multiplication of decimals, we do not place the decimal points directly under each other as in addition and subtraction. We pay no attention for the time being to the decimal points. Place the multiplier under the multiplicand, so that the *right-hand* figure of the one is under the *right-hand* figure of the other, and proceed exactly as in multiplication of whole numbers. After multiplying, *count the number of decimal places in both multiplicand and multiplier, and point off the same number in the product.*

EXAMPLE.—Multiply .825 by 13.

SOLUTION.—

multiplicand	.8 2 5
multiplier	1 3
	2 4 7 5
	8 2 5
product	1 0.7 2 5 Ans.

In this example there are 3 decimal places in the multiplicand and none in the multiplier; therefore, 3 decimal places are pointed off in the product.

17. EXAMPLE.—What is the product of 426 and the decimal .005?

SOLUTION.—

multiplicand	4 2 6
multiplier	.0 0 5
product	2.1 3 0, or 2.13 Ans.

In this example there are 3 decimal places in the multiplier and none in the multiplicand; therefore, 3 decimal places are pointed off in the product.

18. It is not necessary to multiply by the ciphers on the *left* of a decimal; they merely *determine the number of decimal places.* Ciphers to the *right* of a decimal should be omitted, as they only make more figures to deal with and do not change the value.

19. EXAMPLE.—Multiply 1.205 by 1.15.

SOLUTION.—

multiplicand	1.2 0 5
multiplier	1.1 5
	6 0 2 5
	1 2 0 5
	1 2 0 5
product	1.3 8 5 7 5 Ans.

In this example there are 3 decimal places in the multiplicand and 2 in the multiplier; therefore, 3 + 2, or 5, decimal places must be pointed off in the product.

20. EXAMPLE.—Multiply .232 by .001.

SOLUTION.—

multiplicand	.2 3 2
multiplier	.0 0 1
product	.0 0 0 2 3 2 Ans.

In this example we multiply the multiplicand by the digit in the multiplier, which gives 232 for the product; but since there are 3 decimal places each in the multiplier and multiplicand, we must prefix 3 ciphers to the 232 to make 3 + 3, or 6, decimal places in the product.

21. **Rule.**—*Place the multiplier under the multiplicand, disregarding the position of the decimal points. Multiply as in whole numbers, and in the product point off as many decimal places as there are decimal places in both multiplier and multiplicand, prefixing ciphers if necessary.*

EXAMPLES FOR PRACTICE.

22. Find the product of:

(*a*)	.000492 × 4.1418.	(*a*)	.0020377656.
(*b*)	4,003.2 × 1.2.	(*b*)	4,803.84.
(*c*)	78.6531 × 1.03.	(*c*)	81.012693.
(*d*)	.3685 × .042.	(*d*)	.015477.
(*e*)	178,352 × .01.	(*e*)	1,783.52.
(*f*)	.00045 × .0045.	(*f*)	.000002025.
(*g*)	.714 × .00002.	(*g*)	.00001428.
(*h*)	.00004 × .008.	(*h*)	.00000032.

Ans.

1. The stroke of an engine was found by measurement to be 2.987 feet. How many feet will the crosshead pass over in 600 revolutions? Ans. 3,584.4 ft.

2. If a steam pump delivers 2.39 gallons of water per stroke and runs at 51 strokes a minute, how many gallons of water would it pump in 58½ minutes? Ans. 7,130.565 gal.

3. Wishing to obtain the weight of a connecting-rod from a drawing, it was calculated that the rod contained 294.8 cubic inches of wrought iron, 63.5 cubic inches of brass, and 10.4 cubic inches of Babbitt. Assuming the weight of wrought iron to be .278 pound per cubic inch, of brass .303 pound, and of Babbitt .264 pound, what was the weight of the rod? Ans. 103.94 lb.

DIVISION OF DECIMALS.

23. In division of decimals we pay no attention to the decimal point until after the division is performed. The *number of decimal places in the dividend must equal (or be made to equal by annexing ciphers) the number of decimal places in the divisor. Divide exactly as in whole numbers. Subtract the number of decimal places in the divisor from the number of decimal places in the dividend, and point off as many decimal places in the quotient as are indicated by the remainder.*

EXAMPLE.—Divide .625 by 25.

SOLUTION.—

divisor dividend quotient

25) .625 (.025 Ans.

50

125

125

remainder 0

In this example there are no decimal places in the divisor and 3 decimal places in the dividend; therefore, there are 3 minus 0, or 3, decimal places in the quotient. One cipher has to be prefixed to the 25 to make the 3 decimal places.

24. EXAMPLE.—Divide 6.035 by .05.

SOLUTION.—

divisor dividend quotient

.0 5) 6.0 3 5 (1 2 0.7 Ans.

5

1 0

1 0

3 5

3 5

remainder 0

In this example we divide by 5, as if the cipher were not before it. There is 1 more decimal place in the dividend than in the divisor; therefore, 1 decimal place is pointed off in the quotient.

25. EXAMPLE.—Divide .125 by .005.

SOLUTION.—

divisor dividend quotient

.0 0 5) .1 2 5 (2 5 Ans.

1 0

2 5

2 5

remainder 0

In this example there are the same number of decimal places in the dividend as in the divisor; therefore, the quotient has no decimal places and is a whole number.

26. EXAMPLE.—Divide 326 by .25.

SOLUTION.—

divisor dividend quotient

.2 5) 3 2 6.0 0 (1 3 0 4 Ans.

2 5

7 6

7 5

1 0 0

1 0 0

remainder 0

In this problem two ciphers were annexed to the dividend, to make the number of decimal places equal to the number in the divisor. The quotient is a whole number.

27. EXAMPLE.—Divide .0025 by 1.25.

divisor dividend quotient

SOLUTION.— 1.2 5) .0 0 2 5 0 (.0 0 2 Ans.

2 5 0

remainder 0

EXPLANATION.—In this example we are to divide .0025 by 1.25. Consider the dividend as a whole number, i. e., as 25 (disregarding the two ciphers at its left, for the present); also, consider the divisor as a whole number, i. e., as 125. It is clearly evident that the dividend, 25, will not contain the divisor, 125; we must, therefore, annex one cipher to the 25, thus making the dividend 250. 125 is contained twice in 250, so we place the figure 2 in the quotient. In pointing off the decimal places in the quotient, it must be remembered that there were only 4 decimal places in the dividend; but one cipher was annexed, thereby making $4 + 1$, or 5, decimal places. Since there are 5 decimal places in the dividend and 2 decimal places in the divisor, we must point off $5 - 2$, or 3, decimal places in the quotient. In order to point off 3 decimal places, two ciphers must be prefixed to the figure 2, thereby making .002 the quotient. It is not necessary to consider the ciphers at the left of a decimal when dividing, except when determining the position of the decimal point in the quotient.

28. **Rule.—I.** *Place the divisor to the left of the dividend and proceed as in division of whole numbers; in the quotient, point off as many decimal places as the number of decimal places in the dividend exceeds the number of decimal places in the divisor, prefixing ciphers to the quotient, if necessary.*

II. *If in dividing one whole number by another there be a remainder, the remainder can be placed over the divisor as a fractional part of the quotient; but it is generally better to annex ciphers to the remainder and continue dividing until there are three or four decimal places in the quotient, and*

then if there still be a remainder, terminate the quotient by the plus sign (+), *which shows that it can be carried farther.*

29. EXAMPLE.—What is the quotient of 199 divided by 15?

divisor dividend quotient

SOLUTION.— 15)199(13 + $\frac{4}{15}$ Ans.

15

49

45

remainder 4

Or, 15)199.000(13.266+ Ans.

15

49

45

40

30

100

90

100

90

remainder 10

$13\frac{4}{15} = 13.266+$

$\frac{4}{15} = .266+$

It very frequently happens, as in the above example, that the division will never terminate. In such cases, decide to how many decimal places the division is to be carried and carry the work one place farther. If the last figure of the quotient thus obtained is 5 or a greater number, increase the preceding figure by 1, and write after it the minus sign (−), thus indicating that the quotient is not quite as large as indicated; if the figure thus obtained is less than 5, write the plus sign (+) after the quotient, thus indicating that the number is slightly greater than as indicated. In the last example, had it been desired to obtain the answer correct to four decimal places, the work would have been carried to five places, obtaining 13.26666, and the answer would have been given as 13.2667−. This remark applies to any other calculation involving decimals, when it is

desired to omit some of the figures in the decimal. Thus, if it is desired to retain three decimal places in the number .2471253, it would be expressed as .247+; if it was desired to retain five decimal places, it would be expressed as .24713−. Both the + and − signs are frequently omitted; they are seldom used outside of arithmetic, except in exact calculations, when it is desired to call particular attention to the fact that the result obtained is not quite exact.

EXAMPLES FOR PRACTICE.

30. Divide:

(*a*)	101.6688 by 2.36.	Ans. (*a*)	43.08.
(*b*)	187.12264 by 123.107.	(*b*)	1.52.
(*c*)	.08 by .008.	(*c*)	10.
(*d*)	.0003 by 3.75.	(*d*)	.00008.
(*e*)	.0144 by .024.	(*e*)	.6.
(*f*)	.00375 by 1.25.	(*f*)	.003.
(*g*)	.004 by 400.	(*g*)	.00001.
(*h*)	.4 by .008.	(*h*)	50.

1. In a steam-engine test of an hour's duration, the horsepower developed was found to be as follows at 10-minute intervals: 25.73, 25.64, 26.13, 25.08, 24.20, 26.7, 26.34. What was the average horsepower? Ans. 25.6886−, average.

NOTE.—Add the different horsepowers together and divide by the number of tests, or 7.

2. There are 31.5 gallons in a barrel. How many barrels are there in 2,787.75 gallons? Ans. 88.5 bbl.

3. A carload of 18.75 tons of coal cost $60.75. How much was it worth per ton? Ans. $3.24 per ton.

4. A keg of $\frac{9}{16}'' \times 1\frac{5}{8}''$ boiler rivets weighs 100 pounds and contains 595 rivets. What is the weight of one of the rivets?

Ans. .168+ lb.

TO REDUCE A FRACTION TO A DECIMAL.

31. EXAMPLE 1.— $\frac{3}{4}$ equals what decimal?

SOLUTION.— 4) 3.0 0

.7 5, or $\frac{3}{4} = .75$ Ans.

EXAMPLE 2.—What decimal is equivalent to $\frac{7}{8}$?

SOLUTION.—
$$\begin{array}{r} 8\,)\,7.0\,0\,0\,(\,.8\,7\,5 \\ 6\,4 \\ \hline 6\,0 \\ 5\,6 \\ \hline 4\,0 \\ 4\,0 \\ \hline \end{array}$$
or $\frac{7}{8} = .875$. Ans.

32. Rule.—*Annex ciphers to the numerator and divide by the denominator. Point off as many decimal places in the quotient as there are ciphers annexed.*

EXAMPLES FOR PRACTICE.

33. Reduce the following common fractions to decimals:

(*a*) $\frac{15}{32}$.
(*b*) $\frac{7}{8}$.
(*c*) $\frac{21}{32}$.
(*d*) $\frac{51}{64}$.
(*e*) $\frac{4}{25}$.
(*f*) $\frac{5}{8}$.
(*g*) $\frac{10}{200}$.
(*h*) $\frac{4}{1000}$.

Ans. (*a*) .46875. (*b*) .875. (*c*) .65625. (*d*) .796875. (*e*) .16. (*f*) .625. (*g*) .05. (*h*) .004.

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34. To reduce inches to decimal parts of a foot:

EXAMPLE.—What decimal part of a foot is 9 inches?

SOLUTION.—Since there are 12 inches in 1 foot, 1 inch is $\frac{1}{12}$ of a foot and 9 inches is $9 \times \frac{1}{12}$, or $\frac{9}{12}$ of a foot. This reduced to a decimal by the above rule shows what decimal part of a foot 9 inches is.

$$\begin{array}{r} 1\,2\,)\,9.0\,0\,(\,.7\,5 \text{ of a foot. Ans.} \\ 8\,4 \\ \hline 6\,0 \\ 6\,0 \\ \hline \end{array}$$

35. Rule.—I. *To reduce inches to a decimal part of a foot, divide the number of inches by 12.*

II. *Should the resulting decimal be an unending one, and it is desired to terminate the division at some point, say the fourth decimal place, carry the division one place farther, and if the fifth figure is 5 or greater, increase the fourth figure by 1, omitting the signs + and −.*

EXAMPLES FOR PRACTICE.

36. Reduce to the decimal part of a foot:

(*a*) 3 in.		(*a*)	.25 ft.
(*b*) 4½ in.		(*b*)	.375 ft.
(*c*) 5 in.	Ans.	(*c*)	.4167 ft.
(*d*) 6⅝ in.		(*d*)	.5521 ft.
(*e*) 11 in.		(*e*)	.9167 ft.

1. The lengths of belting required to connect three countershafts with the main line shaft were found with the tape measure to be 27 feet 4 inches, 23 feet 8 inches, and 38 feet 6 inches. How many feet of belting were necessary? Ans. 89.5 ft.

2. The stroke of an engine is 14 inches. What is the length of the crank in feet measured from center of shaft to center of crankpin, knowing that length of crank is one-half the stroke? Ans. .58̇33+ ft.

3. A steam pipe fitted with an expansion joint was found to expand 1.668 inches when steam was admitted into it. How much was its expansion in decimal parts of a foot? Ans. .139 ft.

TO REDUCE A DECIMAL TO A FRACTION.

37. EXAMPLE 1.—Reduce .125 to a fraction.

SOLUTION.— $.125 = \frac{125}{1000} = \frac{5}{40} = \frac{1}{8}$. Ans.

EXAMPLE 2.—Reduce .875 to a fraction.

SOLUTION.— $.875 = \frac{875}{1000} = \frac{35}{40} = \frac{7}{8}$. Ans.

38. **Rule.**—*Under the figures of the decimal, place 1 with as many ciphers at its right as there are decimal places in the decimal, and reduce the resulting fraction to its lowest terms by dividing both numerator and denominator by the same number.*

EXAMPLES FOR PRACTICE.

39. Reduce the following to common fractions:

(*a*) .375.
(*b*) .625.
(*c*) .3125.
(*d*) .04.
(*e*) .06.
(*f*) .75.
(*g*) .15625.
(*h*) .875.

Ans. (*a*) $\frac{3}{8}$. (*b*) $\frac{5}{8}$. (*c*) $\frac{5}{16}$. (*d*) $\frac{1}{25}$. (*e*) $\frac{3}{50}$. (*f*) $\frac{3}{4}$. (*g*) $\frac{5}{32}$. (*h*) $\frac{7}{8}$.

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40. To express a decimal approximately as a fraction having a given denominator:

EXAMPLE 1.—Express .5827 in 64ths.

SOLUTION.— $.5827 \times \frac{64}{64} = \frac{37.2928}{64}$, say $\frac{37}{64}$.

Hence, $.5827 = \frac{37}{64}$, nearly. Ans.

EXAMPLE 2.—Express .3917 in 12ths.

SOLUTION.— $.3917 \times \frac{12}{12} = \frac{4.7004}{12}$, say $\frac{5}{12}$.

Hence, $.3917 = \frac{5}{12}$, nearly. Ans.

41. Rule.—*Reduce 1 to a fraction having the given denominator. Multiply the given decimal by the fraction so obtained, and the result will be the fraction required.*

EXAMPLES FOR PRACTICE.

42. Express:

(*a*) .625 in 8ths.
(*b*) .3125 in 16ths.
(*c*) .15625 in 32ds.
(*d*) .77 in 64ths.
(*e*) .81 in 48ths.
(*f*) .923 in 96ths.

Ans. (*a*) $\frac{5}{8}$. (*b*) $\frac{5}{16}$. (*c*) $\frac{5}{32}$. (*d*) $\frac{49}{64}$. (*e*) $\frac{39}{48}$. (*f*) $\frac{89}{96}$.

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43. The sign for dollars is $. It is read *dollars.* $25 is read *25 dollars.*

Since there are 100 cents in a dollar, 1 cent is 1 one-hundredth of a dollar; the first two figures of a decimal part of a dollar represent *cents*. Since a mill is $\frac{1}{10}$ of a cent, or $\frac{1}{1000}$ of a dollar, the third figure represents *mills*.

Thus, \$25.16 is read *twenty-five dollars and sixteen cents;* \$25.168 is read *twenty-five dollars sixteen cents and eight mills.*

SIGNS OF AGGREGATION.

44. The **vinculum** ——, **parenthesis** ( ), **brackets** [ ], and **brace** { } are called **symbols of aggregation,** and are used to include numbers which are to be considered together; thus, $13 \times \overline{8 - 3}$, or $13 \times (8 - 3)$, shows the 3 is to be taken from 8 before multiplying by 13.

$$13 \times \overline{8 - 3} = 13 \times 5 = 65. \quad \text{Ans.}$$
$$13 \times (8 - 3) = 13 \times 5 = 65. \quad \text{Ans.}$$

When the vinculum or parenthesis is not used, we have

$$13 \times 8 - 3 = 104 - 3 = 101. \quad \text{Ans.}$$

45. In any series of numbers connected by the signs +, −, ×, and ÷, the operations indicated by the signs must be performed in order from left to right, *except* that no addition or subtraction may be performed if a sign of multiplication or division *follows* the number on the *right* of a sign of addition or subtraction until the indicated multiplication or division has been performed. In all cases the sign of multiplication takes the precedence, the reason being that when two or more numbers or expressions are connected by the sign of multiplication, the numbers thus connected are regarded as factors of the product indicated, and not as separate numbers.

EXAMPLE.—What is the value of $4 \times 24 - 8 + 17$?

SOLUTION.—Performing the operations in order from left to right, $4 \times 24 = 96$; $96 - 8 = 88$; $88 + 17 = 105$. Ans.

46. EXAMPLE.—What is the value of the following expression: $1{,}296 \div 12 + 160 - 22 \times 3\frac{1}{2}$?

SOLUTION.— $1{,}296 \div 12 = 108$; $108 + 160 = 268$; here we cannot subtract 22 from 268 because the sign of multiplication follows 22; hence, multiplying 22 by $3\frac{1}{2}$, we get 77, and $268 - 77 = 191$. Ans.

Had the above expression been written $1,296 \div 12 + 160 - 22 \times 3\frac{1}{2} \div 7 + 25$, it would have been necessary to have divided $22 \times 3\frac{1}{2}$ by 7 before subtracting, and the final result would have been $22 \times 3\frac{1}{2} = 77$; $77 \div 7 = 11$; $268 - 11 = 257$; $257 + 25 = 282$. Ans. In other words, it is necessary to perform *all* the multiplication or division included between the signs + and −, or − and +, before adding or subtracting. Also, had the expression been written $1,296 \div 12 + 160 - 24\frac{1}{2} \div 7 \times 3\frac{1}{2} + 25$, it would have been necessary to have multiplied $3\frac{1}{2}$ by 7 before dividing $24\frac{1}{2}$, since the sign of multiplication takes the precedence, and the final result would have been $3\frac{1}{2} \times 7 = 24\frac{1}{2}$; $24\frac{1}{2} \div 24\frac{1}{2} = 1$; $268 - 1 = 267$; $267 + 25 = 292$. Ans.

It likewise follows that if a succession of multiplication and division signs occurs, the indicated operations must not be performed in order, from left to right—the multiplication must be performed first. Thus, $24 \times 3 \div 4 \times 2 \div 9 \times 5 = \frac{1}{6}$. Ans. In order to obtain the same result that would be obtained by performing the indicated operations in order, from left to right, symbols of aggregation must be used. Thus, by using two vinculums, the last expression becomes $\overline{24 \times 3} \div \overline{4 \times 2} \div 9 \times 5 = 20$, the same result that would be obtained by performing the indicated operations in order, from left to right.

EXAMPLES FOR PRACTICE.

47. Find the values of the following expressions:

(*a*)	$(8 + 5 - 1) \div 4$.	Ans.	(*a*) 3.
(*b*)	$5 \times 24 - 32$.		(*b*) 88.
(*c*)	$5 \times 24 \div 15$.		(*c*) 8.
(*d*)	$144 - 5 \times 24$.		(*d*) 24.
(*e*)	$(1,691 - 540 + 559) \div 3 \times 57$.		(*e*) 10.
(*f*)	$2,080 + 120 - 80 \times 4 - 1,670$.		(*f*) 210.
(*g*)	$\overline{(90 + 60} \div 25) \times 5 - 29$.		(*g*) 1.
(*h*)	$\overline{90 + 60} \div 25 \times 5$.		(*h*) 1.2.

ARITHMETIC.

(PART 4.)

PERCENTAGE.

DEFINITIONS AND PRINCIPLES.

1. In certain operations, particularly those pertaining to business, it is very convenient to regard the quantity on which we are to operate as being divided into 100 equal parts; thus, instead of using the ordinary fractions $\frac{1}{4}$, $\frac{3}{5}$, $\frac{2}{7}$, we use the equivalent fractions $\frac{25}{100}$, $\frac{60}{100}$, $\frac{28\frac{4}{7}}{100}$, or their equivalent decimals, .25, .60, .28$\frac{4}{7}$. This practice is a very convenient one in all computations involving United States money, because, since \$1 equals 100 cents, it is easier to comprehend what part of the whole $\frac{35}{100}$ is than some other equivalent fraction, as $\frac{49}{140}$; it is also much easier to compute with fractions whose denominators are 100 than it is to compute with fractions whose denominators are composed of other figures.

2. **Percentage** is a term applied to those arithmetical operations in which the number or quantity to be operated upon is supposed to be divided into 100 equal parts.

3. The term **per cent.** means *by the hundred.* Thus, 8 per cent. of a number means 8 hundredths; i. e., $\frac{8}{100}$, or .08, of that number; 8 per cent. of 250 is $250 \times \frac{8}{100}$, or $250 \times .08 = 20$; 47 per cent. of 75 tons is $75 \times \frac{47}{100} = 75 \times .47 = 35.25$ tons. The statement that the population of a city has increased 22 per cent. in a given time, say from

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1880 to 1890, is equivalent to saying that the increase is 22 in every hundred; that is, for every 100 in 1880 there are 22 more, or 122, in 1890.

4. The **sign** of per cent. is %, and is read per cent. Thus, 6% is read *six per cent.;* $12\frac{1}{2}\%$ is read *twelve and one-half per cent.*, etc.

5. When expressing the per cent. of a number to use in calculations, it is customary to express it decimally instead of fractionally. Thus, instead of expressing 6%, 25%, and 43% as $\frac{6}{100}$, $\frac{25}{100}$, and $\frac{43}{100}$, it is usual to express them as .06, .25, and .43.

6. The following table will show how any per cent. can be expressed either as a decimal or as a fraction:

1%	.01	$\frac{1}{100}$	$\frac{1}{4}\%$	.0025	$\frac{\frac{1}{4}}{100}$ or $\frac{1}{400}$
2%	.02	$\frac{2}{100}$ or $\frac{1}{50}$	$\frac{1}{2}\%$	.005	$\frac{\frac{1}{2}}{100}$ or $\frac{1}{200}$
5%	.05	$\frac{5}{100}$ or $\frac{1}{20}$	$1\frac{1}{2}\%$	.015	$\frac{1\frac{1}{2}}{100}$ or $\frac{3}{200}$
10%	.10	$\frac{10}{100}$ or $\frac{1}{10}$	$6\frac{1}{4}\%$	$.06\frac{1}{4}$	$\frac{6\frac{1}{4}}{100}$ or $\frac{1}{16}$
25%	.25	$\frac{25}{100}$ or $\frac{1}{4}$	$8\frac{1}{3}\%$	$.08\frac{1}{3}$	$\frac{8\frac{1}{3}}{100}$ or $\frac{1}{12}$
50%	.50	$\frac{50}{100}$ or $\frac{1}{2}$	$12\frac{1}{2}\%$	.125	$\frac{12\frac{1}{2}}{100}$ or $\frac{1}{8}$
75%	.75	$\frac{75}{100}$ or $\frac{3}{4}$	$16\frac{2}{3}\%$	$.16\frac{2}{3}$	$\frac{16\frac{2}{3}}{100}$ or $\frac{1}{6}$
100%	1.00	$\frac{100}{100}$ or 1	$33\frac{1}{3}\%$	$.33\frac{1}{3}$	$\frac{33\frac{1}{3}}{100}$ or $\frac{1}{3}$
125%	1.25	$\frac{125}{100}$ or $1\frac{1}{4}$	$37\frac{1}{2}\%$	$.37\frac{1}{2}$	$\frac{37\frac{1}{2}}{100}$ or $\frac{3}{8}$
150%	1.50	$\frac{150}{100}$ or $1\frac{1}{2}$	$62\frac{1}{2}\%$	.625	$\frac{62\frac{1}{2}}{100}$ or $\frac{5}{8}$
500%	5.00	$\frac{500}{100}$ or 5	$87\frac{1}{2}\%$	.875	$\frac{87\frac{1}{2}}{100}$ or $\frac{7}{8}$

7. The names of the different terms used in percentage are: the *base*, the *rate* or *rate per cent.*, the *percentage*, the *amount*, and the *difference*.

8. The **base** is the number or quantity which is supposed to be divided into 100 equal parts.

9. The **rate per cent.** is that number of the 100 equal parts into which the base is supposed to be divided which is taken or considered. The **rate** is the number of hundredths of the base that is taken or considered. The distinction between the rate per cent. and the rate is this: The *rate per cent.* is always 100 times the *rate*. Thus, 7% of 125 and .07 of 125 amount in the end to the same thing; the former, 7, is the *rate per cent.*—the *number* of hundredths of 125 intended; the latter, .07, is the *rate*, the *part* of 125 that is to be found; 7% is used in *speech*, .07 is the form used in *computation*. So, also, $12\frac{1}{2}\% = .125$, $\frac{1}{2}\% = .005$, $1\frac{3}{4}\% = .0175$. In the table just given, the numbers in the first column are rates per cent.; those in the second column are rates.

10. The **percentage** is the result obtained by multiplying the base by the rate. Thus, 7% of $125 = 125 \times .07 = 8.75$, the percentage.

11. The **amount** is the sum of the base and the percentage.

12. The **difference** is the remainder obtained when the percentage is subtracted from the base.

13. The terms amount and difference are ordinarily used when there is an increase or a decrease in the base. For example, suppose the population of a village is 1,500 and it increases 25 per cent. This means that for every 100 of the original 1,500 there is an increase of 25, or a total increase of $15 \times 25 = 375$. This increase added to the original population gives the amount, or the population after the increase. If the population had decreased 375,

the final population would have been 1,500 − 375 = 1,125, and this would be the **difference.** The original population, 1,500, is the base on which the percentage is computed; the 25 is the rate per cent., and the increase or decrease, 375, is the percentage. If the base increases, the final value is the amount; and if it decreases, its final value is the difference.

CALCULATIONS INVOLVING PERCENTAGE.

14. From the foregoing it is evident that to find the percentage, the base must be multiplied by the rate. Hence, the following

Rule.—*To find the percentage, multiply the base by the rate.*

EXAMPLE.—Out of a lot of 300 boiler tubes 76% was used in a boiler. How many tubes were used?

SOLUTION.—The rate is .76; the base is 300; hence, the number of tubes used, or the percentage, is by the above rule

$$300 \times .76 = 228 \text{ tubes.} \quad \text{Ans.}$$

Expressing the rule as a

Formula, *percentage* = *base* × ***rate.***

15. When the percentage and rate are given, the base may be found by dividing the percentage by the rate. For, suppose that 12 is 6%, or $\frac{6}{100}$, of some number; then 1%, or $\frac{1}{100}$, of the number, is 12 ÷ 6, or 2. Consequently, if 2 = 1%, or $\frac{1}{100}$, 100%, or $\frac{100}{100} = 2 \times 100 = 200$. But as the same result may be arrived at by dividing 12 by .06, since 12 ÷ .06 = 200, it follows that:

Rule.—*When the percentage and rate are given, to find the base, divide the percentage by the rate.*

Formula, *base* = *percentage* ÷ *rate.*

EXAMPLE.— 76% of a lot of boiler tubes was used in the construction of a boiler. If the number of tubes used was 228, how many tubes were in the lot?

SOLUTION.—Here 228 is the percentage and .76 is the rate; hence, applying the rule,

$$228 \div .76 = 300 \text{ tubes.} \quad \text{Ans.}$$

16. When the base and percentage are given, to find the rate, the rate may be found by dividing the percentage by the base. For, suppose that it is desired to find what per cent. 12 is of 200. 1% of 200 is $200 \times .01 = 2$. Now, if 1% is 2, 12 is evidently as many per cent. as 2 is contained times in 12, or $12 \div 2 = 6\%$. But the same result may be obtained by dividing 12, the percentage, by 200, the base, since $12 \div 200 = .06 = 6\%$. Hence,

Rule.—*When the percentage and base are given, to find the rate, divide the percentage by the base, and the result will be the rate.*

Formula, *rate* = *percentage* ÷ *base*.

EXAMPLE 1.—Out of a lot of 300 boiler tubes 228 were used. What per cent. of the total number was used?

SOLUTION.—Here 300 is the base and 228 is the percentage; hence, applying rule,

$$\text{Rate} = 228 \div 300 = .76 = 76\%. \quad \text{Ans.}$$

EXAMPLE 2.—What per cent. of 875 is 25?

SOLUTION.—Here 875 is the base and 25 is the percentage; hence, applying rule,

$$\text{Rate} = 25 \div 875 = .02\tfrac{2}{7} = 2\tfrac{2}{7}\%. \quad \text{Ans.}$$

PROOF.— $875 \times .02\tfrac{2}{7} = 25$.

EXAMPLES FOR PRACTICE.

17. What per cent of:

(*a*)	360 is 90?	Ans. (*a*)	25%.
(*b*)	900 is 360?	(*b*)	40%.
(*c*)	125 is 25?	(*c*)	20%.
(*d*)	150 is 750?	(*d*)	500%.
(*e*)	280 is 112?	(*e*)	40%.
(*f*)	400 is 200?	(*f*)	50%.
(*g*)	47 is 94?	(*g*)	200%.
(*h*)	500 is 250?	(*h*)	50%.

18. The amount may be found, when the base and rate are given, by multiplying the base by 1 plus the rate expressed decimally. For, suppose that it is desired to find the amount when 200 is the base and .06 is the rate. The percentage is $200 \times .06 = 12$, and, according to definition, Art. **11**, the amount is $200 + 12 = 212$. But the same result may be obtained by multiplying 200 by $1 + .06$, or 1.06, since $200 \times 1.06 = 212$. Hence,

Rule.—*When the base and rate are given, to find the amount, multiply the base by 1 plus the rate.*

Formula, *amount* = *base* × (1 + *rate*).

EXAMPLE.—If a man earned $725 in a year, and the next year 10% more, how much did he earn the second year?

SOLUTION.—Here 725 is the base and .10 is the rate, and the amount is required. Hence, applying the rule,

$725 \times 1.10 = \$797.50$. **Ans.**

19. When the base and rate are given, the difference may be found by multiplying the base by 1 minus the rate expressed decimally. For, suppose that it is desired to find the difference when the base is 200 and the rate is 6%. The percentage is $200 \times .06 = 12$; and, according to definition, Art. **12**, the difference $= 200 - 12 = 188$. But the same result may be obtained by multiplying 200 by $1 - .06$, or .94, since $200 \times .94 = 188$. Hence,

Rule.—*When the base and rate are given, to find the difference, multiply the base by 1 minus the rate.*

Formula, *difference* = *base* × (1 − *rate*).

EXAMPLE.—Out of a lot of 300 boiler tubes all but 24% were used in one boiler; how many tubes were used?

SOLUTION.—Here 300 is the base, .24 is the rate, and it is desired to find the difference. Hence, applying the rule,

$300 \times (1 - .24) = 228$ tubes. **Ans.**

20. When the amount and rate are given, the base may be found by dividing the amount by 1 plus the rate. For, suppose that it is known that 212 equals some number

increased by 6% of itself. Then, it is evident that 212 equals 106% of the number (base) that it is desired to find. Consequently, if $212 = 106\%$, $1\% = \frac{212}{106} = 2$, and $100\% = 2 \times 100 = 200 =$ the base. But the same result may be obtained by dividing 212 by $1 + .06$ or 1.06, since $212 \div 1.06 = 200$. Hence,

Rule.—*When the amount and rate are given, to find the base, divide the amount by 1 plus the rate.*

Formula, *base* = *amount* ÷ (1 + *rate*).

EXAMPLE.—The theoretical discharge of a certain pump when running at a piston speed of 100 feet per minute is 278,910 gallons per day of 10 hours. Owing to leakage and other defects, this value is 25% greater than the actual discharge. What is the actual discharge?

SOLUTION.—Here 278,910 equals the actual discharge (base) increased by 25% of itself. Consequently, 278,910 is the amount and 25% is the rate. Applying rule,

Actual discharge = 278,910 ÷ 1.25 = 223,128 gal. Ans.

21. When the difference and rate are given, the base may be found by dividing the difference by 1 minus the rate. For, suppose that 188 equals some number less 6% of itself. Then, 188 evidently equals $100 - 6 = 94\%$ of some number. Consequently, if $188 = 94\%$, $1\% = 188 \div 94 = 2$, and $100\% = 2 \times 100 = 200$. But the same result may be obtained by dividing 188 by $1 - .06$, or .94, since $188 \div .94 = 200$. Hence,

Rule.—*When the difference and rate are given, to find the base, divide the difference by 1 minus the rate.*

Formula, *base* = *difference* ÷ (1 − *rate*).

EXAMPLE 1.—From a lot of boiler tubes 76% was used in the construction of a boiler. If there were 72 tubes unused, how many tubes were in the lot?

SOLUTION.—Here 72 is the difference and .76 is the rate. Applying rule,

72 ÷ (1 − .76) = 300 tubes. Ans.

EXAMPLE 2.—The theoretical number of foot-pounds of work per minute required to operate a boiler feed-pump is 127,344. If 30% of the total number actually required be allowed for friction, leakage, etc., how many foot-pounds are actually required to work the pump?

SOLUTION.—Here the number actually required is the base; hence, 127,344 is the difference and .30 is the rate. Applying the rule,

$$127{,}344 \div (1 - .30) = 181{,}920 \text{ foot-pounds.} \quad \text{Ans.}$$

22. EXAMPLE.—A certain chimney gives a draft of 2.76 inches of water. By increasing the height 20 feet, the draft was increased to 3 inches of water. What was the gain per cent.?

SOLUTION.—Here it is evident that 3 inches is the amount and that 2.76 inches is the base. Consequently, $3 - 2.76 = .24$ inch is the percentage, and it is required to find the rate. Hence, applying the rule given in Art. **16**,

$$\text{Gain per cent.} = .24 \div 2.76 = .087 = 8.7\%. \quad \text{Ans.}$$

23. EXAMPLE.—A certain chimney gave a draft of 3 inches of water. After an economizer had been put in, the draft was reduced to 1.2 inches of water. What was the loss per cent.?

SOLUTION.—Here it is evident that 1.2 inches is the difference (since it equals 3 inches diminished by a certain per cent. loss of itself) and 3 inches is the base. Consequently, $3 - 1.2 = 1.8$ inches is the percentage. Hence, applying the rule given in Art. **16**,

$$\text{Loss per cent.} = 1.8 \div 3 = .60 = 60\%. \quad \text{Ans.}$$

24. To find the gain or loss per cent.:

Rule.—*Find the difference between the initial and the final value; divide this difference by the initial value.*

EXAMPLE.—If a man buys a steam engine for $1,860 and some time afterwards purchases a condenser for 25% of the cost of the engine, does he gain or lose, and how much per cent., if he sells both engine and condenser for $2,100?

SOLUTION.—The cost of the condenser was $\$1{,}860 \times .25 = \$465$; consequently, the initial value, or cost, was $\$1{,}860 + \$465 = \$2{,}325$. Since he sold them for $2,100, he lost $\$2{,}325 - \$2{,}100 = \$225$. Hence, applying rule,

$$225 \div 2{,}325 = .0968 = 9.68\% \text{ loss.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

25. Solve the following:

(*a*)	What is 12½% of $900?	(*a*)	$112.50.
(*b*)	What is ⅛% of 627?	(*b*)	5.016.
(*c*)	What is 33⅓% of 54?	(*c*)	18.
(*d*)	101 is 68¾% of what number?	(*d*)	146$\frac{10}{11}$.
(*e*)	784 is 83⅓% of what number?	(*e*)	940.8.
(*f*)	What % of 960 is 160?	(*f*)	16⅔%.
(*g*)	What % of $3,606 is $450¾?	(*g*)	12½%.
(*h*)	What % of 280 is 112?	(*h*)	40%.

Ans.

1. A steam plant consumed an average of 3,640 pounds of coal per day. The engineer made certain alterations which resulted in a saving of 250 pounds per day. What was the per cent. of coal saved?

Ans. 7%, nearly.

2. If the speed of an engine running at 126 revolutions per minute should be increased 6½%, how many revolutions per minute would it then make? Ans. 134.19 rev.

3. The list price of an engine was $1,400; of a boiler, $1,150; and of the necessary fittings for the two, $340. If 25% discount was allowed on the engine, 22% on the boiler, and 12½% on the fittings, what was the actual cost of the plant? Ans. $2,244.50.

4. If I lend a man $1,100, and this is 18½% of the amount that I have on interest, how much money have I on interest? Ans. $5,945.95.

5. A test showed that an engine developed 190.4 horsepower, 15% of which was consumed in friction. How much power was available for use? Ans. 161.84 H. P.

6. By adding a condenser to a steam engine, the power was increased 14% and the consumption of coal per horsepower per hour was decreased 20%. If the engine could originally develop 50 horsepower and required 3½ pounds of coal per horsepower per hour, what would be the total weight of coal used in an hour with the condenser, assuming the engine to run full power? Ans. 159.6 lb.

DENOMINATE NUMBERS.

26. A **denominate number** is a concrete number, and may be either simple or compound; as, 8 quarts; 5 feet 10 inches, etc. Denominate numbers are also called **compound numbers.**

27. A **simple denominate number** consists of units of but one denomination; as, 16 cents; 10 hours; 5 dollars, etc.

28. A **compound denominate number** consists of units of two or more denominations of a similar kind; as, 3 yards 2 feet 1 inch.

29. In *whole numbers* and in *decimals*, the law of increase and decrease is on the scale of 10, but in *compound*, or *denominate*, *numbers* the scale varies.

MEASURES.

30. A **measure** is a *standard unit*, established by *law* or *custom*, by which quantity of any kind is measured. The standard unit of **dry measure** is the Winchester bushel; of **weight**, the pound; of **liquid measure**, the gallon, etc.

31. Measures are of six kinds:

1. Extension.
2. Weight.
3. Capacity.
4. Time.
5. Angles.
6. Money or value.

MEASURES OF EXTENSION.

32. **Measures of extension** are used in measuring lengths, distances, surfaces, and solids.

LINEAR MEASURE.

TABLE.

Abbreviation.			in.	ft.	yd.	rd.	fur.	mi.
12 inches (in.)	= 1 foot	ft.	36	= 3	= 1			
3 feet	= 1 yard	yd.	198	= 16½	= 5.5	= 1		
5.5 yards	= 1 rod	rd.	7,920	= 660	= 220	= 40	= 1	
40 rods	= 1 furlong	fur.	63,360	= 5,280	= 1,760	= 320	= 8	= 1
8 furlongs	= 1 mile	mi.						

SQUARE MEASURE.

TABLE.

144 square inches (sq. in.)	= 1 square foot	sq. ft.
9 square feet	= 1 square yard	sq. yd.
30¼ square yards	= 1 square rod	sq. rd.
160 square rods	= 1 acre	A.
640 acres	= 1 square mile	sq. mi.

sq. mi.	A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
1	= 640	= 102,400	= 3,097,600	= 27,878,400	= 4,014,489,600

CUBIC MEASURE.

TABLE.

1,728	cubic inches (cu. in.) . .	=	1 cubic foot	cu. ft.
27	cubic feet	=	1 cubic yard	cu. yd.
128	cubic feet	=	1 cord	cd.
24¾	cubic feet	=	1 perch	P.

cu. yd.		cu. ft.		cu. in.
1	=	27	=	46,656

MEASURES OF WEIGHT.

AVOIRDUPOIS WEIGHT.

TABLE.

16 ounces (oz.)	=	1 pound	lb.
100 pounds	=	1 hundredweight . . .	cwt.
20 cwt., or 2,000 lb.	=	1 ton	T.

T.		cwt.		lb.		oz.
1	=	20	=	2,000	=	32,000

33. The ounce is divided into halves, quarters, etc. Avoirdupois weight is used for weighing coarse and heavy articles.

LONG TON TABLE.

16 ounces	=	1 pound	lb.
112 pounds	=	1 hundredweight . . .	cwt.
20 cwt., or 2,240 lb.	=	1 ton	T.

34. In all the calculations hereafter, 2,000 pounds will be considered 1 ton, unless the long ton (2,240 pounds) is especially mentioned.

TROY WEIGHT.

TABLE.

24 grains (gr.)	=	1 pennyweight	pwt.
20 pennyweights	=	1 ounce	oz.
12 ounces	=	1 pound	lb.

lb.		oz.		pwt.		gr.
1	=	12	=	240	=	5,760

Troy weight is used in weighing gold and silver ware, jewels, etc. It is used by jewelers.

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40 rods	= 1 furlong	fur.	7,920 =	660 =	220 =	40 =	1	
8 furlongs	= 1 mile	mi.	63,360 =	5,280 =	1,760 =	320 =	8 =	1

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cu. yd.		cu. ft.		cu. in.
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MEASURES OF WEIGHT.

AVOIRDUPOIS WEIGHT.

TABLE.

16 ounces (oz.)	=	1 pound	lb.
100 pounds	=	1 hundredweight . . .	cwt.
20 cwt., or 2,000 lb.	=	1 ton	T.

T.		cwt.		lb.		oz.
1	=	20	=	2,000	=	32,000

33. The ounce is divided into halves, quarters, etc. Avoirdupois weight is used for weighing coarse and heavy articles.

LONG TON TABLE.

16 ounces	=	1 pound	lb.
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5.5 yards	= 1 rod	rd.	198 =	16½ =	5.5 =	1		
40 rods	= 1 furlong	fur.	7,920 =	660 =	220 =	40 =	1	
8 furlongs	= 1 mile	mi.	63,360 =	5,280 =	1,760 =	320 =	8 =	1

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sq. mi.	A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
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In an expression like 3^5, the exponent 5 shows how often 3 is used as a factor. Hence, if the exponent of a number is unity, the number is used once as a factor; thus, $3^1 = 3$, $4^1 = 4$, $5^1 = 5$.

3. If the side of a square contains 5 inches, the area of the square contains 5×5, or 5^2, square inches. If the edge of a cube contains 5 inches, the volume of the cube contains $5 \times 5 \times 5$, or 5^3, cubic inches. It is for this reason that 5^2 and 5^3 are called the square and cube of 5, respectively.

4. To find any power of a number:

EXAMPLE 1.—What is the third power, or cube, of 35?

SOLUTION.— $35 \times 35 \times 35$,

or

$$\begin{array}{r} 35 \\ 35 \\ \hline 175 \\ 105 \\ \hline 1225 \\ 35 \\ \hline 6125 \\ 3675 \\ \hline 42875 \end{array}$$

cube = 42875 Ans.

EXAMPLE 2.—What is the fourth power of 15?

SOLUTION.— $15 \times 15 \times 15 \times 15$,

or

$$\begin{array}{r} 15 \\ 15 \\ \hline 75 \\ 15 \\ \hline 225 \\ 15 \\ \hline 1125 \\ 225 \\ \hline 3375 \\ 15 \\ \hline 16875 \\ 3375 \\ \hline 50625 \end{array}$$

fourth power = 50625 Ans.

EXAMPLE 3.— 1.2^3 = what?

SOLUTION.— $1.2 \times 1.2 \times 1.2$,

or

$$\begin{array}{r} 1.2 \\ 1.2 \\ \hline 1.44 \\ 1.2 \\ \hline 288 \\ 144 \\ \hline \end{array}$$

cube = 1.728 Ans.

EXAMPLE 4.—What is the third power, or cube, of $\frac{3}{8}$?

SOLUTION.— $(\frac{3}{8})^3 = \frac{3^3}{8^3} = \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} = \frac{3 \times 3 \times 3}{8 \times 8 \times 8} = \frac{27}{512}$. Ans.

5. Rule.—I. *To raise a whole number or a decimal to any power, use it as a factor as many times as there are units in the exponent.*

II. *To raise a fraction to any power, raise both the numerator and denominator to the power indicated by the exponent.*

EXAMPLES FOR PRACTICE.

6. Raise the following to the powers indicated:

(*a*) 85^2.
(*b*) $(\frac{12}{13})^2$.
(*c*) 6.5^2.
(*d*) 14^4.
(*e*) $(\frac{3}{4})^3$.
(*f*) $(\frac{5}{6})^3$.
(*g*) $(\frac{7}{2})^3$.
(*h*) 1.4^5.

Ans. (*a*) 7,225. (*b*) $\frac{144}{169}$. (*c*) 42.25. (*d*) 38,416. (*e*) $\frac{27}{64}$. (*f*) $\frac{125}{216}$. (*g*) $\frac{343}{8}$. (*h*) 5.37824.

EVOLUTION.

7. Evolution is the reverse of involution. It is the process of finding the root of a number which is considered as a power.

8. The **square root** of a number is that number which, when used twice as a factor, produces the number.

Thus, 2 is the square root of 4, since 2×2, or $2^2 = 4$.

9. The **cube root** of a number is that number which, when used three times as a factor, produces the number.

Thus, 3 is the cube root of 27, since $3 \times 3 \times 3$, or $3^3 = 27$.

10. The **radical sign** $\surd$, when placed before a number, indicates that some root of that number is to be found. The vinculum is almost always used in connection with the radical sign, as shown in Art. **11.**

11. The **index** of the root is a *small figure* placed *over* and to the *left* of the *radical sign*, to show what root is to be found.

Thus, $\sqrt[2]{100}$ denotes the *square root* of 100.
$\sqrt[3]{125}$ denotes the *cube root* of 125.
$\sqrt[4]{256}$ denotes the *fourth root* of 256, and so on.

12. When the square root is to be extracted, the index is generally omitted. Thus, $\surd\overline{100}$ indicates the square root of 100. Also, $\surd\overline{225}$ indicates the square root of 225.

SQUARE ROOT.

13. The *largest* number that can be written with *one* figure is 9, and $9^2 = 81$; the *largest* number that can be written with *two* figures is 99, and $99^2 = 9{,}801$; with *three* figures 999, and $999^2 = 998{,}001$; with *four* figures 9,999, and $9{,}999^2 = 99{,}980{,}001$, etc.

In *each* of the above it will be noticed that the square of the number contains just *twice* as many figures as the number.

In order to find the square root of a number, the first step is to find how many figures there will be in the root. This is done by pointing off the number into *periods* of *two* figures each, *beginning at the right.* The number of periods will indicate the number of figures in the root.

Thus, the square root of 83,740,801 must contain 4 figures, since, pointing off the periods, we get 83'74'08'01, or 4 periods; consequently, there must be 4 figures in the root. In like manner, the square root of 50,625 must contain 3 figures,

since there are (5'06'25) 3 periods. The extreme left-hand period may contain either 1 or 2 figures, according to the size of the number squared.

14. EXAMPLE.—Find the square root of 31,505,769.

SOLUTION.—

				root	
				3 1'5 0'5 7'6 9 (5 6 1 3	Ans.
(*a*)	5				
	5	(*b*)	2 5		
(*d*)	1 0 0	(*c*)	6 5 0		
	6		6 3 6		
	1 0 6	(*e*)	1 4 5 7		
	6		1 1 2 1		
	1 1 2 0		3 3 6 6 9		
	1		3 3 6 6 9		
→	1 1 2 1				
	1				
	1 1 2 2 0				
	3				
	1 1 2 2 3				

EXPLANATION.—Pointing off into periods of two figures each, it is seen that there are 4 figures in the root. Now, find the largest single number whose square is less than or equal to 31, the first period. This is evidently 5, since $6^2 = 36$, which is greater than 31. Write it to the right, as in long division, and also to the left, as shown at (*a*). This is the first figure of the root. Now, multiply the 5 at (*a*) by the 5 in the root, and write the result under the first period, as shown at (*b*). Subtract and obtain 6 as a remainder.

Bring down the next period, 50, and annex it to the remainder 6, as shown at (*c*), which we call the **dividend.** Add the root already found to the 5 at (*a*), getting 10, and annex a cipher to this 10, thus making it 100, as shown at (*d*), which we call the **trial divisor.** Divide the dividend (*c*) by the trial divisor (*d*) and obtain 6, which is *probably* the next figure of the root. Write 6 in the root, as shown, and also add it to 100, the trial divisor, making it 106. This is called the **complete divisor.**

Multiply this by 6, the second figure in the root, and subtract the result from the dividend (*c*). The remainder is 14, to which annex the next period, making it 1457, as shown at (*e*), which we call the **new dividend.** Add the second figure of the root to the complete divisor, 106, and annex a cipher, thus getting 1120. Dividing 1457 by 1120, we get 1 as the next figure of the root. Adding this last figure of the root to 1120, multiplying the result by it, and subtracting from 1457, the remainder is 336.

Annexing the next and last period, 69, the result is 33669. Now, adding the last figure of the root to 1121 and annexing a cipher as before, the result is 11220. Dividing 33669 by 11220, the result is 3, the fourth figure in the root. Adding it to 11220 and multiplying the sum by it, the result is 33669. Subtracting, there is no remainder; hence, $\sqrt{31,505,769} = 5,613$.

15. The square of any number wholly decimal always contains twice as many figures as the number squared. For example, $.1^2 = .01$, $.13^2 = .0169$, $.751^2 = .564001$, etc.

16. It will also be noticed that the number squared is always less than the decimal. Hence, the square root of a number wholly decimal is greater than the number itself. If it be required to find the square root of a decimal, and the decimal has not an even number of figures in it, annex a cipher. The best way to determine the number of figures in the root of a decimal is to begin at the decimal point, and, going towards the *right*, point off the decimal into periods of two figures each. Then, if the last period contains but one figure, annex a cipher.

17. EXAMPLE.—What is the square root of .000576?

SOLUTION.—

		root	
	2	.00'05'76 (.024	Ans.
	2	4	
	40	176	
	4	176	
	44		

EXPLANATION.—Beginning at the decimal point and pointing off the number into periods of two figures each, it is seen that the first period is composed of ciphers; hence, the first figure of the root must be a cipher. The remaining portion of the solution should be perfectly clear from what has preceded.

18. If the number is not a perfect power, the root will consist of an interminable number of decimal places. The result may be carried to any required number of decimal places by annexing periods of two ciphers each to the number.

19. EXAMPLE.—What is the square root of 3? Find the result to five decimal places.

SOLUTION.—

		root	
1	3.00'00'00'00'00	1.732 05+	Ans.
1	1		
20	200		
7	189		
27	1100		
7	1029		
340	7100		
3	6924		
343	176 0000		
3	173 2025		
3460	27975		
2			
3462			
2			
346400			
5			
346405			

EXPLANATION.—Annexing 5 periods of two ciphers each to the right of the decimal point, the first figure of the root is 1. To get the second figure, we find that, in dividing 200 by 20, it is 10. This is evidently too large.

Trying 9, we add 9 to 20 and multiply 29 by 9; the result is 261, a result which is considerably larger than 200; hence,

9 is too large. In the same way, it is found that 8 is also too large. Trying 7, 7 times 27 are 189, a result smaller than 200; therefore, 7 is the second figure of the root. The next two figures, 3 and 2, are easily found. The fifth figure in the root is a cipher, since the trial divisor, 34640, is greater than the new dividend, 17600. In a case of this kind we annex another cipher to 34640, thereby making it 346400, and bring down the next period, making the 17600, 1760000. The next figure of the root is 5, and, as we now have five decimal places, we will stop.

The square root of 3 is, then, 1.73205+.

20. EXAMPLE.—What is the square root of .3 to five decimal places?

SOLUTION.—

			root
5	.3 0'0 0'0 0'0 0'0 0	(.5 4 7 7 2+	Ans.
5	2 5		
100	5 0 0		
4	4 1 6		
104	8 4 0 0		
4	7 6 0 9		
1080	7 9 1 0 0		
7	7 6 6 2 9		
1087	2 4 7 1 0 0		
7	2 1 9 0 8 4		
10940	2 8 0 1 6		
7			
10947			
7			
109540			
2			
109542			

EXPLANATION.—In the above example we annex a cipher to .3, making the first period .30, since every period of a decimal, as was mentioned before, must have two figures in it. The remainder of the work should be perfectly clear.

21. If it is required to find the square root of a mixed number, begin at the decimal point and point off the periods

both to the right and to the left. The manner of finding the root will then be exactly the same as in the previous cases.

22. EXAMPLE.—What is the square root of 258.2449?

SOLUTION.—

			root
1		2'5 8.2 4'4 9	(1 6.0 7 Ans.
1		1	
20		158	
6		156	
26		22449	
6		22449	
3200			
	7		
3207			

EXPLANATION.—In the above example, since 320 is greater than 224, we place a cipher for the third figure of the root and annex a cipher to 320, making it 3200. Then, bringing down the next period, 49, 7 is found to be the fourth figure of the root. Since there is no remainder, the square root of 258.2449 is 16.07.

23. Proof. — *To prove square root, square the result obtained. If the number is an exact power, the square of the root will equal it; if it is not an exact power, the square of the root will very nearly equal it.*

24. Rule.—I. *Begin at units place and separate the number into periods of two figures each, proceeding from left to right with the decimal part, if there be any.*

II. *Find the greatest number whose square is contained in the first, or left-hand, period. Write this number as the first figure in the root; also, write it at the left of the given number.*

Multiply this number at the left by the first figure of the root, and subtract the result from the first period; then, annex the second period to the remainder.

III. *Add the first figure of the root to the number in the first column on the left and annex a cipher to the result; this is the trial divisor. Divide the dividend by the trial*

divisor for the second figure in the root and add this figure to the trial divisor to form the complete divisor. Multiply the complete divisor by the second figure in the root and subtract this result from the dividend. (*If this result is larger than the dividend, a smaller number must be tried for the second figure of the root.*) *Now bring down the third period and annex it to the last remainder for a new dividend. Add the second figure of the root to the complete divisor and annex a cipher for a new trial divisor.*

IV. *Continue in this manner to the last period, after which, if any additional places in the root are required, bring down cipher periods and continue the operation.*

V. *If at any time the trial divisor is not contained in the dividend, place a cipher in the root, annex a cipher to the trial divisor, and bring down another period.*

VI. *If the root contains an interminable decimal and it is desired to terminate the operation at some point, say, the fourth decimal place, carry the operation one place farther, and if the fifth figure is 5 or greater, increase the fourth figure by 1 and omit the sign* $+$.

25. Short Method.—If the number whose root is to be extracted is not an exact square, the root will be an interminable decimal. It is then usual to extract the root to a certain number of decimal places. In such cases, the work may be greatly shortened as follows: Determine to how many decimal places the work is to be carried, say 5, for example; add to this the number of places in the integral part of the root, say 2, for example, thus determining the number of *figures* in the root, in this case $5 + 2 = 7$. Divide this number by 2 and take the next higher number. In the above case, we have $7 \div 2 = 3\frac{1}{2}$; hence, we take 4, the next higher number. Now extract the root in the usual manner until the same number of figures have been obtained as was expressed by the number obtained above, in this case 4. Then form the trial divisor in the usual manner, but omitting to annex the cipher; divide the last remainder by the trial divisor, as in long division, obtaining as many figures

of the quotient as there are remaining figures of the root, in this case $7 - 4 = 3$. The quotient so obtained is the remaining figures of the root.

Consider the example in Art. **20.** Here there are 5 figures in the root. We therefore extract the root to 3 figures in the usual manner, obtaining .547 for the first three root figures. The next trial divisor is 1094 (with the cipher omitted) and the last remainder is 791. Then, $791 \div 1094 = .723$, and the next two figures of the root are 72, the whole root being .54772+. Always carry the division one place farther than desired, and if the last figure is 5 or greater, increase the preceding figure by 1. This method should not be used unless the root contains five or more figures.

NOTE.—If the last figure of the root found in the regular manner is a cipher, carry the process one place farther before dividing as described above.

EXAMPLES FOR PRACTICE.

26. Find the square root of :

(*a*)	186,624.	(*a*)	432.
(*b*)	2,050,624.	(*b*)	1,432.
(*c*)	29,855,296.	(*c*)	5,464.
(*d*)	.0116964.	(*d*)	.1081+.
(*e*)	198.1369.	(*e*)	14.0761.
(*f*)	994,009.	(*f*)	997.
(*g*)	2.375 to four decimal places.	(*g*)	1.5411.
(*h*)	1.625 to three decimal places.	(*h*)	1.275.
(*i*)	.3025.	(*i*)	.55.
(*j*)	.571428.	(*j*)	.7559+.
(*k*)	.78125.	(*k*)	.8839.

Ans.

CUBE ROOT.

27. By a method similar to the foregoing for extracting square root, the cube root of any number may be found. The labor involved in finding the cube root of a number is so great and the necessity of extracting the cube root so seldom arises that we have deemed it best to omit the exact method and substitute instead an approximate method,

which will answer the student's purposes fully as well as the exact method, and is far easier to learn and apply.

28. In applying this approximate method, we find the first three or four figures of the root by means of a table in a manner to be described presently and then find more figures, if necessary, as described in Arts. **35** and **36.**

29. In any number, the figures beginning with the first digit* at the left and ending with the last digit at the right are called the **significant figures** of the number. Thus, the number 405,800 has the four significant figures 4, 0, 5, 8; and the number .000090067 has the five significant figures 9, 0, 0, 6, and 7.

The part of a number consisting of its significant figures is called the **significant part** of the number. Thus, in the number 28,070, the significant part is 2807; in the number .00812, the significant part is 812; and in the number 170.3, the significant part is 1703.

In speaking of the significant figures or of the significant part of a number, we consider the figures, in their proper order, from the first digit at the left to the last digit at the right, but we pay no attention to the position of the decimal point. Hence, *all numbers that differ only in the position of the decimal point have the same significant part.* For example, .002103, 21.03, 21,030, and 210,300 have the same significant figures 2, 1, 0, and 3, and the same significant part 2103.

30. In the same manner as in the case of square root, it can be shown that the periods into which a number is divided whose cube root is to be extracted, must contain three figures, except that the first (left-hand) period of a whole or mixed number may contain one, two, or three figures. Hence, the first step in extracting the cube root is to point off the number into periods of three figures each, beginning at the decimal point and proceeding to the left and to the right. The remaining steps are best illustrated by an example.

* A cipher is not a digit.

SQUARES AND CUBES.

No.	Square.	Cube.	No.	Square.	Cube.	No.	Square.	Cube.
1	1	1	34	11'56	39'304	67	44'89	300'763
2	4	8	35	12'25	42'875	68	46'24	314'432
3	9	27	36	12'96	46'656	69	47'61	328'509
4	16	64	37	13'69	50'653	70	49'00	343'000
5	25	125	38	14'44	54'872	71	50'41	357'911
6	36	216	39	15'21	59'319	72	51'84	373'248
7	49	343	40	16'00	64'000	73	53'29	389'017
8	64	512	41	16'81	68'921	74	54'76	405'224
9	81	729	42	17'64	74'088	75	56'25	421'875
10	1'00	1'000	43	18'49	79'507	76	57'76	438'976
11	1'21	1'331	44	19'36	85'184	77	59'29	456'533
12	1'44	1'728	45	20'25	91'125	78	60'84	474'552
13	1'69	2'197	46	21'16	97'336	79	62'41	493'039
14	1'96	2'744	47	22'09	103'823	80	64'00	512'000
15	2'25	3'375	48	23'04	110'592	81	65'61	531'441
16	2'56	4'096	49	24'01	117'649	82	67'24	551'368
17	2'89	4'913	50	25'00	125'000	83	68'89	571'787
18	3'24	5'832	51	26'01	132'651	84	70'56	592'704
19	3'61	6'859	52	27'04	140'608	85	72'25	614'125
20	4'00	8'000	53	28'09	148'877	86	73'96	636'056
21	4'41	9'261	54	29'16	157'464	87	75'69	658'503
22	4'84	10'648	55	30'25	166'375	88	77'44	681'472
23	5'29	12'167	56	31'36	175'616	89	79'21	704'969
24	5'76	13'824	57	32'49	185'193	90	81'00	729'000
25	6'25	15'625	58	33'64	195'112	91	82'81	753'571
26	6'76	17'576	59	34'81	205'379	92	84'64	778'688
27	7'29	19'683	60	36'00	216'000	93	86'49	804'357
28	7'84	21'952	61	37'21	226'981	94	88'36	830'584
29	8'41	24'389	62	38'44	238'328	95	90'25	857'375
30	9'00	27'000	63	39'69	250'047	96	92'16	884'736
31	9'61	29'791	64	40'96	262'144	97	94'09	912'673
32	10'24	32'768	65	42'25	274'625	98	96'04	941'192
33	10'89	35'937	66	43'56	287'496	99	98'01	970'299

EXAMPLE.—Extract the cube root of 160,524.

SOLUTION. — Pointing off into periods of three figures each, we have 160'524, two periods of three figures each; hence, the whole-number part of the root will contain two figures. Referring to the table of Squares and Cubes, and looking in the columns headed "*Cube*" (these columns contain the cubes of the numbers in the

columns headed "No."), we see that the given number falls between 157,464, the cube of 54, and 166,375, the cube of 55. Hence, the required cube root is 54+. Now find the difference between the two numbers in the table between which the given number falls, and call the result the **first difference,** in this case 166,375 − 157,464 = 8,911. Also find the difference between the given number and the smaller of the two numbers in the table between which it falls, and call the result the **second difference,** in this case 160,524 − 157,464 = 3,060. Now divide the second difference by the first difference and carry the result to three decimal places, increasing the second figure of the decimal by 1 if the third figure is 5 or greater, thus: 3,060 ÷ 8,911 = .343+, or .34. The decimal .34 is the next two figures of the root. Hence, the cube root of 160,524, or $\sqrt[3]{160,524}$, is 54.34 to four significant figures. Ans.

31. Had the given number been 160.524, .160524, .000160524, or 160,524,000, the figures of the root would have been exactly the same, the position of the decimal point only being changed, the reason being that there are the *same number of figures in the left-hand period of the significant part of the given number.* Hence, $\sqrt[3]{160.524} = 5.434+$, $\sqrt[3]{.160524} = .5434+$, $\sqrt[3]{.000160524} = .05434+$, and $\sqrt[3]{160,524,000} = 543.4+$.

Suppose, however, that the given number had been 16,052.4, 16.0524, or .0160524, etc.; then, pointing if off into periods, in the usual manner, we should have 16'052.400, 16.052'400, or .016'052'400, etc., and the left-hand period of the significant part of the number contains only two significant figures, instead of three, as in the former case; in other words, the left-hand period is 16 instead of 160. Since the cube root of 16 is entirely different from the cube root of 160, it is evident that great care must be taken to point off the periods correctly.

EXAMPLE.— $\sqrt[3]{16,052.4} = ?$

SOLUTION.—Pointing off into periods of three figures each, we have 16'052.400, annexing two ciphers to complete the right-hand decimal period. Referring to the table, we see that only two periods are given; we therefore drop the third period and find the cube root of 16'052. The period dropped will in no case affect the root when only four figures are found; but had the period dropped been 500 or larger, 1 would have been added to the last figure of the second period. Referring to the table, 16'052 lies between 15'625, the cube of 25, and 17'576, the cube

of 26. The first difference is 17,576 − 15,625 = 1,951; the second difference is 16,052 − 15,625 = 427; and 427 ÷ 1,951 = .218+, or .22. Hence, $\sqrt[3]{16,052.4} = 25.22$ to four figures. Ans.

32. EXAMPLE.—What is the cube root of .003 ?

SOLUTION.—Since the number is entirely decimal, the root is entirely decimal. The given number has but one period and but one figure in that period to be considered, since the two ciphers on the left are not significant figures. The figures of the root will be exactly the same as those in the cube root of 3 or the cube root of 3,000; hence, for convenience, we neglect the decimal point for the present and find the cube root of 3,000. Referring to the table, 3'000 lies between 2'744, the cube of 14, and 3'375, the cube of 15; the first two figures of the root are, therefore, 14. The first difference is 3,375 − 2,744 = 631; the second difference is 3,000 − 2,744 = 256; and 256 ÷ 631 = .405+ or .41. Therefore, the first four figures of the root are 1441. To locate the decimal point, we employ the following principle: *The cube root of any number wholly decimal will have as many ciphers between the decimal and the first significant figure of the root as there are cipher periods of three ciphers each between decimal point and the first significant figure of the given number.* In the present example there are no cipher periods between the decimal point of the given number, .003, and the first significant figure; hence, $\sqrt[3]{.003} = .1441$. Ans.

33. We extracted the cube root of .003 by assuming it to be 3,000 instead of 3, in order to obtain two periods and get two figures of the root from the table. The fourth figure of the root just found is not quite correct—it ought to have been 2 instead of 1—in other words, the fourth figure cannot always be relied upon as being absolutely exact; it may be 1 or 2 less.

34. Rule.—*Point off the number into periods of three figures each, beginning at the decimal point, proceeding to the left and right. Consider the first two periods and find in the table in the columns headed "Cube" the two numbers between which the first two periods of the given number falls. The first two figures of the root will be found opposite the smaller of the two numbers in the nearest column to the left, headed "No." Find the difference between the two numbers in the column headed "Cube"; also find the difference between the smaller of these numbers and the first two periods of the given number. Divide the second difference by the first, carrying*

the quotient to three figures and increasing the second figure by 1, if the third figure is 5 or greater. The first two figures of this quotient will be the third and fourth figures of the root.

35. As stated in Art. **33,** the fourth figure cannot always be depended upon; hence, three figures of the root are all that we can be absolutely sure of as being correct. If it should be necessary to find more than three or four figures of the root, it can be easily done as follows:

Find the first four figures as above described and locate the decimal point. Divide the given number by the root as found and carry the quotient to 6 or 7 figures. Divide this quotient by the root, and carry the quotient to 6 figures. Then add the last quotient and the two divisors together and divide the sum by 3; the result will be the root correct to 5 or 6 figures.

36. EXAMPLE.—Find the cube root of 160,524 to 7 figures.

SOLUTION.—In Art. **30,** the root to four figures was found to be 54.34. Proceeding as in Art. **35,** $160{,}524 \div 54.34 = 2{,}954.067$; $2{,}954.067 \div 54.34 = 54.3626$; $54.34 + 54.34 + 54.36266 = 163.04266$; $163.0426 \div 3 = 54.34755$. Therefore, $\sqrt[3]{160{,}524} = 54.34755$, correct to 7 figures. Ans.

37. The method illustrated in the example of Art. **37** is perfectly general, but if 5 figures only of the root are required, the fourth and fifth figures may be determined in an easier manner by adding a correction to the root as found by applying the rule of Art. **34.** The correction is determined as follows: When dividing the second difference by the first difference carry the quotient to 4 decimal places, increasing the fourth figure by 1 if the fifth figure is 5 or greater. Consider the quotient as a decimal and subtract it from 1. Multiply the remainder by the quotient and divide the product by the first two figures of the root, as found from the table, carrying the quotient to four decimal places. Then add this quotient to the quotient previously found, and the sum will be the third, fourth, and fifth figures of the root. Thus, referring to Art. **30,** the quotient is .3434; $1 - .3434 = .6566$; $.3434 \times .6566 \div 54 = .0042$; $.3434 + .0042 = .3476$. The root is, therefore, 54.348 to five figures.

RATIO.

38. Suppose that it is desired to compare two numbers, say 20 and 4. If we wish to know how many times larger 20 is than 4, we divide 20 by 4 and obtain 5 for the quotient; thus, $20 \div 4 = 5$. Hence, we say that 20 is 5 times as large as 4, i. e., 20 contains 5 times as many units as 4. Again, suppose we desire to know what part of 20 is 4. We then divide 4 by 20 and obtain $\frac{1}{5}$; thus, $4 \div 20 = \frac{1}{5}$, or .2. Hence, 4 is $\frac{1}{5}$, or .2, of 20. This operation of comparing two numbers is termed *finding the ratio* of the two numbers. Ratio, then, is a comparison. It is evident that the two numbers to be compared must be expressed in the same unit; in other words, the two numbers must both be abstract numbers or concrete numbers of the same kind. For example, it would be absurd to compare 20 horses with 4 birds, or 20 horses with 4. Hence, **ratio** may be defined as a comparison between two numbers of the same kind.

39. A ratio may be *expressed* in three ways; thus, if it is desired to compare 20 and 4 and express this comparison as a ratio, it may be done as follows: $20 \div 4$; $20 : 4$, or $\frac{20}{4}$. All three are read *the ratio of 20 to 4*. The ratio of 4 to 20 would be expressed thus: $4 \div 20$; $4 : 20$, or $\frac{4}{20}$. The first method of expressing a ratio, although correct, is seldom or never used; the second form is the one oftenest met with, while the third form, called the fractional form, possesses great advantages to students of algebra and of higher mathematical subjects. The second form is better adapted to arithmetical subjects and is one we shall ordinarily adopt.

40. The **terms** of a ratio are the two numbers to be compared; thus, in the above ratio, 20 and 4 are the terms. When both terms are considered together, they are called a **couplet**; when considered separately, the first term is called the **antecedent** and the second term the **consequent**. Thus, in the ratio 20 : 4, 20 and 4 form a couplet, and 20 is the antecedent and 4 the consequent.

41. A ratio may be **direct** or **inverse.** The *direct ratio* of 20 to 4 is 20 : 4, while the *inverse ratio* of 20 to 4 is 4 : 20. The direct ratio of 4 to 20 is 4 : 20, and the inverse ratio is 20 : 4. An inverse ratio is sometimes called a **reciprocal ratio.** The **reciprocal** of a number is 1 divided by the number. Thus, the reciprocal of 17 is $\frac{1}{17}$; of $\frac{3}{8}$ is $1 \div \frac{3}{8} = \frac{8}{3}$; i. e., the reciprocal of a fraction is the fraction inverted. Hence, the inverse ratio of 20 to 4 may be expressed as 4 : 20, or as $\frac{1}{20} : \frac{1}{4}$. Both have equal values; for, $4 \div 20 = \frac{1}{5}$, and $\frac{1}{20} \div \frac{1}{4} = \frac{1}{20} \times \frac{4}{1} = \frac{1}{5}$.

42. The term **vary** implies a ratio. When we say that two numbers vary as some other two numbers, we mean that the ratio between the first two numbers is the same as the ratio between the other two numbers.

43. The **value** of a ratio is the result obtained by performing the division indicated. Thus, the value of the ratio 20 : 4 is 5—it is the quotient obtained by dividing the antecedent by the consequent. The value of a ratio is always an abstract number, regardless of whether the terms are abstract or concrete numbers

44. When a ratio is expressed in words, as the ratio of 20 to 4, the first number named is always regarded as the antecedent and the second as the consequent, without regard to whether the ratio itself is direct or inverse. *When not otherwise specified, all ratios are understood to be direct.* To express an inverse ratio, the simplest way of doing it is to express it as if it were a direct ratio, with the first number named as the antecedent, and then transpose the antecedent to the place occupied by the consequent and the consequent to the place occupied by the antecedent; or if expressed in the fractional form, invert the fraction. Thus, to express the inverse ratio of 20 to 4, first write it 20 : 4, and then, transposing the terms, as 4 : 20; or as $\frac{20}{4}$, and then inverting, as $\frac{4}{20}$. Or, the reciprocals of the numbers may be taken, as explained above. To **invert** a ratio is to transpose its terms.

45. Instead of expressing the value of a ratio by a single number, as above, it is convenient to express it by means of another ratio in which the consequent is 1. Thus, suppose that it is desired to find the ratio of the weights of two pieces of iron, one weighing 45 pounds and the other weighing 30 pounds. The ratio of the heavier to the lighter is then 45 : 30, an inconvenient expression. Using the fractional form, we have $\frac{45}{30}$. Dividing both terms by 30,* the consequent, we obtain $\frac{1\frac{1}{2}}{1}$, or $1\frac{1}{2} : 1$. This is the same result as obtained above, for $1\frac{1}{2} \div 1 = 1\frac{1}{2}$, and $45 \div 30 = 1\frac{1}{2}$.

PROPORTION.

46. **Proportion** is an equality of ratios, the equality being indicated by the double colon (::) or by the sign of equality (=). Thus, to write in the form of a proportion the two equal ratios, 8 : 4 and 6 : 3, which both have the same value, 2, we may employ one of the three following forms:

$$8 : 4 :: 6 : 3 \qquad (1)$$

$$8 : 4 = 6 : 3 \qquad (2)$$

$$\frac{8}{4} = \frac{6}{3} \qquad (3)$$

47. The first form is the one most extensively used, by reason of its having been exclusively employed in all the older works on mathematics. The second and third forms are being adopted by all modern writers on mathematical subjects, and in time will probably entirely supersede the first form. In this arithmetic we shall adopt the second form, unless some statement can be made clearer by using the third form.

* This evidently does not alter the value of the ratio, since by the laws of fractions, both numerator and denominator may be divided by the same number without changing the value of the fraction.

48. A proportion may be *read* in two ways. The old way to read the above proportion was: *8 is to 4 as 6 is to 3;* the new way is: *the ratio of 8 to 4 equals the ratio of 6 to 3.* The student may read it either way, but we recommend the latter.

49. Each ratio of a proportion is termed a **couplet.** In the above proportion, 8 : 4 is a couplet, and so is 6 : 3.

50. The numbers forming the proportion are called **terms;** and they are numbered consecutively from left to right, thus:

first second third fourth

8 : 4 = 6 : 3

Hence, in any proportion, the ratio of the first term to the second term equals the ratio of the third term to the fourth term.

51. The first and fourth terms of a proportion are called the **extremes,** and the second and third terms the **means.** Thus, in the foregoing proportion, 8 and 3 are the extremes and 4 and 6 are the means.

52. A **direct proportion** is one in which both couplets are direct ratios.

53. An **inverse proportion** is one which requires one of the couplets to be expressed as an inverse ratio. Thus, 8 is to 4 inversely as 3 is to 6 must be written 8 : 4 = 6 : 3; i. e., the second ratio (couplet) must be inverted.

54. Proportion forms one of the most useful sections of arithmetic. In our grandfathers' arithmetics, it was called "The rule of three."

55. **Rule.**—*In any proportion, the product of the extremes equals the product of the means.*

Thus, in the proportion,

17 : 51 = 14 : 42.

$17 \times 42 = 51 \times 14$, since both products equal 714.

56. **Rule.**—*The product of the extremes divided by either mean gives the other mean.*

EXAMPLE.—What is the third term of the proportion 17 : 51 = : 42?

SOLUTION.—Applying the rule, $17 \times 42 = 714$, and $714 \div 51 = 14$.

Ans.

57. Rule.—*The product of the means divided by either extreme gives the other extreme.*

EXAMPLE.—What is the first term of the proportion : 51 = 14 : 42?

SOLUTION.—Applying the rule, $51 \times 14 = 714$, and $714 \div 42 = 17$.

Ans.

58. When stating a proportion in which one of the terms is unknown, represent the missing term by a letter, as x. Thus, the last example would be written

$$x : 51 = 14 : 42,$$

and for the value of x we have $x = \frac{51 \times 14}{42} = 17$.

59. The principle of all calculations in proportion is this: *Three of the terms are always given and the remaining one is to be found.*

60. EXAMPLE.—If 4 men can earn $25 in one week, how much can 12 men earn in the same time?

SOLUTION.—The required term must bear the same relation to the given term of the same kind as one of the remaining terms bears to the other remaining term. We can then form a proportion by which the required term may be found.

The first question the student must ask himself in every calculation by proportion is: "What is it I want to find?" In this case it is dollars. We have two sets of men, one set earning $25, and we want to know how many dollars the other set earns. It is evident that the *amount* 12 men earn bears the same relation to the *amount* that 4 men earn as 12 men bears to 4 men. Hence, we have the proportion, the amount 12 men earn is to $25 as 12 men is to 4 men; or, since either extreme equals the product of the means divided by the other extreme, we have

The amount 12 men earn : $25 = 12 men : 4 men,

or the amount 12 men earn $= \frac{\$25 \times 12}{4} = \$75$. Ans.

Since it matters not which place x, or the required term, occupies, the problem could be stated in any of the following forms, the value of x being the same in each:

(*a*) \$25 : the amount 12 men earn = 4 men : 12 men; or the amount 12 men earn $= \frac{\$25 \times 12}{4}$, or \$75, since either mean equals the product of the extremes divided by the other mean.

(*b*) 4 men : 12 men = \$25 : the amount 12 men earn; or the amount that 12 earn $= \frac{\$25 \times 12}{4}$, or \$75, since either extreme equals the product of the means divided by the other extreme.

(*c*) 12 men : 4 men = the amount 12 men earn : \$25; or the amount that 12 men earn $= \frac{\$25 \times 12}{4}$, or \$75, since either mean equals the product of the extremes divided by the other mean.

61. If the proportion is an inverse one, first form it as though it were a direct proportion and then invert one of the couplets.

EXAMPLES FOR PRACTICE.

62. Find the value of x in each of the following:

(*a*)	\$16 : \$64 = x : \$4.	Ans.	(*a*) x = \$1.
(*b*)	x : 85 = 10 : 17.		(*b*) x = 50.
(*c*)	24 : x = 15 : 40.		(*c*) x = 64.
(*d*)	18 : 94 = 2 : x.		(*d*) $x = 10\frac{4}{9}$.
(*e*)	\$75 : \$100 = x : 100.		(*e*) x = 75.
(*f*)	15 pwt. : x = 21 : 10.		(*f*) $x = 7\frac{1}{7}$ pwt.
(*g*)	x : 75 yd. = \$15 : \$5.		(*g*) x = 225 yd.

1. If 75 pounds of lead cost \$2.10, what would 125 pounds cost at the same rate? Ans. \$3.50.

2. If A does a piece of work in 4 days and B does it in 7 days, how long will it take A to do what B does in 63 days? Ans. 36 days.

3. The circumferences of any two circles are to each other as their diameters. If the circumference of a circle 7 inches in diameter is 22 inches, what will be the circumference of a circle 31 inches in diameter? Ans. $97\frac{3}{7}$ in.

INVERSE PROPORTION.

63. In Art. **53**, an inverse proportion was defined as one which required one of the couplets to be expressed as an inverse ratio. Sometimes the word *inverse* occurs in the statement of the example; in such cases the proportion can be written directly, merely inverting one of the couplets.

But it frequently happens that only by carefully studying the conditions of the example can it be ascertained whether the proportion is direct or inverse. When in doubt, the student can always satisfy himself as to whether the proportion is direct or inverse by first ascertaining what is required, and stating the proportion as a direct proportion. Then, in order that the proportion may be true, *if the first term is smaller than the second term, the third term must be smaller than the fourth; or if the first term is larger than the second term, the third term must be larger than the fourth term.* Keeping this in mind, the student can always tell whether the required term will be larger or smaller than the other term of the couplet to which the required term belongs. Having determined this, the student then refers to the example and ascertains from its conditions whether the required term is to be larger or smaller than the other term of the same kind. If the two determinations agree, the proportion is direct; otherwise, it is inverse, and one of the couplets must be inverted.

64. EXAMPLE.—A's *rate* of doing work is to B's as 5 : 7; if A does a piece of work in 42 days, in what time will B do it?

SOLUTION.—The required term is the number of days it will take B to do the work. Hence, stating as a direct proportion,

$$5 : 7 = 42 : x.$$

Now, since 7 is greater than 5, x will be greater than 42. But, referring to the statement of the example, it is easy to see that B works faster than A; hence it will take B a *less* number of days to do the work than A. Therefore, the proportion is an inverse one, and should be stated

$$5 : 7 = x : 42,$$

from which $x = \frac{5 \times 42}{7} = 30$ days. Ans.

Had the example been stated thus: The time that A requires to do a piece of work is to the time that B requires, as 5 : 7; A can do it in 42 days, in what time can B do it? it is evident that it would take B a longer time to do the work than it would A; hence, x would be greater than 42, and the proportion would be direct, the value of x being $\frac{7 \times 42}{5} = 58.8$ days.

MENSURATION AND USE OF LETTERS IN FORMULAS.

FORMULAS.

1. The term **formula,** as used in mathematics and in technical books, may be defined as *a rule in which symbols are used instead of words;* in fact, a formula may be regarded as a shorthand method of expressing a rule. Any formula can be expressed in words, and when so expressed it becomes a rule.

Formulas are much more convenient than rules; they show at a glance all the operations that are to be performed; they do not require to be read three or four times, as is the case with most rules, to enable one to understand their meaning; they take up much less space, both in the printed book and in one's note book, than rules; in short, whenever a rule can be expressed as a formula, the formula is to be preferred.

As the term "quantity" is a very convenient one to use, we will define it. In mathematics, the word **quantity** is applied to anything that it is desired to subject to the ordinary operations of addition, subtraction, multiplication, etc., when we do not wish to be more specific and state exactly what the thing is. Thus, we can say "two or more numbers," or "two or more quantities"; the word quantity is more general in its meaning than the word number.

2. The signs used in formulas are the ordinary signs indicative of operations and the signs of aggregation. All

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7. There is one other sign used, which is neither a sign of aggregation nor a sign indicative of an operation to be performed; it is (=), and is called the sign of **equality**; it means that all on one side of it is exactly equal to all on the other side. For example, $2 = 2$, $5 - 3 = 2$, $5 \times (14 - 9) = 25$.

8. Having called particular attention to certain signs used in formulas, the formulas themselves will now be explained. First, consider the well-known rule for finding the horsepower of a steam engine, which may be stated as follows:

Divide the continued product of the mean effective pressure in pounds per square inch, the length of the stroke in feet, the area of the piston in square inches, and the number of strokes per minute, by 33,000; the result will be the horsepower.

This is a very simple rule, and very little, if anything, will be saved by expressing it as a formula, so far as clearness is concerned. The formula, however, will occupy a great deal less space, as we shall show.

An examination of the rule will show that four quantities (viz., the mean effective pressure, the length of the stroke, the area of the piston, and the number of strokes) are multiplied together, and the result is divided by 33,000. Hence, the rule might be expressed as follows:

$$\text{Horsepower} = \frac{\text{mean effective pressure (in pounds per square inch)} \times \text{stroke (in feet)} \times \text{area of piston (in square inches)} \times \text{number of strokes (per minute)}}{33{,}000}.$$

This expression could be shortened by representing each quantity by a single letter; thus, representing horsepower by the letter "H," the mean effective pressure in pounds per square inch by "P," the length of stroke in feet by "L," the area of the piston in square inches by "A," the number of strokes per minute by "N," and substituting these letters for the quantities that they represent, the above expression would reduce to

$$H = \frac{P \times L \times A \times N}{33{,}000},$$

a much simpler and shorter expression. This last expression is called a *formula.*

9. The formula just given shows, as we stated in the beginning, that a formula is really a shorthand method of expressing a rule. It is customary, however, to omit the sign of multiplication between two or more quantities when they are to be multiplied together, or between a number and a letter representing a quantity, it being always understood that, when two letters are adjacent with no sign between them, the quantities represented by these letters are to be multiplied. Bearing this fact in mind, the formula just given can be further simplified to

$$H = \frac{PLAN}{33{,}000}.$$

10. The sign of multiplication, evidently, cannot be omitted between two or more numbers, as it would then be impossible to distinguish the numbers. A near approach to this, however, may be attained by placing a dot between the numbers which are to be multiplied together, and this is frequently done in works on mathematics when it is desired to economize space. In such cases it is usual to put the dot higher than the position occupied by the decimal point. Thus, $2 \cdot 3$ means the same as 2×3; $542 \cdot 749 \cdot 1{,}006$ indicates that the numbers 542, 749, and 1,006 are to be multiplied together.

It is also customary to omit the sign of multiplication in expressions similar to the following: $a \times \sqrt{b+c}$, $3 \times (b+c)$, $(b+c) \times a$, etc., writing them $a\sqrt{b+c}$, $3\,(b+c)$, $(b+c)\,a$, etc. The sign is not omitted when several quantities are included by a vinculum and it is desired to indicate that the quantities so included are to be multiplied by another quantity. For example, $3 \times \overline{b+c}$, $\overline{b+c} \times a$, $\sqrt{\overline{b+c}} \times a$, etc. are always written as here printed.

11. Before proceeding further, we will explain one other device that is used by formula makers and which is apt to puzzle one who encounters it for the first time—it is the use of what mathematicians call *primes* and *subs.*, and

what printers call *superior* and *inferior* characters. As a rule, formula makers designate quantities by the initial letters of the names of the quantities. For example, they represent volume by v, pressure by p, height by h, etc. This practice is to be commended, as the letter itself serves in many cases to identify the quantity which it represents. Some authors carry the practice a little further and represent all quantities of the same nature by the same letter throughout the book, always having the same letter represent the same thing. Now, this practice necessitates the use of the primes and subs. above mentioned when two quantities have the same name but represent different things. Thus, consider the word *pressure* as applied to steam at different stages between the boiler and the condenser. First, there is *absolute* pressure, which is equal to the gauge pressure in pounds per square inch plus the pressure indicated by the barometer reading (usually assumed in practice to be 14.7 pounds per square inch, when a barometer is not at hand). If this be represented by p, how shall we represent the gauge pressure? Since the absolute pressure is always greater than the gauge pressure, suppose we decide to represent it by a capital letter and the gauge pressure by a small (lower-case) letter. Doing so, P represents absolute pressure and p, gauge pressure. Further, there is usually a "drop" in pressure between the boiler and the engine, so that the initial pressure, or pressure at the beginning of the stroke, is less than the pressure at the boiler. How shall we represent the initial pressure? We may do this in one of three ways and still retain the letter p or P to represent the word pressure: First, by the use of the prime mark; thus, p' or P' (read *p prime* and *P major prime*) may be considered to represent the initial gauge pressure or the initial absolute pressure. Second, by the use of sub. figures; thus, p_1 or P_1 (read *p sub. one* and *P major sub. one*). Third, by the use of sub. letters; thus, p_i or P_i (read *p sub. i* and *P major sub. i*). In the same manner p'' (read *p second*), p_2, or p_r might be used to represent the gauge pressure at release, etc. The sub. letters have the advantage of still further identifying the quantity represented; in many instances,

however, it is not convenient to use them, in which case primes and subs. are used instead. The prime notation may be continued as follows: p''', p^{iv}, p^{v}, etc.; it is inadvisable to use superior figures, for example, p^1, p^2, p^3, p^4, etc., as they are liable to be mistaken for exponents.

12. The main thing to be remembered by the student is that *when a formula is given in which the same letters occur several times, all like letters having the same primes or subs. represent the same quantities, while those which differ in any respect represent different quantities.* Thus, in the formula

$$t = \frac{w_1 s_1 t_1 + w_2 s_2 t_2 + w_3 s_3 t_3}{w_1 s_1 + w_2 s_2 + w_3 s_3},$$

w_1, w_2, and w_3 represent the weights of three different bodies; s_1, s_2, and s_3, their specific heats; and t_1, t_2, and t_3, their temperatures; while t represents the final temperature after the bodies have been mixed together. It should be noted that those letters having the *same* subs. refer to the same bodies. Thus, w_1, s_1, and t_1 all refer to one of the three bodies; w_2, s_2, t_2, to another body, etc.

It is very easy to apply the above formula when the values of the quantities represented by the different letters are known. All that is required is to substitute the numerical values of the letters and then perform the indicated operations. Thus, suppose that the values of w_1, s_1, and t_1 are, respectively, 2 pounds, .0951, and 80°; of w_2, s_2, and t_2, 7.8 pounds, 1, and 80°; and of w_3, s_3, and t_3, $3\frac{1}{4}$ pounds, .1138, and 780°; then, the final temperature t is, substituting these values for their respective letters in the formula,

$$t = \frac{2 \times .0951 \times 80 + 7.8 \times 1 \times 80 + 3\frac{1}{4} \times .1138 \times 780}{2 \times .0951 + 7.8 \times 1 + 3\frac{1}{4} \times .1138}$$

$$= \frac{15.216 + 624 + 288.483}{.1902 + 7.8 + .36985} = \frac{927.699}{8.36005} = 110.97°.$$

In substituting the numerical values, the signs of multiplication are, of course, written in their proper places; all the multiplications are performed before adding, according to the rule previously given.

13. The student should now be able to apply any formula involving only algebraic expressions that he may meet with, and which does not require the use of logarithms for its solution. We will, however, call his attention to one or two other facts that he may have forgotten.

Expressions similar to $\frac{160}{\frac{660}{25}}$ sometimes occur, the heavy line indicating that 160 is to be divided by the quotient obtained by dividing 660 by 25. If both lines were light, it would be impossible to tell whether 160 was to be divided by $\frac{660}{25}$, or whether $\frac{160}{660}$ was to be divided by 25. If this latter result were desired, the expression would be written $\frac{\frac{160}{660}}{25}$. In every case the heavy line indicates that all above it is to be divided by all below it.

In an expression like the following, $\frac{160}{7+\frac{660}{25}}$, the heavy line is not necessary, since it is impossible to mistake the operation that is required to be performed. But, since $7+\frac{660}{25}=\frac{175+660}{25}$, if we substitute $\frac{175+660}{25}$ for $7+\frac{660}{25}$, the heavy line becomes necessary in order to make the resulting expression clear. Thus,

$$\frac{160}{7+\frac{660}{25}}=\frac{160}{\frac{175+660}{25}}=\frac{160}{\frac{835}{25}}.$$

14. Fractional exponents are sometimes used instead of the radical sign. That is, instead of indicating the square, cube, fourth root, etc. of some quantity, as 37, by $\sqrt{37}$, $\sqrt[3]{37}$, $\sqrt[4]{37}$, etc., these roots are indicated by $37^{\frac{1}{2}}$, $37^{\frac{1}{3}}$, $37^{\frac{1}{4}}$, etc. Should the numerator of the fractional exponent be some quantity other than 1, this quantity, whatever it may be, indicates that the quantity affected by the exponent is to be raised to the power indicated by the numerator; the

denominator is *always* the index of the root. Hence, instead of writing $\sqrt[3]{37^2}$ for the cube root of the square of 37, it may be written $37^{\frac{2}{3}}$, the denominator being the index of the root; in other words, $\sqrt[3]{37^2} = 37^{\frac{2}{3}}$. Likewise, $\sqrt[5]{(1 + a^2 b)^3}$ may also be written $(1 + a^2 b)^{\frac{3}{5}}$, a much simpler expression.

15. We will now give several examples showing how to apply some of the more difficult formulas that the student may encounter.

1. The area of any segment of a circle that is less than (or equal to) a semicircle is expressed by the formula

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r - h),$$

in which A = area of segment;
$\pi = 3.1416$;
r = radius;
E = angle obtained by drawing lines from the center to the extremities of arc of segment;
c = chord of segment;
h = height of segment.

EXAMPLE.—What is the area of a segment whose chord is 10 inches long, angle subtended by chord is 83.46°, radius is 7.5 inches, and height of segment is 1.91 inches?

SOLUTION.—Applying the formula just given,

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r - h) = \frac{3.1416 \times 7.5^2 \times 83.46}{360} - \frac{10}{2}(7.5 - 1.91)$$

$$= 40.968 - 27.95 = 13.018 \text{ sq. in., nearly. Ans.}$$

2. The area of any triangle may be found by means of the following formula, in which A = the area, and a, b, and c represent the lengths of the sides:

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2}.$$

EXAMPLE.—What is the area of a triangle whose sides are 21 feet, 46 feet, and 50 feet long?

SOLUTION.—In order to apply the formula, suppose we let a represent the side that is 21 feet long; b, the side that is 50 feet long; and c, the side that is 46 feet long. Then, substituting in the formula

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2} = \frac{50}{2}\sqrt{21^2 - \left(\frac{21^2 + 50^2 - 46^2}{2 \times 50}\right)^2}$$

$$= \frac{50}{2}\sqrt{441 - \left(\frac{441 + 2{,}500 - 2{,}116}{100}\right)^2} = 25\sqrt{441 - \left(\frac{825}{100}\right)^2}$$

$$= 25\sqrt{441 - 8.25^2} = 25\sqrt{441 - 68.0625} = 25\sqrt{372.9375}$$

$$= 25 \times 19.312 = 482.8 \text{ sq. ft., nearly. Ans.}$$

The operations in the above examples have been extended much farther than was necessary; it was done in order to show the student every step of the process. The last formula is perfectly general, and the same answer would have been obtained had the 50-foot side been represented by a, the 46-foot side by b, and the 21-foot side by c.

3. The Rankine-Gordon formula for determining the least load in pounds that will cause a long column to break is

$$P = \frac{SA}{1 + q\frac{l^2}{G^2}},$$

in which P = load (pressure) in pounds;

S = ultimate strength (in pounds per square inch) of the material composing the column;

A = area of cross-section of column in square inches;

q = a factor (multiplier) whose value depends upon the shape of the ends of the column and on the material composing the column;

l = length of column in inches;

G = least radius of gyration of cross-section of column.

The values of S, q, and G^2 are given in printed tables in books in which this formula occurs.

EXAMPLE.—What is the least load that will break a hollow steel column whose outside diameter is 14 inches; inside diameter, 11 inches; length, 20 feet, and whose ends are flat?

SOLUTION.—For steel, $S = 150{,}000$, and $q = \frac{1}{25{,}000}$ for flat-ended steel columns; A, the area of the cross-section, $= .7854\,(d_1^2 - d_2^2)$ $= .7854\,(14^2 - 11^2)$, d_1 and d_2 being the outside and inside diameters,

respectively; $l = 20 \times 12 = 240$ inches; and $G^2 = \frac{d_1^2 + d_2^2}{16} = \frac{14^2 + 11^2}{16}$. Substituting these values in the formula

$$P = \frac{SA}{1 + q\frac{l^2}{G^2}} = \frac{150{,}000 \times .7854\,(14^2 - 11^2)}{1 + \frac{1}{25{,}000} \times \frac{240^2}{\frac{14^2 + 11^2}{16}}}$$

$$= \frac{150{,}000 \times 58.905}{1 + .1163} = \frac{8{,}835{,}750}{1.1163} = 7{,}915{,}211 \text{ lb.} \quad \text{Ans.}$$

4. EXAMPLE.—When $A = 10$, $B = 8$, $C = 5$, and $D = 4$, what is the value of E in the following?

(*a*) $E = \sqrt[3]{\frac{BCD}{A\left(2 + \frac{D^2}{C^2}\right)}}$; (*b*) $E = \frac{A - \frac{1}{4}D + \frac{4B^2}{A + C}}{A - \sqrt{\frac{2B^2}{A + 22}}}$.

SOLUTION.—(*a*) Substituting,

$$E = \sqrt[3]{\frac{8 \times 5 \times 4}{10\left(2 + \frac{4^2}{5^2}\right)}}.$$

To simplify the denominator, square the 4 and 5, add the resulting fraction to 2, and multiply by 10. Simplifying, we have

$$E = \sqrt[3]{\frac{160}{10\left(2 + \frac{16}{25}\right)}} = \sqrt[3]{\frac{160}{10 \times \frac{66}{25}}} = \sqrt[3]{\frac{160}{\frac{660}{25}}} = \sqrt[3]{\frac{200}{33}}.$$

Reducing the fraction to a decimal before extracting the cube root,

$$E = \sqrt[3]{6.0606} = 1.823. \quad \text{Ans.}$$

(*b*) Substituting,

$$E = \frac{10 - \frac{1}{4} \times 4 + \frac{4 \times 8^2}{10 + 5}}{10 - \sqrt{\frac{2 \times 8^2}{10 + 22}}} = \frac{10 - 3 + \frac{4 \times 64}{15}}{10 - \sqrt{\frac{2 \times 64}{32}}}$$

$$= \frac{7 + 17.066+}{10 - \sqrt{4}} = \frac{24.066+}{8} = 3.008+. \quad \text{Ans.}$$

16. In the preceding pages, the unknown quantity has always been represented by the single letter at the left of the sign of equality, while the letters at the right have represented known values from which the required values could be found. It is possible, however, to find the value of the quantity represented by any letter in a formula, if the values represented by all the others are known. For example, let it be required to find how many strokes per minute an

engine having a piston area of 78.54 square inches must make in order to develop 60 horsepower, if the mean effective pressure is 40 pounds per square inch and the length of stroke is $1\frac{1}{4}$ ft. By substituting the given values in the formula $H = \frac{PLAN}{33{,}000}$, we have

$$60 = \frac{40 \times 1\frac{1}{4} \times 78.54 \times N}{33{,}000},$$

in which N, the number of strokes, is to be found.

But it is evident that the expression on the right of the sign of equality is equal to $\frac{40 \times 1\frac{1}{4} \times 78.54}{33{,}000} \times N$, a fraction whose numerator is composed of three factors. Reducing the numerator to a single number by performing the indicated multiplications, we obtain, after canceling,

$$60 = \frac{119}{1{,}000} \times N = .119\,N.$$

If 60 equals .119 N, then N equals 60 divided by .119; hence,

$$N = \frac{60}{.119} = 504.2 \text{ strokes per minute.}$$

The method of procedure is essentially the same when the unknown quantity occurs in the denominator of a formula. Thus, in the formula $f = \frac{mv^2}{r}$, suppose that $f = 375$, $m = 1.25$, and $v = 60$. Then, substituting,

$$375 = \frac{1.25 \times 60^2}{r} = \frac{4{,}500}{r}.$$

But, if 375 equals 4,500 divided by r, then $375 \times r = 4{,}500$; hence, r must equal 4,500 divided by 375, or $r = \frac{4{,}500}{375} = 12$.

EXAMPLES FOR PRACTICE.

Find the numerical values of x in the following formulas, when $A = 9$, $B = 8$, $d = 10$, $e = 3$, and $c = 2$:

1. $x = \frac{d + c^2}{d^2 - 40}$. Ans. $x = \frac{7}{10}$.

2. $x = \frac{\frac{3}{8}(A + e)}{c\,e}$. Ans. $x = 1\frac{1}{2}$.

3. $x = \sqrt{\frac{d^2}{2c}} + \sqrt{AB^2}$. Ans. $x = 29$.

4. $x = \frac{Ae}{\sqrt{16Bc}} + \frac{5}{16}$. Ans. $x = 2$.

5. $x = (c + 2e)\left(\sqrt{B} - \frac{1}{c}\right) + \frac{e^2 - c^2}{e^2 + c^2}$. Ans. $x = 12\frac{5}{13}$.

6. $x = \sqrt{\frac{Bcd}{A\left(2 + \frac{d^2}{e^2}\right)}}$ Ans. $x = .396+$.

MENSURATION.

17. **Mensuration** treats of the measurement of lines, angles, surfaces, and solids.

LINES AND ANGLES.

18. A **straight line** is one that does not change its direction throughout its whole length. To distinguish one straight line from another, two of its points are designated by letters. The line shown in Fig. 1 would be called the line *A B*.

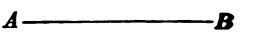

FIG. 1.

19. A **curved line** changes its direction at every point. Curved lines are designated by three or more letters, as the curved line *A B C*, Fig. 2.

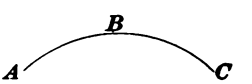

FIG. 2.

20. **Parallel lines** (Fig. 3) are those which are equally distant from each other at all points.

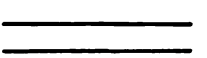
FIG. 3.

21. A line is **perpendicular** to another (see Fig. 4) when it meets that line so as not to incline towards it on either side.

FIG. 4.

22. A **vertical line** is one that points towards the center of the earth, and is also known as a *plumb*-line.

23. A **horizontal line** (see Fig. 5) is one that makes a right angle with any vertical line.

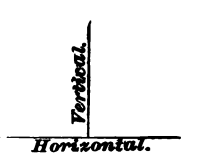

FIG. 5.

24. An **angle** is the opening between two lines which intersect or meet; the point of meeting is called the **vertex** of the angle. Angles are distinguished by naming the vertex and a point on each line. Thus, in Fig. 6, the angle formed by the lines $A\,B$ and $C\,B$ is called the angle $A\,B\,C$, or the angle $C\,B\,A$; the letter at the vertex is always placed in the middle. When an angle stands alone so that it cannot be mistaken for any other angle, only the vertex letter need be used. Thus, the angle referred to might be designated simply as the angle B.

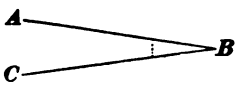

FIG. 6.

25. If one straight line meets another straight line at a point between its ends, as in Fig. 7, two angles, $A\,B\,C$ and $A\,B\,D$, are formed, which are called **adjacent angles.**

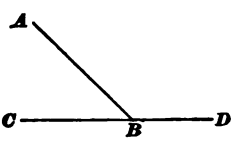

FIG. 7.

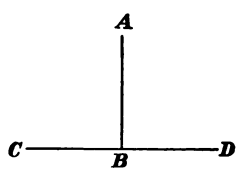

FIG. 8.

26. When these adjacent angles, $A\,B\,C$ and $A\,B\,D$, are equal, as in Fig. 8, they are called **right angles.**

27. An **acute angle** is less than a right angle. $A\,B\,C$, Fig. 9, is an acute angle.

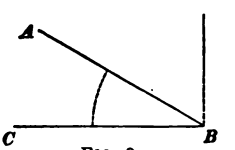

FIG. 9.

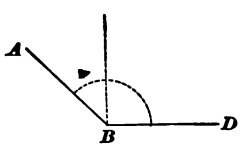

FIG. 10.

28. An **obtuse angle** is greater than a right angle. $A\,B\,D$ (Fig. 10) is an obtuse angle.

29. A **circle** (see Fig. 11) is a figure bounded by a curved line, called the **circumference,** every point of which is equally distant from a point within, called the **center.**

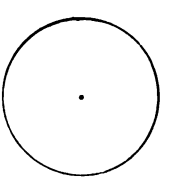
FIG. 11.

30. An **arc** of a circle is any part of its circumference; thus *a e b*, Fig. 12, is an arc of the circle.

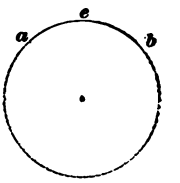

FIG. 12.

31. The circumference of every circle is considered to be divided into 360 equal parts, or arcs, called **degrees**; every degree is subdivided into 60 equal parts, called **minutes**, and every minute is again divided into 60 equal parts, called **seconds.**

Since 1 degree is $\frac{1}{360}$ of any circumference, it follows that the length of a degree will be different in circles of different sizes, but the proportion of the length of an arc of 1 degree to the whole circumference will always be the same, viz., $\frac{1}{360}$ of the circumference.

Degrees, minutes, and seconds are denoted by the symbols °, ′, ″. Thus, the expression 37° 14′ 44″ is read 37 degrees 14 minutes 44 seconds.

32. The arcs of circles are used to measure angles. An angle having its vertex at the center of a circle is measured by the arc included between its sides; thus, in Fig. 13, the arc $F B$ measures the angle $F O B$. If the arc $F B$ contains 20°, or $\frac{20}{360}$ of the circumference, the angle $F O B$ would be an angle of 20°; if it contained 20° 14′ 18″, it would be an angle of 20° 14′ 18″, etc.

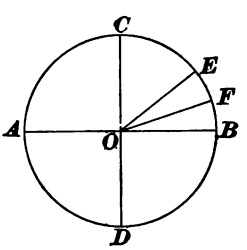

FIG. 13.

In the figure, if the line $C D$ be drawn perpendicular to $A B$, the adjacent angles will be equal, and the circle will be divided into four equal angles, each of which will be a right angle. A right angle, therefore, is an angle of $\frac{360°}{4}$, or 90°; two right angles are measured by 180°, or half the circumference, and four right angles by the whole circumference, or 360°. One-half of a right angle, as $E O B$, is an angle of 45°. An *acute* angle may now be defined as an angle of *less* than 90°, and an *obtuse* angle as one of *more*

than 90°. These values are important and should be remembered.

33. From the foregoing it will be evident that if a number of straight lines on the same side of a given straight line meet at the same point, the sum of all the angles formed is equal to two right angles, or 180°. Thus, in Fig. 14, angles $COB + DOC + EOD + FOE + AOF$ = 2 right angles, or 180°.

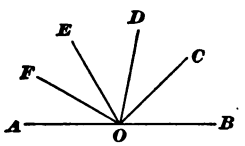

FIG. 14.

34. Also, if through a given point any number of straight lines be drawn, the sum of all the angles formed about the points of intersection equals four right angles, or 360°. Thus, in Fig. 15, angles $HOF + FOC + COA + AOG + GOE + EOD + DOB + BOH$ = 4 right angles, or 360°.

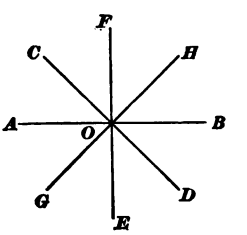

FIG. 15.

EXAMPLE.—In a flywheel with 12 arms, how many degrees are there in the angle included between the center lines of any two arms, the arms being spaced equally?

SOLUTION.—Since there are 12 arms, there are 12 angles, which together equal 360°. Hence, one angle equals $\frac{1}{12}$ of 360°, or $\frac{360°}{12} = 30°$. Ans.

EXAMPLES FOR PRACTICE.

1. How many seconds are in 32° 14′ 6″? Ans. 116,046 sec.

2. How many degrees, minutes, and seconds do 38,582 seconds amount to? Ans. 10° 43′ 2″.

3. How many right angles are there in an angle of 170°? Ans. $1\frac{8}{9}$ right angles.

4. In a pulley with five arms, what part of a right angle is included between the center lines of any two arms? Ans. $\frac{4}{5}$ of a right angle.

5. If one straight line meets another so as to form an angle of 20° 10′, what does its adjacent angle equal? Ans. 159° 50′.

6. If a number of straight lines meet a given straight line at a given point, all being on the same side of the given line, so as to form six equal angles, how many degrees are there in each angle? Ans. 30°.

QUADRILATERALS.

35. A **plane figure** is any part of a plane or flat surface bounded by straight or curved lines.

36. A **quadrilateral** is a plane figure bounded by four straight lines.

37. A **parallelogram** is a quadrilateral whose opposite sides are parallel.

There are four kinds of parallelograms: the **square,** the **rectangle,** the **rhombus,** and the **rhomboid.**

38. A **rectangle** (Fig. 16) is a parallelogram whose angles are all right angles.

FIG. 16.

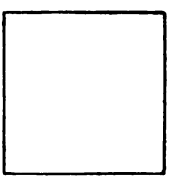

FIG. 17.

39. A **square** (Fig. 17) is a rectangle whose sides are all of the same length.

40. A **rhomboid** (Fig. 18) is a parallelogram whose opposite sides are equal and parallel, and whose angles are not right angles.

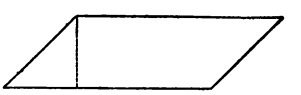

FIG. 18.

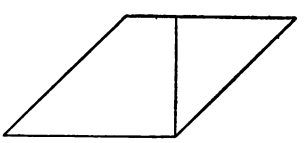

FIG. 19.

41. A **rhombus** (Fig. 19) is a parallelogram having equal sides, and whose angles are not right angles.

42. A **trapezoid** (Fig. 20) is a quadrilateral which has only two of its sides parallel.

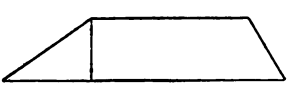

FIG. 20.

43. The **altitude** of a parallelogram or a trapezoid is the perpendicular distance between the parallel lines, as shown by the vertical lines in Figs. 18, 19, and 20.

44. The **base** of *any* plane figure is the side on which it is supposed to stand.

45. The **area** of a surface is expressed by the number of unit squares it will contain.

46. A **unit square** is the square having a unit for its side. For example, if the unit is 1 inch, the unit square is the square each of whose sides measures 1 inch in length, and the area of a surface would be expressed by the number of square inches it would contain. If the unit were 1 foot, the unit square would measure 1 foot on each side, and the area of the given surface would be the number of square feet it would contain, etc.

The square that measures 1 inch on a side is called a **square inch,** and the one that measures 1 foot on a side is called a **square foot.** Square inch and square foot are abbreviated to sq. in. and sq. ft.

47. To find the area of any parallelogram:

Rule 1.—*Multiply the base by the altitude.*

NOTE.—Before multiplying, the base and altitude must be reduced to the same kind of units; that is, if the base should be given in feet and the altitude in inches, they could not be multiplied together until either the altitude had been reduced to feet or the base to inches. This principle holds throughout the subject of mensuration.

EXAMPLE 1.—The sides of a square piece of sheet iron are each $10\frac{1}{4}$ inches long. How many square inches does it contain?

SOLUTION.— $10\frac{1}{4}$ inches = 10.25 inches when reduced to a decimal. The base and altitude are each 10.25 inches. Multiplying them together, $10.25 \times 10.25 = 105.06+$ sq. in. Ans.

EXAMPLE 2.—What is the area in square rods of a piece of land in the shape of a rhomboid, one side of which is 8 rods long and whose length, measured on a line perpendicular to this side, is 200 feet?

SOLUTION.—The base is 8 rods and the altitude 200 feet. As the answer is to be in rods, the 200 feet should be reduced to rods. Reducing $200 \div 16\frac{1}{2} = 200 \div \frac{33}{2} = 12.12$ rods. Hence, area $= 8 \times 12.12$ $= 96.96$ sq. rd. Ans.

48. To find the area of a trapezoid:

Rule 2.—*Multiply one-half the sum of the parallel sides by the altitude.*

EXAMPLE.—A board 14 feet long is 20 inches wide at one end and 16 inches wide at the other. If the ends are parallel, how many square feet does the board contain?

SOLUTION.—One-half the sum of the parallel sides $= \frac{20 + 16}{2}$ $= 18$ inches $= 1\frac{1}{2}$ feet. The length of the board corresponds to the altitude of a trapezoid. Hence, $14 \times 1\frac{1}{2} = 21$ sq. ft. Ans.

49. Having given the area of a parallelogram and one dimension, to find the other dimension:

Rule 3.—*Divide the area by the given dimension.*

EXAMPLE.—What is the width of a parallelogram whose area is 212 square feet and whose length is $26\frac{1}{2}$ feet?

SOLUTION.— $212 \div 26\frac{1}{2} = 212 \div \frac{53}{2} = 8$ ft. Ans.

The following examples illustrate a few special cases:

EXAMPLE 1.—An engine room is 22 feet by 32 feet. The engine bed occupies a space of 3 feet by 12 feet; the flywheel pit, a space of 2 feet by 6 feet, and the outer bearing a space of 2 feet by 4 feet. How many square feet of flooring will be required for the room?

SOLUTION.—Area of engine bed $= 3 \times 12 = 36$ sq. ft.
Area of flywheel pit $= 2 \times 6 = 12$ sq. ft.
Area of outer bearing $= 2 \times 4 = 8$ sq. ft.
Total, 56 sq. ft.
Area of engine room $= 22 \times 32 = 704$ sq. ft.
$704 - 56 = 648$ sq. ft. of flooring required. Ans.

EXAMPLE 2.—How many square yards of plaster will it take to cover the sides and ceiling of a room 16×20 feet and 11 feet high, having four windows, each 7×4 feet, and three doors, each 9×4 feet over all, the baseboard coming 6 inches above the floor?

SOLUTION.—

Area of ceiling $= 16 \times 20 =$ 320 sq. ft.
Area of end walls $= 2(16 \times 11) =$ 352 sq. ft.
Area of side walls $= 2(20 \times 11) =$ 440 sq. ft.
Total area = 1,112 sq. ft.

From the above must be deducted:

Windows $= 4(7 \times 4) = 112$ sq. ft.
Doors $= 3(9 \times 4) = 108$ sq. ft.

Baseboard less the width of three doors $= (72 - 12) \times \frac{6}{12} = 30$ sq. ft.

Total number of feet to be deducted $= 112 + 108 + 30 = 250$ sq. ft.

Hence, number of square feet to be plastered $= 1{,}112 - 250 = 862$ sq. ft., or $95\frac{7}{9}$ sq. yd. Ans.

EXAMPLE 3.—How many acres are contained in a rectangular tract of land 800 rods long and 520 rods wide?

SOLUTION.— $800 \times 520 = 416{,}000$ sq. rd. Since there are 160 square rods in 1 acre, the number of acres $= 416{,}000 \div 160 = 2{,}600$ acres. Ans.

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EXAMPLES FOR PRACTICE.

1. What is the area in square feet of a rhombus whose base is 84 inches and whose altitude is 3 feet? Ans. 21 sq. ft.

2. A flat roof, 46 feet by 80 feet in size, is covered by tin roofing weighing one-half pound per square foot; what is the total weight of the roofing? Ans. 1,840 lb.

3. One side of a room measures 16 feet. If the floor contains 240 square feet, what is the length of the other side? Ans. 15 ft.

4. How many square feet in a board 12 feet long, 18 inches wide at one end, and 12 inches wide at the other end? Ans. 15 sq. ft.

5. How much would it cost to lay a sidewalk a mile long and 8 feet 6 inches wide, at the rate of 20 cents per square foot? How much at the rate of $1.80 per square yard? Ans. $8,976 in each case.

6. How many square yards of plastering will be required for the ceiling and walls of a room 10 ft. × 15 ft. and 9 feet high; the room contains one door 3½ ft. × 7 ft., three windows 3½ ft. × 6 ft., and a baseboard 8 inches high? Ans. 53.5 sq. yd.

THE TRIANGLE.

50. A **triangle** is a plane figure having three sides.

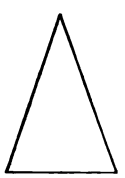

FIG. 21.

51. An **isosceles** triangle is one having two of its sides equal; see Fig. 21.

52. An **equilateral** triangle (Fig. 22) is one having all of its sides of the same length.

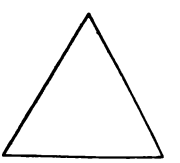

FIG. 22.

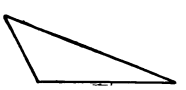

FIG. 23.

53. A **scalene** triangle (Fig. 23) is one having no two of its sides equal.

54. A **right-angled** triangle (Fig. 24) is any triangle having one right angle. The side opposite the right angle is called the **hypotenuse.** A right-angled triangle is now usually called a **right triangle.**

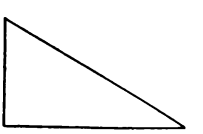

FIG. 24.

55. In any triangle the sum of the three angles equals two right angles, or 180°. Thus, in Fig. 25, the sum of the angles *A*, *B*, and *C* equals two right angles, or 180°. Hence, if any two angles of a triangle are given and it is required to find the third angle:

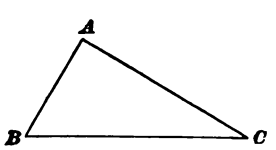

FIG. 25.

Rule 4.—*Add the two given angles and subtract their sum from 180°; the remainder will be the third angle.*

EXAMPLE.—If two angles of a triangle are 48° 16′ and 47° 50′, what does the third angle equal?

SOLUTION.—First reduce 48° 16′ and 47° 50′ to minutes, for convenience in adding and subtracting the angles. $48° = 48 \times 60' = 2{,}880'$; $2{,}880' + 16' = 2{,}896'$; hence, $48° 16' = 2{,}896'$. In like manner, $47° 50' = 47 \times 60' + 50' = 2{,}820' + 50' = 2{,}870'$. Adding the two angles and subtracting the sum from 180° reduced to minutes, $2{,}896' + 2{,}870' = 5{,}766'$; $180° = 180 \times 60' = 10{,}800'$; $10{,}800 - 5{,}766 = 5{,}034'$. Reducing this last number to degrees and minutes, $\frac{5{,}034}{60} = 83\frac{54}{60}° = 83° 54'$. Hence, the third angle in the triangle $= 83° 54'$. Ans.

56. In any right triangle there can be but one right angle, and since the sum of all the angles is two right angles, it is evident that the sum of the two acute angles must equal one right angle, or 90°. Therefore, if in any right triangle one acute angle is known, to find the other acute angle:

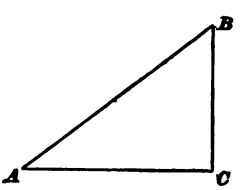

FIG. 26.

Rule 5.—*Subtract the known acute angle from 90°; the result will be the other acute angle.*

EXAMPLE.—If one acute angle, as A, of the right triangle $A\,B\,C$, Fig. 26, equals 30°, what does the angle B equal?

SOLUTION.— $90° - 30° = 60°$. Ans.

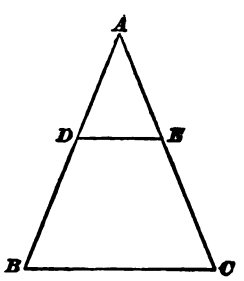

FIG. 27.

57. If a straight line be drawn through two sides of a triangle, parallel to the third side, a second triangle will be formed whose sides will be proportional to the corresponding sides of the first triangle. Thus, in the triangle $A\,B\,C$, Fig. 27, if the line $D\,E$ be drawn parallel to the side $B\,C$, the triangle $A\,D\,E$ will be formed and we shall have

(1) Side $A\,D$: side $D\,E$ = side $A\,B$: side $B\,C$; and,

(2) Side $A\,E$: side $D\,E$ = side $A\,C$: side $B\,C$; also,

(3) Side $A\,D$: side $A\,E$ = side $A\,B$: side $A\,C$.

EXAMPLE.—In Fig. 27, if $A\,B = 24$, $B\,C = 18$, and $D\,E = 8$, what does $A\,D$ equal?

SOLUTION.—Writing these values for the sides in (1),

$$A\,D : 8 = 24 : 18;\ \text{whence, } A\,D = \frac{24 \times 8}{18} = 10\tfrac{2}{3}.\quad \text{Ans.}$$

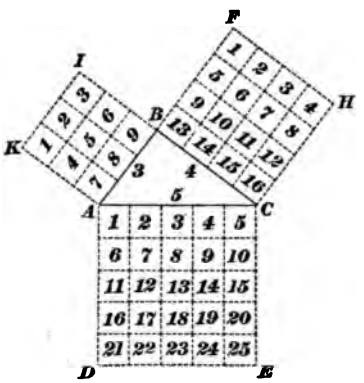

FIG. 28.

58. In any right triangle, the square described on the hypotenuse is equal to the sum of the squares described upon the other two sides. If $A\,B\,C$, Fig. 28, is a right triangle right-angled at B, then the square described upon the hypotenuse $A\,C$ is equal to the sum of the squares described upon the sides $A\,B$ and $B\,C$. Hence, having given the two sides forming the right angle in a right triangle, to find the hypotenuse:

Rule 6.—*Square each of the sides forming the right angle; add the squares together and take the square root of the sum.*

EXAMPLE.—If $A\,B = 3$ inches and $B\,C = 4$ inches, what is the length of the hypotenuse $A\,C$?

SOLUTION.—Squaring each of the given sides, $3^2 = 9$ and $4^2 = 16$. Taking the square root of the sum of 9 and 16, the hypotenuse $= \sqrt{9 + 16} = \sqrt{25} = 5$ in. Ans.

59. If the hypotenuse and one side are given, the other side can be found as follows:

Rule 7.—*Subtract the square of the given side from the square of the hypotenuse, and extract the square root of the remainder.*

EXAMPLE 1.—The side given is 3 inches, the hypotenuse is 5 inches; what is the length of the other side?

SOLUTION.— $3^2 = 9$; $5^2 = 25$. $25 - 9 = 16$, and $\sqrt{16} = 4$ in. Ans.

EXAMPLE 2.—If from a church steeple which is 150 feet high a rope is to be attached to the top and to a stake in the ground, which is 85 feet from the center of the base (the ground being supposed to be level), what must be the length of the rope?

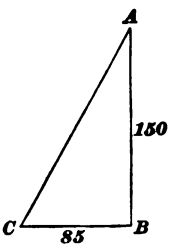

FIG. 29.

SOLUTION.—In Fig. 29, $A\ B$ represents the steeple, 150 feet high; C a stake 85 feet from the foot of the steeple, and $A\ C$ the rope. Here we have a right triangle right-angled at B, and $A\ C$ is the hypotenuse. The square of $A\ B = 150^2 = 22{,}500$; of $C\ B$, $85^2 = 7{,}225$. $22{,}500 + 7{,}225 = 29{,}725$; $\sqrt{29{,}725} = 172.4$ ft., nearly. Ans.

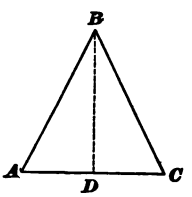

FIG. 30.

60. The **altitude** of any triangle is a line, as $B\ D$, drawn from the vertex B of the angle opposite the base $A\ C$, perpendicular to the base, as in Fig. 30, or to the base extended, as in Fig. 31.

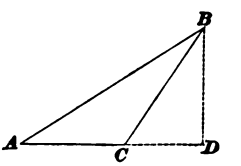

FIG. 31.

61. If in any parallelogram a straight line, called the **diagonal,** be drawn, connecting two opposite corners, it will divide the parallelogram into two equal triangles, as $A\ D\ B$ and $D\ B\ C$ in Fig. 32. The area of each triangle will equal one-half the area of the parallelogram, i. e., one-half the product of the base and the altitude. Hence, to find the area of any triangle:

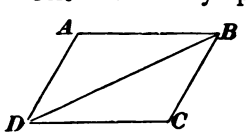

FIG. 32.

Rule 8.—*Multiply the base by the altitude and divide the product by 2.*

EXAMPLE.—What is the area in square feet of a triangle whose base is 18 feet and whose altitude is 7 feet 9 inches?

SOLUTION.— 7 ft. 9 in. $= 7\frac{3}{4}$ ft. $= \frac{31}{4}$ ft. $18 \times \frac{31}{4} = 139\frac{1}{2}$, and one-half of $139\frac{1}{2} = 69\frac{3}{4}$ sq. ft. Ans.

To find the altitude or base of a triangle, having given the area and the base or altitude:

Rule 9.—*Multiply the area by 2 and divide by the given dimension.*

EXAMPLE.—What must be the height of a triangular piece of sheet metal to contain 100 square inches, if the base is 10 inches long?

SOLUTION.— $100 \times 2 = 200$; $200 \div 10 = 20$ in. Ans.

EXAMPLES FOR PRACTICE.

1. What is the area of a triangle whose base is 18 feet long and whose altitude is 10 feet 6 inches? Ans. 94.5 sq. ft.

2. Two angles of a scalene triangle together equal 100° 4′. What is the size of the third angle? Ans. 79° 56′.

3. One angle of a right triangle equals 20° 10′ 5″. What is the size of the other acute angle? Ans. 69° 49′ 55″.

4. A ladder 65 feet long reaches to the top of a wall when its foot is 25 feet from the wall. How high is the wall? Ans. 60 ft.

5. Draw a triangle, and through two of its sides draw a line parallel to the base. Letter the different lines, and then, without referring to the text, write out the proportions existing between the sides of the two triangles.

6. A triangular piece of sheet metal weighs 24 pounds. If the base of the triangle is 4 feet and its height 6 feet, how much does the metal weigh per square foot? Ans. 2 lb.

7. The area of a triangle is 16 square inches. If the altitude is 4 inches, what does the base measure? Ans. 8 in.

8. Two sides of a right triangle are 92 feet and 69 feet long. How long is the hypotenuse? Ans. 115 ft.

POLYGONS.

62. A **polygon** is a plane figure bounded by straight lines. The term is usually applied to a figure having more than four sides. The bounding lines are called the **sides**, and the sum of the lengths of all the sides is called the **perimeter** of the polygon.

63. A **regular polygon** is one in which all the sides and all the angles are equal.

64. A polygon of five sides is called a **pentagon**; one of six sides, a **hexagon**; one of seven sides, a **heptagon**,

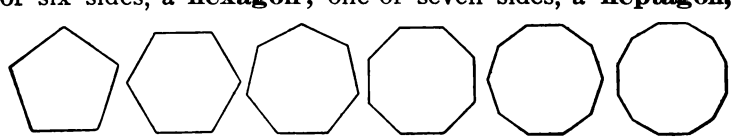

FIG. 33.

etc. Regular polygons having from five to twelve sides are

shown in Fig. 33. In any polygon, the sum of all the interior angles, as $A + B + C + D + E$, Fig. 34, equals 180° multiplied by a number which is two less than the number of sides in the polygon. Hence, to find the size of any one of the interior angles of a *regular* polygon:

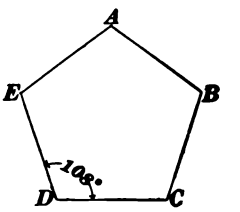

FIG. 34.

Rule 10.—*Multiply 180° by the number of sides less two and divide the result by the number of sides; the quotient will be the number of degrees in each interior angle.*

EXAMPLE 1.—If Fig. 34 is a regular pentagon, how many degrees are there in each interior angle?

SOLUTION.—In a pentagon there are five sides; hence, $5 - 2 = 3$ and $180 \times 3 = 540$; $540 \div 5 = 108°$ in each angle. Ans.

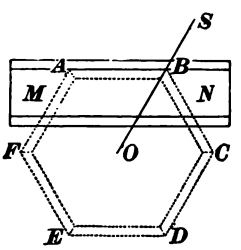

FIG. 35.

EXAMPLE 2.—It is desired to make a miter-box in which to cut a strip of molding to fit around a column having the shape of a regular hexagon. At what angle should the saw run across the miter-box?

SOLUTION.—In Fig. 35, let $A\,B$, $B\,C$, $C\,D$, etc. represent the pieces of molding as they will fit around the column. First find the size of one of the equal angles of the polygon by the above rule. Number of sides $= 6$; $6 - 2 = 4$; hence, $180 \times 4 = 720$, and $720 \div 6 = 120°$ in each angle. Now, let $M\,N$ represent the miter-box and $O\,S$ the direction in which the saw should run; then, $A\,B\,O$ is the angle made by the saw with the side of the miter-box; but as the polygon is a regular one, this angle is one-half the interior angle $A\,B\,C$, which we have found to be 120°. Hence, the saw should run at an angle of $\frac{120}{2} = 60°$ with the side of the miter-box. Ans.

65. The area of any regular polygon may be found by drawing lines from the center to each angle and computing the area of each triangle thus formed. Hence, to find the area of any regular polygon:

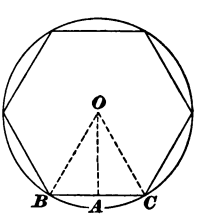

FIG. 36.

Rule 11.—*Multiply the length of a side by half the distance from the side to the center, and that product by the number of sides. The last product will be the area of the figure.*

EXAMPLE.—In Fig. 36 the side BC of the regular hexagon is 12 inches and the distance AO is 10.4 inches; required the area of the polygon.

SOLUTION.— $10.4 + 2 = 5.2$; $12 \times 5.2 \times 6 = 374.4$ sq. in. Ans.

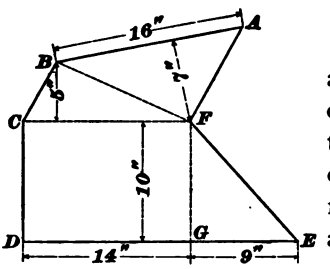

FIG. 37.

66. To obtain the area of any irregular polygon, draw diagonals dividing the polygon into triangles and quadrilaterals, and compute the areas of these separately; their sum will be the area of the figure.

EXAMPLE.—It is required to find the area of the polygon $ABCDEF$, Fig. 37.

SOLUTION.—Draw the diagonals BF and CF and the line FG perpendicular to DE, dividing the figure into the triangles ABF, BCF, and FGE and the rectangle $FCDG$. Let it be supposed that the altitudes of the figures and the lengths of the sides AB, DG, and GE are as indicated in the polygon above. Then,

$$\text{Area } ABF = \frac{16 \times 7}{2} = 56 \text{ sq. in.}$$

$$\text{Area } BCF = \frac{14 \times 5}{2} = 35 \text{ sq. in.}$$

$$\text{Area } FCDG = 14 \times 10 = 140 \text{ sq. in.}$$

$$\text{Area } FGE = \frac{9 \times 10}{2} = 45 \text{ sq. in.}$$

Total area $= 56 + 35 + 140 + 45 = 276$ sq. in. Ans.

EXAMPLES FOR PRACTICE.

1. How many degrees are there in one of the angles of a regular octagon? Ans. 135°.

2. Find the area of the polygon $ABCDEF$ (see Fig. 37), supposing each of the given dimensions to be increased to $1\frac{1}{2}$ times the length given in the figure. Ans. 621 sq. in.

3. What is the area of a regular heptagon whose sides are 4 inches long, the distance from one side to the center being 4.15 inches? Ans. 58.1 sq. in.

4. At what angle should the saw run in a miter-box to cut strips to fit around the edge of a table top made in the shape of a regular pentagon? Ans. 54°.

THE CIRCLE.

67. A **circle** (Fig. 38) is a figure bounded by a curved line, called the **circumference,** every point of which is equally distant from a point within, called the **center.**

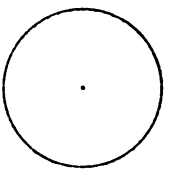

FIG. 38.

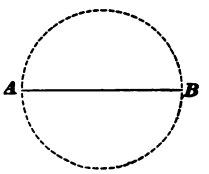

FIG. 39.

68. The **diameter** of a circle is a straight line passing through the center and terminated at both ends by the circumference; thus, $A\ B$ (Fig. 39) is a diameter of the circle.

69. The **radius** of a circle, $A\ O$ (Fig. 40), is a straight line drawn from the center O to the circumference. It is equal in length to one-half the diameter. The plural of radius is **radii,** and all radii of a circle are equal.

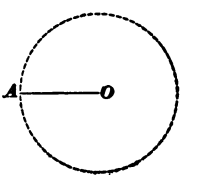

FIG. 40.

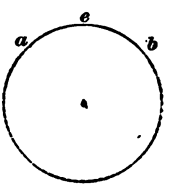

FIG. 41.

70. An **arc** of a circle (see $a\ c\ b$, Fig. 41) is any part of its circumference.

71. A **chord** is a straight line joining any two points in a circumference; or it is a straight line joining the extremities of an arc; thus, the straight line $A\ B$, Fig. 42, is a chord of the circle whose corresponding arc is $A\ E\ B$.

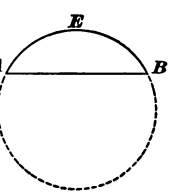

FIG. 42.

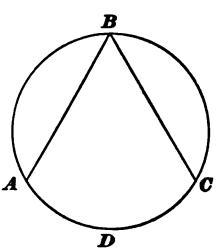

FIG. 43.

72. An **inscribed angle** is one whose vertex lies on the circumference of a circle and whose sides are chords. It is measured by one-half the intercepted arc. Thus, in Fig. 43, $A\ B\ C$ is an inscribed angle, and it is measured by one-half the arc $A\ D\ C$.

EXAMPLE.—If in Fig. 43, the arc $A\ D\ C = \frac{2}{5}$ of the circumference, what is the measurement of the inscribed angle $A\ B\ C$?

SOLUTION.—Since the angle is an inscribed angle, it is measured by one-half the intercepted arc, or $\frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$ of the circumference. The whole circumference = 360°; hence, $360° \times \frac{1}{5} = 72°$; therefore, angle $A\ B\ C$ is an angle of 72°.

73. If a circle is divided into halves, each half is called a **semicircle,** and each half circumference is called a **semi-circumference.**

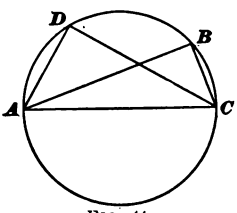

FIG. 44.

Any angle inscribed in a semicircle is a right angle, since it is measured by one-half a semi-circumference, or $180° \div 2 = 90°$. Thus, the angles $A\ D\ C$ and $A\ B\ C$, Fig. 44, are right angles, since they are inscribed in a semicircle.

74. An **inscribed** polygon is one whose vertexes lie on the circumference of a circle and whose sides are chords, as $A\ B\ C\ D\ E$, Fig. 45.

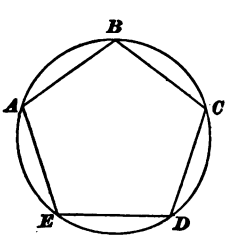

FIG. 45.

The sides of an inscribed regular hexagon have the same length as the radius of the circle.

If, in any circle, a radius be drawn perpendicular to any chord, it bisects (cuts in halves) the chord. Thus, if the radius $O\ C$, Fig. 46, is perpendicular to the chord $A\ B$, $A\ D = D\ B$.

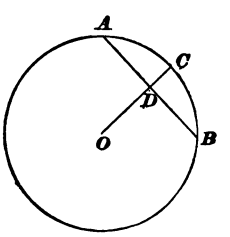

FIG. 46.

EXAMPLE.—If a regular pentagon is inscribed in a circle and a radius is drawn perpendicular to one of the sides, what are the lengths of the two parts of the side, the perimeter of the pentagon being 27 inches?

SOLUTION.—A pentagon has five sides, and since it is a regular pentagon, all the sides are of equal lengths; the perimeter of the pentagon, which equals the distance around it, or equals the sum of all the sides, is 27 inches. Therefore, the length of one side $= 27 \div 5 = 5\frac{2}{5}$ inches. Since the pentagon is an inscribed pentagon, its sides are chords, and as a radius perpendicular to a chord bisects it, we have $5\frac{2}{5} \div 2 = 2\frac{7}{10}$ inches, which equals the length of each of the parts of the side cut by a radius perpendicular to it. Ans.

75. If, from any point on the circumference of a circle, a perpendicular is let fall upon a diameter, it will divide the diameter into two parts, one of which will be in the same ratio to the perpendicular as the perpendicular is to the other part. That is, the perpendicular will be a *mean proportional* between the two parts of the diameter.

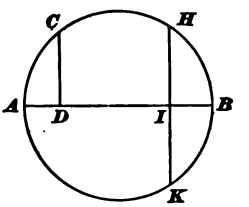

FIG. 47.

If AB, Fig. 47, is the given diameter and C any point on the circumference, then $AD : CD = CD : DB$, CD being a mean proportional between AD and DB.

EXAMPLE.—If $HK = 30$ feet and $IB = 8$ feet, what is the diameter of the circle, HK being perpendicular to AB?

SOLUTION.— 30 feet ÷ 2 feet = 15 feet = IH. And $BI : IH = IH : IA$, or $8 : 15 = 15 : IA$.

Therefore, $IA = \frac{15^2}{8} = \frac{225}{8} = 28\frac{1}{8}$ feet and $IA + IB = 28\frac{1}{8} + 8 = 36\frac{1}{8}$ feet $= AB$, the diameter of the circle. Ans.

76. When the diameter of a circle and the lengths of the two parts into which it is divided are given, the length of the perpendicular may be found by multiplying the lengths of the two parts together and extracting the square root of the product.

EXAMPLE.—In Fig. 47, the diameter of the circle AB is $36\frac{1}{8}$ feet and the distance BI is 8 feet; what is the length of the line HK?

SOLUTION.—As the diameter of the circle is $36\frac{1}{8}$ feet and as BI is 8 feet, IA is equal to $36\frac{1}{8} - 8 = 28\frac{1}{8}$ feet. The two parts, therefore, are 8 and $28\frac{1}{8}$ feet, and their product $= 8 \times 28\frac{1}{8} = 8 \times \frac{225}{8} = 225$; the square root of their product $= \sqrt{225} = 15$ feet, and as $HK = IH + IK$, or $2\,IH$, $HK = 15 \times 2 = 30$ ft. Ans.

77. To find the circumference of a circle, the diameter being given:

Rule 12.—*Multiply the diameter by 3.1416.*

EXAMPLE.—What is the circumference of a circle whose diameter is 15 inches?

SOLUTION.— $15 \times 3.1416 = 47.124$ in. Ans.

78. To find the diameter of a circle, the circumference being given:

Rule 13.—*Divide the circumference by 3.1416.*

EXAMPLE.—What is the diameter of a circle whose circumference is 65.973 inches?

SOLUTION.— $65.973 \div 3.1416 = 21$ in. Ans.

79. To find the length of an arc of a circle:

Rule 14.—*Multiply the length of the circumference of the circle of which the arc is a part by the number of degrees in the arc and divide by 360.*

EXAMPLE.—What is the length of an arc of 24°, the radius of the arc being 18 inches?

SOLUTION. — $18 \times 2 = 36$ in. = the diameter of the circle. $36 \times 3.1416 = 113.1$ in., the circumference of the circle of which the arc is a part.

$113.1 \times \frac{24}{360} = 7.54$ in., or the length of the arc. Ans.

80. To find the area of a circle:

Rule 15.—*Square the diameter and multiply by .7854.*

EXAMPLE.—What is the area of a circle whose diameter is 15 inches?

SOLUTION.— $15^2 = 225$; and $225 \times .7854 = 176.72$ sq. in. Ans.

81. Given the area of a circle, to find its diameter:

Rule 16.—*Divide the area by .7854 and extract the square root of the quotient.*

EXAMPLE 1.—The area of a circle = 17,671.5 square inches. What is its diameter in feet?

SOLUTION.— $\sqrt{\frac{17,671.5}{.7854}} = 150$ inches.

$\frac{150}{12} = 12\frac{1}{2}$ feet, or the diameter. Ans.

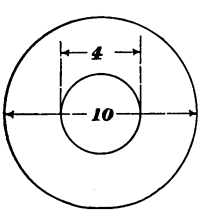

FIG. 48.

EXAMPLE 2.—What is the area of a flat circular ring, Fig. 48, whose outside diameter is 10 inches and inside diameter is 4 inches?

SOLUTION.—The area of the large circle = $10^2 \times .7854 = 78.54$ sq. in.; the area of the small circle = $4^2 \times .7854 = 12.57$ sq. in. The area of the ring is the difference between these areas, or $78.54 - 12.57 = 65.97$ sq. in. Ans.

82. To find the area of a sector (a **sector** of a circle is the area included between two radii and the circumference, as, for example, the area $BACO$, Fig. 36):

Rule 17.—*Divide the number of degrees in the arc of the sector by 360. Multiply the result by the area of the circle of which the sector is a part.*

EXAMPLE.—The number of degrees in the angle formed by drawing radii from the center of a circle to the extremities of the arc of the circle is 75°. The diameter of the circle is 12 inches; what is the area of the sector?

SOLUTION.— $\frac{75}{360} = \frac{5}{24}$; and $12^2 \times .7854 = 113.1$ sq. in.

$$113.1 \times \frac{5}{24} = 23.56 \text{ sq. in., the area. Ans.}$$

83. To find the area of a segment of a circle (a **segment** of a circle is the area included between a chord and its arc; for example, the area *A B C*, Fig. 49) when its chord and height are given. There is no exact method, except by applying principles of trigonometry. The following rule gives results that are exact enough for practical purposes.

Rule 18.—*Divide the diameter by the height of the segment; subtract .608 from the quotient and extract the square root of the remainder. This result multiplied by 4 times the square of the height of the segment and then divided by 3 will give the area, very nearly.*

The rule, expressed as a formula, is as follows, where D = the diameter of the circle and h = the height of the segment (see Fig. 49):

$$\text{Area of } A\,B\,C\,A = \frac{4\,h^2}{3}\sqrt{\frac{D}{h} - .608}.$$

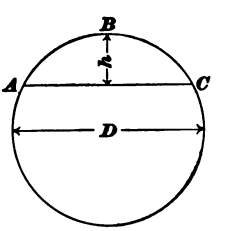

FIG. 49.

EXAMPLE.—What is the area of the segment of a circle whose diameter is 54 inches, the height of the segment being 20 inches?

SOLUTION.—Substituting in the formula,

$$\text{Area} = \frac{4 \times 20^2}{3}\sqrt{\frac{54}{20} - .608}. \quad \frac{4 \times 20^2}{3} = \frac{4 \times 400}{3}$$

$$= \frac{1{,}600}{3}; \quad \sqrt{\frac{54}{20} - .608} = \sqrt{2.092} = 1.447; \quad \frac{1{,}600}{3} \times 1.447 = 771.7 \text{ sq. in.}$$

Ans.

NOTE.—Had the chord *A C*, Fig. 49, been given instead of the diameter, the diameter would have been found as explained in Art. **75.**

EXAMPLES FOR PRACTICE.

1. An angle inscribed in a circle intercepts one-third of the circumference. How many degrees are there in the angle? Ans. 60°.

2. Suppose that in Fig. 47, the diameter $A\,B = 15$ feet and the distance $B\,I = 3$ feet. What is the length of the line $H\,K$? Ans. 12 ft.

3. The diameter of a flywheel is 18 feet. What is the distance around it to the nearest 16th of an inch? Ans. 56 ft. $6\frac{9}{16}$ in.

4. A carriage wheel was observed to make $71\frac{2}{3}$ turns while going 300 yards. What was its diameter? Ans. 4 ft., nearly.

5. What is the length of an arc of 64°, the radius of the arc being 30 inches? Ans. 33.51 in.

6. Find the area of a circle 2 feet 3 inches in diameter. Ans. 3.976 sq. ft.

7. What must be the diameter of a circle to contain 100 square inches? Ans. 11.28 in.

8. Compute the area of a segment whose height is 11 inches and the radius of whose arc is 21 inches. Ans. 289.04 sq. in.

9. Find the area of a flat circular ring whose outside diameter is 12 inches and whose inside diameter is 6 inches. Ans. 84.82 sq. in.

THE PRISM AND CYLINDER.

84. A **solid,** or body, has three dimensions: length, breadth, and thickness. The sides which enclose it are called the **faces,** and their lines of intersection are called the **edges.**

85. A **prism** is a solid whose ends are equal and parallel polygons and whose sides are parallelograms. Prisms take their names from the form of their bases. Thus, a triangular prism is one having a triangle for its base; a hexagonal prism is one having a hexagon for its base, etc.

86. A **cylinder** is a body of uniform diameter whose ends are equal parallel circles.

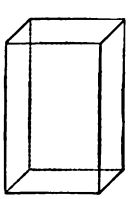

FIG. 50.

87. A **parallelopipedon** (Fig. 50) is a prism whose bases (ends) are parallelograms.

88. A **cube** (Fig. 51) is a prism whose faces and ends are squares. All the faces of a cube are equal.

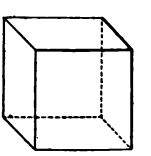

FIG. 51.

In the case of plane figures, we are concerned with perimeters and areas. In the case of solids, we are

concerned with the areas of their outside surfaces and with their contents or volumes.

89. The **entire surface** of any solid is the area of the whole outside of the solid, including the ends.

The **convex surface** of a solid is the same as the entire surface, except that the areas of the ends are not included.

90. A **unit of volume** is a cube each of whose edges is equal in length to the unit. The **volume** is expressed by the number of times it will contain a *unit of volume.*

Thus, if the unit of length is 1 inch, the unit of volume will be the cube whose edges each measure 1 inch, this cube being 1 *cubic inch;* and the number of cubic inches the solid contains will be its volume. If the unit of length is 1 foot, the unit of volume will be 1 *cubic foot*, etc. Cubic inch, cubic foot, and cubic yard are abbreviated to cu. in., cu. ft., and cu. yd., respectively.

Instead of the word *volume*, the expression **cubical contents** is sometimes used.

91. To find the area of the convex surface of a prism or cylinder:

Rule 19.—*Multiply the perimeter of the base by the altitude.*

EXAMPLE 1.—A block of marble is 24 inches long and its ends are 9 inches square. What is the area of its convex surface?

SOLUTION. — $9 \times 4 = 36$ = the perimeter of the base; $36 \times 24 = 864$ sq. in., the convex area. Ans.

To find the entire area of the outside surface, add the areas of the two ends to the convex area. Thus, the area of the two ends $= 9 \times 9 \times 2 = 162$ sq. in.; $864 + 162 = 1{,}026$ sq. in. Ans.

EXAMPLE 2.—How many square feet of sheet iron will be required for a pipe $1\frac{1}{2}$ feet in diameter and 10 feet long, neglecting the amount necessary for lapping?

SOLUTION.—The problem is to find the convex surface of a cylinder $1\frac{1}{2}$ feet in diameter and 10 feet long. The perimeter, or circumference, of the base $= 1\frac{1}{2} \times 3.1416 = 1.5 \times 3.1416 = 4.712$ ft. The convex surface $= 4.712 \times 10 = 47.12$ sq. ft. of metal. Ans.

92. To find the volume of a prism or a cylinder:

Rule 20.—*Multiply the area of the base by the altitude.*

EXAMPLE 1.—What is the weight of a length of wrought-iron shafting 16 feet long and 2 inches in diameter? Wrought iron weighs .28 pound per cubic inch.

SOLUTION.—The shaft is a cylinder 16 ft. long. The area of one end, or the base, $= 2^2 \times .7854 = 3.1416$ sq. in. Since the weight of the iron is given per cubic inch, the contents of the shaft must be found in cubic inches. The length, 16 ft., reduced to inches $= 16 \times 12 = 192$ in.; $3.1416 \times 192 = 603.19$ cu. in. = the volume. The weight $= 603.19 \times .28 = 168.89$ lb. Ans.

EXAMPLE 2.—Find the cubical contents of a hexagonal prism, Fig. 52, 12 inches long, each edge of the base being 1 inch long.

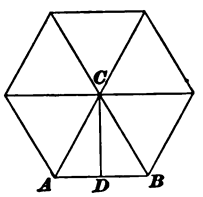

FIG. 52.

SOLUTION.—In order to obtain the area of one end, the distance $C\,D$ from the center C to one side must be found.

In the right triangle $C\,D\,A$, side $A\,D = \frac{1}{2}\,A\,B$, or $\frac{1}{2}$ inch, and since the polygon is a hexagon, side $C\,A$ = distance $A\,B$, or 1 inch (Art. **74**). Hence, $C\,A$ being the hypotenuse, the length of side $C\,D = \sqrt{1^2 - (\frac{1}{2})^2} = \sqrt{1^2 - .5^2} = \sqrt{.75}$, or .866 inch. Area of triangle $A\,C\,B = \frac{1 \times .866}{2} = .433$ sq. in.; area of the whole polygon $= .433 \times 6 = 2.598$ sq. in. Hence, the contents of the prism $= 2.598 \times 12 = 31.176$ cu. in. Ans.

EXAMPLE 3.—It is required to find the number of cubic feet of steam space in the boiler shown in Fig. 53. The boiler is 16 feet long between

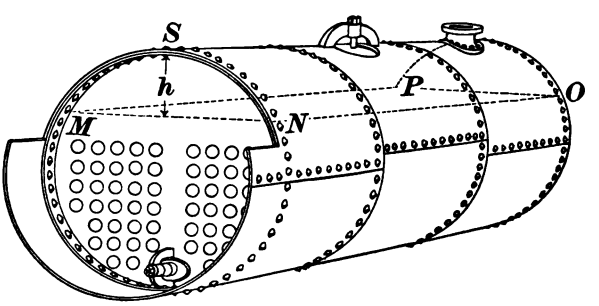

FIG. 53.

heads, 54 inches in diameter, and the mean water-line $M\,N$ is at a distance of 16 inches from the top of the boiler. The volume of the steam outlet casting may be neglected.

SOLUTION.—The volume of the steam space, which is that space within the boiler above the surface $M\,N\,O\,P$ of the water, is found by the rule for finding the volume of a prism or cylinder, the area $M\,N\,S$ being the base and the length $N\,O$ the altitude. First obtain the area of the segment $M\,N\,S$, whose height h is 16 inches, in square feet; then multiply the result by 16, the length of the boiler.

By the formula given in Art. **83**, the area of the segment =

$$\frac{4\,h^2}{3}\sqrt{\frac{D}{h}-.608}=\frac{4\times16^2}{3}\sqrt{\frac{54}{16}-.608}.$$

$$\frac{4\times16^2}{3}=341.33;\ \sqrt{\frac{54}{16}-.608}=\sqrt{2.767}=1.663.$$

Hence, the area $= 341.33 \times 1.663 = 567.63$ sq. in. This reduced to sq. ft. $= 567.63 \div 144 = 3.942$ sq. ft., and the volume therefore $= 3.942 \times 16 = 63.07$ cu. ft. Ans.

In the above solution, the space occupied by the stays is not considered, for sake of simplicity. They are not shown in the figure.

EXAMPLE 4.—In the above boiler there are 60 tubes, 3¼ inches outside diameter. How many gallons of water will it take to fill the boiler up to the mean water level, there being 231 cubic inches in a gallon?

SOLUTION.—Find the volume in cubic inches of that part of the boiler below the surface of the water $M\,N\,O\,P$, since the contents of a gallon is given in cubic inches, and from it subtract the volume of the tubes in cubic inches.

This may be done by first finding the *total* area of one end of the boiler in square inches, from it subtracting the area of the segment $M\,N\,S$, and the areas of the ends of the tubes in square inches, and then by multiplying the result by the length of the boiler *in inches*.

Total area of one end $= 54^2 \times .7854 = 2{,}290.23$ sq. in.

Area of segment $M\,N\,S$, as found in last example, $= 567.63$ sq. in.

Area of the end of one tube $= 3.25^2 \times .7854 = 8.2958$ sq. in.

Area of the ends of the 60 tubes $= 8.2958 \times 60 = 497.75$ sq. in.

Hence, the area to be subtracted $= 567.63 + 497.75 = 1{,}065.38$ sq. in. Subtracting, $2{,}290.23 - 1{,}065.38 = 1{,}224.85$ sq. in. $=$ net area.

The cubical contents $= 1{,}224.85 \times 16 \times 12 = 235{,}171.2$ cu. in. This divided by 231 will give the number of gallons; whence, $235{,}171.2 \div 231 = 1{,}018.06$ gal. of water. Ans.

EXAMPLES FOR PRACTICE.

1. Find the area in square inches of the convex surface of a bar of iron 4¼ inches in diameter and 8 feet 5 inches long. Ans. 1,348.53 sq. in.

2. Find the area of the entire surface of the above bar.

Ans. 1,376.9 sq. in.

3. What is the area of the entire surface of the hexagonal prism whose base is shown in Fig. 52? Ans. 77.196 sq. in.

4. A multitubular boiler has the following dimensions: diameter, 50 inches; length between heads, 15 feet; number of tubes, 56; outside diameter of tubes, 3 inches; distance of mean water-line from top of boiler, 16 inches. (*a*) Compute the steam space in cubic feet. (*b*) Find the number of gallons of water required to fill the boiler up to the mean water-line.

Ans. (*a*) 56.4 cu. ft.
(*b*) 800 gal.

THE PYRAMID AND CONE.

93. A **pyramid** (Fig. 54) is a solid whose base is a polygon and whose sides are triangles uniting at a common point, called the **vertex.**

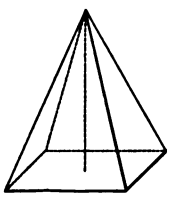

FIG. 54.

94. A **cone** (Fig. 55) is a solid whose base is a circle and whose convex surface tapers uniformly to a point called the **vertex.**

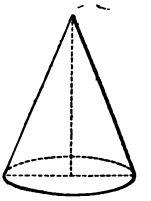

FIG. 55.

95. The **altitude** of a pyramid or cone is the perpendicular distance from the vertex to the base.

96. The **slant height** of a *pyramid* is a line drawn from the vertex perpendicular to one of the sides of the base. The slant height of a *cone* is any straight line drawn from the vertex to the circumference of the base.

97. To find the convex area of a pyramid or cone:

Rule 21.—*Multiply the perimeter of the base by one-half the slant height.*

EXAMPLE 1.—What is the convex area of a pentagonal pyramid if one side of the base measures 6 inches and the slant height is 14 inches?

SOLUTION.—The base of a pentagonal pyramid is a pentagon, and, consequently, has fives sides.

$6 \times 5 = 30$ inches, or the perimeter of the base. $30 \times \frac{14}{2} = 210$ sq. in., or the convex area. Ans.

EXAMPLE 2.—What is the entire area of a right cone whose slant height is 17 inches and whose base is 8 inches in diameter?

SOLUTION.—The perimeter of the base $= 8 \times 3.1416 = 25.1328$ in.

$$\text{Convex area} = 25.1328 \times \frac{17}{2} = 213.63 \text{ sq. in.}$$

$$\text{Area of base} = 8^2 \times .7854 = 50.27 \text{ sq. in.}$$

$$\text{Entire area} = 263.90 \text{ sq. in.} \quad \text{Ans.}$$

98. To find the volume of a pyramid or cone:

Rule 22.—*Multiply the area of the base by one-third of the altitude.*

EXAMPLE 1.—What is the volume of a triangular pyramid, each edge of whose base measures 6 inches and whose altitude is 8 inches?

SOLUTION.—Draw the base as shown in Fig. 56; it will be an equilateral triangle, all of whose sides are 6 inches long.

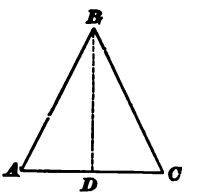

FIG. 56.

Draw a perpendicular BD from the vertex to the base; it will divide the base into two equal parts, since an equilateral triangle is also isosceles, and will be the altitude of the triangle. In order to obtain the area of the base, this altitude must be determined.

In the right triangle BDA, the hypotenuse $BA = 6$ inches and side $AD = 3$ inches, to find the other side,

$$BD = \sqrt{6^2 - 3^2} = 5.2 \text{ in., nearly.}$$

Area of the base, or BAC, $= \frac{6 \times 5.2}{2} = 15.6$ sq. in. Hence, the volume $= 15.6 \times \frac{8}{3} = 41.6$ cu. in. Ans.

EXAMPLE 2.—What is the volume of a cone whose altitude is 18 inches and whose base is 14 inches in diameter?

SOLUTION.—Area of the base $= 14^2 \times .7854 = 153.94$ sq. in. Hence, the volume $= 153.94 \times \frac{18}{3} = 923.64$ cu. in. Ans.

EXAMPLES FOR PRACTICE.

1. Find the convex surface of a square pyramid whose slant height is 28 inches and one edge of whose base is $7\frac{1}{2}$ inches long.

Ans. 420 sq. in.

2. What is the volume of a triangular pyramid, one edge of whose base measures 3 inches and whose altitude is 4 inches? Ans. 5.2 cu. in.

3. Find the volume of a cone whose altitude is 12 inches and the circumference of whose base is 31.416 inches. Ans. 314.16 cu. in.

NOTE.—Find the diameter of the base and then its area.

THE FRUSTUM OF A PYRAMID OR CONE.

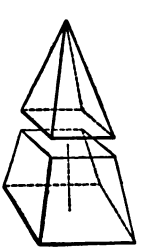
FIG. 57.

99. If a pyramid be cut by a plane, parallel to the base, so as to form two parts, as in Fig. 57, the lower part is called the **frustum** of the pyramid.

If a cone be cut in a similar manner, as in Fig. 58, the lower part is called the **frustum** of the cone.

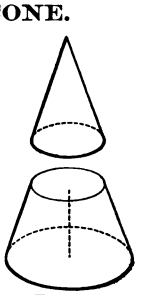
FIG. 58.

100. The upper end of the frustum of a pyramid or cone is called the **upper base,** and the lower end the **lower base.** The **altitude** of a frustum is the perpendicular distance between the bases.

101. To find the convex surface of a frustum of a pyramid or cone:

Rule 23.—*Multiply one-half the sum of the perimeters of the two bases by the slant height of the frustum.*

EXAMPLE 1.—Given, the frustum of a triangular pyramid, in which one side of the lower base measures 10 inches, one side of the upper base measures 6 inches, and whose slant height is 9 inches; find the area of the convex surface.

SOLUTION.— 10 in. $\times$ 3 = 30 in., the perimeter of the lower base.
6 in. $\times$ 3 = 18 in., the perimeter of the upper base.
$\frac{30 + 18}{2} = 24$ in., or one-half the sum of the perimeters of the two bases. $24 \times 9 = 216$ sq. in., the convex area. Ans.

EXAMPLE 2.—If the diameters of the two bases of a frustum of a cone are 12 inches and 8 inches, respectively, and the slant height is 12 inches, what is the entire area of the frustum?

SOLUTION.— $\frac{(12 \times 3.1416) + (8 \times 3.1416)}{2} \times 12 = 376.99$ sq. in., the area of the convex surface.
Area of the upper base = $8^2 \times .7854 = 50.27$ sq. in.
Area of the lower base = $12^2 \times .7854 = 113.1$ sq. in.
The entire area of the frustum = 376.99 + 50.27 + 113.1 = 540.36 sq. in. Ans.

102. To find the volume of the frustum of a pyramid or cone:

Rule 24.—*Add together the areas of the upper and lower bases and the square root of the product of the two areas; multiply the sum by one-third of the altitude.*

EXAMPLE 1.—Given, a frustum of a square pyramid (one whose base is a square); each edge of the lower base is 12 inches, each edge of the upper base is 5 inches, and its altitude is 16 inches; what is its volume?

SOLUTION.—Area of upper base = $5 \times 5 = 25$ sq. in.; area of lower base = $12 \times 12 = 144$ sq. in.; the square root of the product of the two areas = $\sqrt{25 \times 144} = 60$. Adding these three results, and multiplying by one-third the altitude, $25 + 144 + 60 = 229$; $229 \times \frac{16}{3} = 1{,}221\frac{1}{3}$ cu. in. = the volume. Ans.

EXAMPLE 2.—How many gallons of water will a round tank hold which is 4 feet in diameter at the top, 5 feet in diameter at the bottom, and 8 feet deep?

SOLUTION.—There are 231 cubic inches in a gallon, and the volume of the tank should be found in cubic inches. The tank is in the shape of the frustum of a cone. The upper diameter $= 4 \times 12 = 48$ inches; the lower diameter $= 5 \times 12 = 60$ inches, and the depth $= 8 \times 12 = 96$ inches. Area of upper base $= 48^2 \times .7854 = 1{,}809.56$ sq. in.; area of lower base $= 60^2 \times .7854 = 2{,}827.44$ sq. in.; $\sqrt{1{,}809.56 \times 2{,}827.44} = 2{,}261.95$.

Whence, $1{,}809.56 + 2{,}827.44 + 2{,}261.95 = 6{,}898.95$; $6{,}898.95 \times \frac{96}{3} = 220{,}766.4$ cu. in. = contents. Now, since there are 231 cu. in. in 1 gallon, the tank will hold $220{,}766.4 \div 231 = 955.7$ gal., nearly. Ans.

EXAMPLES FOR PRACTICE.

1. Find the convex surface of the frustum of a square pyramid, one edge of whose lower base is 15 inches long, one edge of whose upper base is 14 inches long, and whose slant height is 1 inch. Ans. 58 sq. in.

2. Find the volume of the above frustum, supposing its altitude to be 3 inches. Ans. 631 cu. in.

3. Find the volume of the frustum of a cone whose altitude is 12 feet and the diameters of whose upper and lower bases are 8 and 10 feet, respectively. Ans. 766.55 cu. ft.

4. If a tank had the dimensions of example 3, how many gallons would it hold? Ans. 5,734.2 gal., nearly.

THE SPHERE AND CYLINDRICAL RING.

103. A **sphere** (Fig. 59) is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within, called the center.

The word **ball,** or **globe,** is generally used instead of sphere.

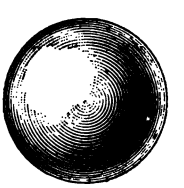

FIG. 59.

104. To find the area of the surface of a sphere:

Rule 25.—*Square the diameter and multiply the result by 3.1416.*

EXAMPLE.—What is the area of the surface of a sphere whose diameter is 14 inches?

SOLUTION. — Diameter squared $\times$ 3.1416 $= 14^2 \times 3.1416 = 14 \times 14 \times 3.1416 = 615.75$ sq. in. Ans.

From this it will be seen that the surface of a sphere equals the circumference of a great circle multiplied by the diameter, a rule often used; a *great circle* of a sphere is the intersection of its surface with a plane passing through its center; for instance, the *great circle* of a sphere 6 inches diameter is a circle of 6 inches diameter. Any number of *great circles* could be described on a given sphere.

105. To find the volume of a sphere:

Rule 26.—*Cube the diameter and multiply the result by .5236.*

EXAMPLE.—What is the weight of a lead ball 12 inches in diameter, a cubic inch of lead weighing .41 pound?

SOLUTION.—Diameter cubed $\times$ .5236 = 12 $\times$ 12 $\times$ 12 $\times$ .5236 = 904.78 cu. in., or the volume of the ball. The weight, therefore, = 904.78 $\times$ .41 = 370.96 lb. Ans.

106. To find the convex area of a cylindrical ring:

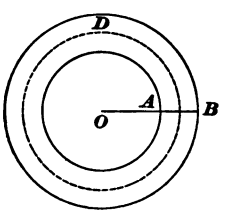

FIG. 60.

A **cylindrical ring** (Fig. 60) is a cylinder bent to a circle. The **altitude** of the cylinder before bending is the same as the length of the dotted center line *D*. The **base** will correspond to a cross-section on the line *A B* drawn from the center *O*. Hence, to find the convex area:

Rule 27.—*Multiply the circumference of an imaginary cross-section on the line A B by the length of the center line D.*

EXAMPLE.—If the outside diameter of the ring is 12 inches and the inside diameter is 8 inches, what is its convex area?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters $= \frac{12 + 8}{2} = 10$, and 10×3.1416 = 31.416 in., the length of the center line.

The radius of the inner circle is 4 inches; of the outside circle, 6 inches; therefore, the diameter of the cross-section on the line *A B* is 2 inches. Then, $2 \times 3.1416 = 6.2832$ in., and 6.2832×31.416 = 197.4 sq. in., the convex area. Ans.

107. To find the volume of a cylindrical ring:

Rule 28.—*The volume will be the same as that of a cylinder whose altitude equals the length of the dotted center line D* (Fig. 61) *and whose base is the same as a cross-section of the ring on the line A B drawn from the center O. Hence, to find the volume of a cylindrical ring, multiply the area of an imaginary cross-section on the line A B by the length of the center line D.*

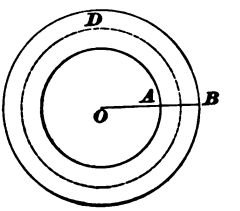

FIG. 61.

EXAMPLE.—What is the volume of a cylindrical ring whose outside diameter is 12 inches and whose inside diameter is 8 inches?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters $= \frac{12 + 8}{2} = 10$.

$10 \times 3.1416 = 31.416$ inches, the length of the center line.

The radius of the outside circle = 6 inches; of the inside circle = 4 inches; therefore, the diameter of the cross-section on the line *A B* = 2 inches.

Then, $2^2 \times .7854 = 3.1416$ sq. in., the area of the imaginary cross-section.

And $3.1416 \times 31.416 = 98.7$ cu. in., the volume. Ans.

EXAMPLES FOR PRACTICE.

1. What is the volume of a sphere 30 inches in diameter?
Ans. 14,137.2 cu. in

2. How many square inches in the surface of the above sphere?
Ans. 2,827.44 sq. in.

3. Required the area of the convex surface of a circular ring, the outside diameter of the ring being 10 inches and the inside diameter $7\frac{1}{2}$ inches. Ans. 107.95 sq. in.

4. Find the cubical contents of the ring in the last example.
Ans. 33.73 cu. in.

5. The surface of a sphere contains 314.16 square inches. What is the volume of the sphere? Ans. 523.6 cu. in.

PRINCIPLES OF MECHANICS.

MATTER AND ITS PROPERTIES.

DEFINITION OF MECHANICS.

1. **Mechanics** is that science which treats of the action of forces on bodies and the effects that they produce; it treats of the laws that govern the movement and equilibrium of bodies and shows how they may be applied.

MATTER.

2. **Matter** is anything that occupies space. It is the substance of which all bodies consist. Matter is composed of *molecules* and *atoms*.

3. A **molecule** is the smallest portion of matter that can exist without changing its nature.

4. An **atom** is an indivisible portion of matter.

Atoms unite to form molecules, and a collection of molecules forms a mass or body.

A drop of water may be divided and subdivided until each particle is so small that it can only be seen by the most powerful microscope, but each particle will still be water.

Now, imagine the division to be carried on still further, until a limit is reached beyond which it is impossible to go without changing the nature of the particle. The particle of water is now so small that, if it be divided again, it will

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cease to be water, and will be something else; we call this particle a *molecule*.

If a molecule of water be divided, it will yield 2 atoms of hydrogen gas and 1 of oxygen gas. If a molecule of sulphuric acid be divided, it will yield 2 atoms of hydrogen, 1 of sulphur, and 4 of oxygen.

5. Bodies are composed of collections of molecules. Matter exists in three conditions or forms: *solid*, *liquid*, and *gaseous*.

6. A **solid body** is one whose molecules change their relative positions with great difficulty; as iron, wood, stone, etc.

7. A **liquid body** is one whose molecules tend to change their relative positions easily. Liquids readily adapt themselves to the shape of vessels that contain them, and their upper surface always tends to become perfectly level. Water, mercury, molasses, etc., are liquids.

8. A **gaseous body,** or gas, is one whose molecules tend to separate from one another; as air, oxygen, hydrogen, etc.

Gaseous bodies are sometimes called **aeriform** (air-like) **bodies.** They are divided into two classes: the so-called "*permanent*" *gases* and *vapors*.

9. A **permanent gas** is one that remains a gas at ordinary temperatures and pressures.

10. A **vapor** is a body that at ordinary temperatures is a liquid or solid, but when heat is applied, becomes a gas, as steam.

One body may, under different conditions, exist in all three states; as, for example, mercury, which at ordinary temperatures is a liquid, becomes a solid (freezes) at 40° below zero, and a vapor (gas) at 600° above zero. By means of great cold, all gases, even hydrogen, have been liquefied, and many solidified.

By means of heat, all solids have been liquefied, and a great many vaporized. It is probable that, if we had the means of producing sufficiently great extremes of heat and

cold, all solids might be converted into gases, and all gases into solids.

11. Every portion of matter possesses certain qualities called *properties.* Properties of matter are divided into two classes: *general* and *special.*

12. **General properties of matter** are those that are common to all bodies. They are as follows: *Extension*, *impenetrability*, *weight*, *indestructibility*, *inertia*, *mobility*, *divisibility*, *porosity*, *compressibility*, *expansibility*, and *elasticity.*

13. **Extension** is the property of occupying space. Since all bodies must occupy space, it follows that extension is a general property.

14. By **impenetrability** we mean that no two bodies can occupy exactly the same space at the same time.

15. **Weight** is the measure of the earth's attraction upon a body. All bodies have weight. In former times it was supposed that gases had no weight, since, if unconfined, they tend to move away from the earth, but, nevertheless, they will finally reach a point beyond which they cannot go, being held in suspension by the earth's attraction. Weight is measured by comparison with a standard. The standard is a bar of platinum weighing 1 pound, owned and kept by the Government.

16. **Inertia** means that a body cannot put itself in motion nor bring itself to rest. To do either it must be acted upon by some force.

17. **Mobility** means that a body can be changed in position by some force acting upon it.

18. **Divisibility** is that property of matter by virtue of which a body may be separated into parts.

19. **Porosity** is the term used to denote the fact that there is space between the molecules of a body. The molecules of a body are supposed to be spherical, and, hence, there is space between them, as there would be between

peaches in a basket. The molecules of water are larger than those of salt; so that when salt is dissolved in water, its molecules wedge themselves between the molecules of the water, and, unless too much salt is added, the water will occupy no more space than it did before. This does not prove that water is penetrable, for the molecules of salt occupy the space that the molecules of water did not.

Water has been forced through iron by pressure, thus proving that iron is porous.

20. Compressibility is a natural consequence of porosity. Since there is space between the molecules, it is evident that by means of force (pressure) they can be brought closer together, and thus the body be made to occupy a smaller space.

21. Expansibility is the term used to denote the fact that the molecules of a body will, under certain conditions (when heated, for example), move farther apart, and so cause the body to *expand*, or occupy a greater space.

22. Elasticity is that property of matter which enables a body when distorted within certain limits to resume its original form when the distorting force is removed. Glass, ivory, and steel are very elastic, clay and putty in their natural state being very slightly so.

23. Indestructibility is the term used to denote the fact that we cannot *destroy* matter. A body may undergo thousands of changes, be resolved into its molecules, and its molecules into atoms, which may unite with other atoms to form other molecules and bodies entirely different in appearance and properties from the original body, but the same number of atoms remain. The whole number of atoms in the universe is exactly the same now as it was millions of years ago, and will always be the same. *Matter is indestructible.*

24. Special properties are those that are not possessed by all bodies. Some of the most important are as

follows: *hardness*, *tenacity*, *brittleness*, *malleability*, and *ductility*.

25. Hardness.—A piece of copper will scratch a piece of wood, steel will scratch copper, and tempered steel will scratch steel in its ordinary state. We express all this by saying that steel is *harder* than copper, and so on. Emery and corundum are extremely hard, and the diamond is the hardest of all known substances. It can only be polished with its own powder.

26. Tenacity is the term applied to the power with which some bodies resist a force tending to pull them apart. Steel is very tenacious.

27. Brittleness. — Some bodies possess considerable power to resist either a pull or a pressure, but they are easily broken when subjected to shocks or jars; for example, good glass will bear a *greater* compressive force than most woods, but may be easily broken when dropped upon a hard floor; this property is called *brittleness*.

28. Malleability is that property which permits of some bodies being hammered or rolled into sheets. Gold is the most malleable of all substances.

29. Ductility is that property which enables some bodies to be drawn into wire. Platinum is the most ductile of all substances.

MOTION AND VELOCITY.

DEFINITIONS.

30. Motion is the opposite of rest and indicates a changing of position in relation to some object which is for that purpose regarded as being fixed. If a large stone is rolled down hill, it is in motion in relation to the hill.

If a person is on a railway train and walks in the opposite direction from that in which the train is moving, and with the same speed, he will be in motion as regards the

train, but at rest with respect to the earth, since, until he gets to the end of the train, he will be directly over the spot at which he was when he started to walk.

31. The **path of a body in motion** is the line described by a certain point in the body called its center of gravity. No matter how irregular the shape of the body may be, nor how many turns and twists it may make, the line that indicates the direction of this point for every instant that it is in motion is the path of the body.

32. **Velocity** is rate of motion. It is measured by a unit of space passed over in a unit of time. When equal spaces are passed over in equal times, the velocity is said to be **uniform.**

If the flywheel of an engine keeps up a constant speed of a certain number of revolutions per minute, the velocity of any point is uniform. A railway train having a constant speed of 40 miles per hour moves 40 miles every hour, or $\frac{40}{60} = \frac{2}{3}$ mile every minute, and since equal spaces are passed over in equal times, the velocity is uniform.

33. **Variable Velocity.**—When a body moves in such a way that the spaces passed over in equal periods of time are unequal, its velocity is said to be **variable.**

34. The rate of motion of a body with a variable velocity may increase or decrease at a uniform rate. When the velocity varies in either of these ways, the body is said to have a **uniformly varying velocity.**

The most familiar example of uniformly varying velocity is a falling weight. Suppose a stone is dropped from a high bridge. It starts from a state of rest with no velocity, but its velocity constantly increases until it strikes. Its increase in velocity during any equal units of time is nearly the same. Thus, at the end of the first second its velocity will have increased from 0 to a rate of 32.16 feet per second, nearly; during the next second the velocity will have increased by the same amount, making the velocity at the end of the second second 64.32 feet. At the end of the third

second a like increase will have taken place, and the velocity will then be $3 \times 32.16 = 96.48$ feet per second.

35. The change in the velocity of a body during a period of time is called its **acceleration** for that period. Thus, in the case of the falling weight just considered, the change of 32.16 feet per second that takes place in its velocity during each second is its acceleration in feet per second for each second considered. If the period of time considered had been 2 seconds, the acceleration would have been the increase in velocity during this time, that is, 64.32 feet per second for each 2 seconds considered.

36. The change in velocity may be from a higher to a lower rate. Thus, when a stone is thrown upwards, it leaves the hand with a given velocity; its upward motion is constantly resisted by the force of gravity and the resistance of the air, and in consequence of these resistances, it moves slower and slower until it finally stops and begins to return to the earth. A change in velocity of this kind is sometimes called **retardation.**

37. The **mean** or **average velocity** of a body moving with a variable velocity can only be given for a stated period of time, and is numerically equal to the uniform velocity that will take the body over the same distance in the same time.

RULES FOR VELOCITY PROBLEMS.

38. Uniform and Average Velocity.—

Let d = distance;
t = time;
v = velocity.

Rule 1.—*To find the uniform or the average velocity that a body must have to pass over a certain distance or space in a given time, divide the distance by the time.*

Or, $$v = \frac{d}{t}.$$

EXAMPLE 1.—The piston of a steam engine travels 3,000 feet in 5 minutes; what is its average velocity in feet per minute?

SOLUTION.—Here 3,000 feet is the distance, and 5 minutes is the time. Applying the rule, $3{,}000 \div 5 = 600$ ft. per min. Ans.

CAUTION.—Before applying the above or any of the succeeding rules, care must be taken to reduce the values given to the denominations required in the answer. Thus, in the above example, if the velocity is required in feet per second instead of in feet per minute, the 5 minutes must be reduced to seconds before dividing. The operation will then be: 5 minutes $= 5 \times 60 = 300$ seconds. Applying the rule, $3{,}000 \div 300 = 10$ ft. per sec. Ans.

If the velocity is required in inches per second, it is necessary to reduce the 3,000 feet to inches and the 5 minutes to seconds, before dividing. Thus, 3,000 feet $\times$ 12 = 36,000 inches. 5 minutes $\times$ 60 = 300 seconds. Now applying the rule, $36{,}000 \div 300 = 120$ in. per sec. Ans.

EXAMPLE 2.—A railroad train travels 50 miles in $1\frac{1}{2}$ hours; what is its average velocity in feet per second?

SOLUTION.—Reducing the miles to feet and the hours to seconds, 50 miles $\times$ 5,280 = 264,000 feet. $1\frac{1}{2}$ hours $\times$ 60 $\times$ 60 = 5,400 seconds. Applying the rule, $264{,}000 \div 5{,}400 = 48\frac{8}{9}$ ft. per sec. Ans.

39. If the uniform velocity (or the average velocity) and the time are given, and it is required to find the distance that a body having the given velocity will travel in the given time:

Rule 2.—*Multiply the velocity by the time.*

Or, $$d = v\,t.$$

EXAMPLE 1.—The velocity of sound in still air is 1,092 feet per second; how many miles will it travel in 16 seconds?

SOLUTION.—Reducing the 1,092 feet to miles, $1{,}092 \div 5{,}280 = \frac{1092}{5280}$. Applying the rule, $\frac{1092}{5280} \times 16 = 3.31$ mi., nearly. Ans.

EXAMPLE 2.—The piston speed of an engine is 11 feet per second, how many miles does the piston travel in 1 hour and 15 minutes?

SOLUTION.— 1 hour and 15 minutes reduced to seconds = 4,500 seconds = the time. 11 feet reduced to miles = $\frac{11}{5280}$ mile = velocity in miles per second. Applying the rule, $\frac{11}{5280} \times 4{,}500 = 9.375$ mi. Ans.

40. If the distance through which a body moves is given, and also its average or uniform velocity, and it is desired to know how long it takes the body to move through the given distance:

Rule 3.—*Divide the distance, or space passed over, by the velocity.*

Or, $$t = \frac{d}{v}.$$

EXAMPLE 1.—Suppose that the radius of the crank of a steam engine is 15 inches and that the shaft makes 120 revolutions per minute; how long will it take the crankpin to travel 18,849.6 feet?

SOLUTION.—Since the radius, or distance from the center of the shaft to the center of the crankpin, is 15 inches, the diameter of the circle it moves in is 15 inches $\times$ 2 = 30 inches = 2.5 feet. The circumference of this circle is 2.5 $\times$ 3.1416 = 7.854 feet. 7.854 $\times$ 120 = 942.48 feet = distance that the crankpin travels in 1 minute = velocity in feet per minute. Applying the rule, 18,849.6 $\div$ 942.48 = 20 min. Ans.

EXAMPLE 2.—A point on the rim of an engine flywheel travels at the rate of 150 feet per second; how long will it take it to travel 45,000 feet?

SOLUTION.—Applying the rule,

45,000 $\div$ 150 = 300 sec. = 5 min. Ans.

EXAMPLES FOR PRACTICE.

1. A locomotive has drivers 80 inches in diameter. If they make 293 revolutions per minute, what is the velocity of the train in (*a*) feet per second? (*b*) miles per hour?

Ans. (*a*) 102.277 ft. per sec.
(*b*) 69.734 mi. per hr.

2. Assuming the velocity of steam as it enters the cylinder to be 900 feet per second, how far can it travel, if unobstructed, during the time the flywheel of an engine revolves 7 times, if the number of revolutions per minute are 120? Ans. 3,150 ft.

3. The average speed of the piston of an engine being 528 feet per minute, how long will it take the piston to travel 4 miles? Ans. 40 min.

4. A speed of 40 miles per hour equals how many feet per second? Ans. 58⅔ ft.

5. The earth turns around once in 24 hours. If the diameter be taken as 8,000 miles, what is the velocity of a point on the equator in miles per minute? Ans. 17.45⅓ mi. per min.

6. The stroke of an engine is 28 inches. If the engine makes 11,400 strokes per hour, (*a*) what is its speed in feet per minute? (*b*) how far will this piston travel in 11 minutes?

Ans. (*a*) 443⅓ ft. per min.
(*b*) 4,876 ft. 8 in.

FORCE.

GENERAL PRINCIPLES.

41. **Force** is known only by the effects it produces on matter. Forces cannot, therefore, be compared in the same way that quantities of matter or distances are compared; the only way in which forces can be examined in reference to one another is by noting their relative effects in the production of motion or a change of state in matter.

The most familiar conception of force is that of a push or pull that tends to produce or destroy motion. If the force is great enough, its effect is seen in a change in the state of motion of the body on which it acts; that is, it either produces motion in the body or destroys some or all of the motion already existing. If, however, the body acted on by a force is so situated that the force applied to it is opposed by a resisting force of equal magnitude, no change in motion is produced. In this case the force is not perceivable, unless some other force is introduced whose effects will reveal the existence of the first force.

42. Forces are called by various names according to the ways in which they manifest themselves. Manifestations of force are: *attraction*, *repulsion*, *cohesion*, *adhesion*, *acceleration*, *retardation*, *resistance*, etc., and the forces producing these manifestations are called *attractive*, *repulsive*, *cohesive*, *adhesive*, *accelerating*, *retarding*, *resisting*, etc. forces.

43. **Comparison of Forces.**—In considering the effects of a force on a body, some standard of comparison must be used. The standard most commonly adopted in English-speaking countries is the **pound**, which is the force required to raise a standard mass of matter from the ground under certain specified conditions.

In practice, force is always regarded as a pressure; that is, a force is considered the equivalent of the pressure exerted by a weight. For example, the effect of a force of

20 pounds acting upon a body is the same as the pressure of 20 pounds exerted by a weight of 20 pounds.

44. In order that we may compare the effect of a force on a body with that of another force on another body, it is necessary that the following *three conditions* be fulfilled in regard to both bodies:

1. The point of application, or point at which the force acts upon the body, must be known.

2. The direction of the force, or, what is the same thing, the straight line along which the force tends to move the point of application, must be known.

3. The magnitude or value of the force in comparison with the given standard must be known.

45. Reduction of Forces.—When a number of forces act upon a body and produce a certain effect, it is often necessary to find a *single* force that, when substituted for the given forces, will produce the *same* effect. In order to find the point of application, the direction, and the magnitude of this single force, it is necessary to know the above three conditions of every one of the given forces.

NEWTON'S LAWS OF MOTION.

46. The fundamental principles of the relations between force and motion were first stated by Sir Isaac Newton, and are called *Newton's Three Laws of Motion.* They are as follows:

1. *All bodies continue in a state of rest, or of uniform motion in a straight line, unless acted upon by some external force that compels a change.*

2. *Every motion or change of motion is proportional to the acting force, and takes place in the direction of the straight line along which the force acts.*

3. *To every action there is always opposed an equal reaction.*

These laws in their accepted form, as just given, have been more or less indirectly derived from experiment, but

they are so comprehensive as to defy complete experimental verification.

47. Exemplification of the First Law.—In the first law of motion, it is stated that a body once set in motion by any force, no matter how small, will move forever in a straight line, and always with the same velocity, unless acted upon by some other force that compels a change. It is not possible to actually verify this law, on account of the earth's attraction for all bodies, but, from astronomical observations, we are certain that the law is true. This law is often called the **law of inertia.**

48. The word **inertia** is so abused that a full understanding of its meaning is important. Inertia is not a force, although it is often so called. If a force acts upon a body and puts it in motion, the effect of the force is stored in the body, and a second body, in stopping the first, will receive a blow equal in every respect to the original force, assuming that there has been no resistance of any kind to the motion of the first body.

It is dangerous for a person to jump from a fast-moving train, for the reason that, since his body has the same velocity as the train, it has the same force stored in it that would cause a body of the same weight to take the same velocity as the train, and the effect of a sudden stoppage is the same as the effect of a blow necessary to give the person that velocity.

By "bracing" himself and jumping in the same direction that the train is moving, and running, he brings himself gradually to rest, and thus reduces the danger. If a body is at rest, it must be acted upon by a force in order to be put in motion, and, no matter how great the force may be, it cannot be *instantly* put in motion.

The resistance thus offered to being put in motion is commonly, but erroneously, called the *resistance of inertia.* It should be called the *resistance due to inertia.*

49. Exemplification of the Second Law.—From the second law, we see that if two or more forces act upon a

body, their final effect on the body will be in proportion to their magnitudes and to the directions in which they act.

Thus, if the wind is blowing due west with a velocity of 50 miles per hour, and a ball is thrown due north with the same velocity, or 50 miles per hour, the wind will carry the ball west while the force of the throw is carrying it north, and the combined effect will be to cause it to move northwest.

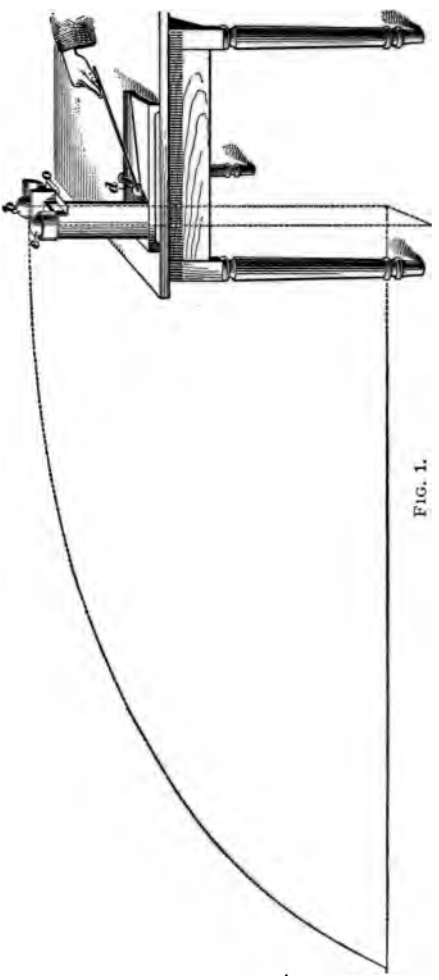

FIG. 1.

The amount of departure from due north will be proportional to the force of the wind and independent of the velocity due to the force of the throw.

50. In Fig. 1 a ball *c* is supported in a cup, the bottom of which is attached to the lever *o* in such a manner that *o* will swing the bottom horizontally and allow the ball to drop. Another ball *b* rests in a horizontal groove that is provided with a slit in the bottom. A swinging arm is actuated by the spring *d* in such a manner that, when drawn back, as shown, and then released, it will strike the lever *o* and the ball *b* at the same time. This gives *b*

an impulse in a horizontal direction, and swings *o* so as to allow *c* to fall.

On trying the experiment, it is found that *b* follows a path shown by the curved dotted line, and reaches the floor at the same instant as *c*, which drops vertically. This shows that the force that gave the first ball its horizontal movement had no effect on the vertical force that compelled both balls to fall to the floor; the vertical force produces the same effect as if the horizontal force had not acted. The second law may also be stated as follows: *A force has the same effect in producing motion, whether it acts upon a body at rest or in motion, and whether it acts alone or with other forces.*

51. Exemplification of the Third Law.—The third law states that action and reaction are equal. A man cannot lift himself by his boot straps for the reason that he presses downwards with the same force that he pulls upwards; the downward reaction equals the upward action, and is opposite to it.

In springing from a boat, we must exercise caution or the reaction will drive the boat from the shore. When we jump from the ground, we tend to push the earth from us, while the earth reacts and pushes us from it.

EXAMPLE.—Two men pull on a rope in opposite directions, each exerting a force of 100 pounds; what is the force that the rope resists?

SOLUTION.—Imagine the rope to be fastened to a tree, and that one man pulls with a force of 100 pounds. The rope evidently resists 100 pounds. According to Newton's third law, the reaction of the tree is also 100 pounds. Now, suppose the rope to be slacked, but that one end is still fastened to the tree; the second man then takes hold of the rope near the tree, and pulls with a force of 100 pounds, the first man pulling as before. The resistance of the rope is 100 pounds, as before, since the second man merely takes the place of the tree. He is obliged to exert a force of 100 pounds to keep the rope from slipping through his fingers. If the rope is passed around the tree, and each man pulls an end with a force of 100 pounds in the same and parallel directions, the stress in the rope is 100 pounds, as before, but the tree must resist the pull of both men, or 200 pounds.

52. **Dynamics,** also called **kinetics,** is that branch of mechanics that treats of forces and their effects when they *produce a change in motion* in the bodies on which they act.

53. **Statics** is that branch of mechanics that treats of forces and their effects when they do *not produce a change in motion* in the bodies on which they act.

GRAVITATION AND WEIGHT.

54. Every body in the universe exerts a certain attractive force on every other body, which tends to draw the two together. To scientists, this attractive force is known as **gravitation.**

If a body is held in the hand, a downward pull is felt, and if the hold is loosened, the body will fall to the ground. This pull, which we commonly call **weight,** is the attraction between the earth and the body.

55. The attraction between the earth and bodies at or near its surface is denoted by the term **force of gravity.** This attraction is generally considered as acting along the line joining the center of gravity of the body and the center of the earth. By **center of gravity** is meant that point of a body at which its whole weight may be assumed to be concentrated.

56. The weight of a body is directly proportional to the force of gravity. From this it follows that the weight of a body can only be uniform everywhere if the force of gravity is uniform. As a matter of fact, the force of gravity varies in different locations; consequently, the weight of the body is not the same at all points on the surface of the earth. This has been conclusively shown by sensitive spring balances.

ACCELERATING AND RETARDING FORCES.

57. According to the first law of motion, if a body is set in motion by a force and the force then ceases to act, the body will continue to move at the rate it had at the

instant the action of the force was discontinued, unless acted upon by some other force.

If, however, a force acts upon a body for a given period of time, say 1 second, and imparts to it a certain amount of motion, and then, instead of ceasing to act, acts with the same intensity during the next second, it will impart to the body an increase in velocity equal to the velocity imparted during the first second. Then, the velocity at the end of the second second will be twice that at the end of the first. During the third second a like increase in velocity will be produced, making the velocity at the end of the third second three times as great as at the end of the first. This uniform increase in velocity will continue as long as the constant force continues to act on the body. A constant force when producing a constant acceleration is called a **constant accelerating force.**

58. If a constant force is applied to a body in motion in such a manner that it opposes the motion, its effect will be to reduce the motion by a certain amount, which will be the same for each second during which it acts. In this case, the force is called a **constant retarding force.**

We thus see that the effect of a constant force acting upon a body in motion is to produce a uniform acceleration or retardation in the velocity of motion of the body, it being assumed that the motion of the body is not opposed by varying resisting forces.

59. Acceleration Due to the Force of Gravity.—If a body falls freely under the action of the force of gravity, its velocity will increase at a uniform rate; in other words, it will be accelerated. Since the force of gravity varies in different localities, it follows that the acceleration produced by it is not everywhere the same. The greatest range in the acceleration due to the force of gravity in the United States is from a minimum of about 32.089 feet per second up to a maximum of about 32.186 feet per second for each second. In the latitude of Scranton, Pa., and at the level of the sea, the acceleration is nearly 32.16 feet per second;

this value will be used in all calculations in this Course that involve the use of acceleration due to the force of gravity. In accordance with the practice of most scientific writers, we will denote the acceleration due to the force of gravity by the letter g.

60. Mass.—If the weight of a body at any place, as determined by a spring balance, is divided by the acceleration due to the force of gravity at that place, a numerical value will be obtained that, for the same body, will be the same wherever it may be weighed. This quotient is called the **mass** of the body, and is generally designated by the letter m.

Rule 4.—*To find the mass of a body, divide its weight by the acceleration due to the force of gravity.*

Let W = the weight of a body;
g = acceleration due to gravity;
m = mass of the body.

Then, $$m = \frac{W}{g}.$$

EXAMPLE.—What is the mass of a body weighing 96.48 pounds?

SOLUTION.—Applying the rule, $m = \frac{96.48}{32.16} = 3$. Ans.

61. Law of Gravitation.—The attractive force by which one body tends to draw another body toward it is directly proportional to its mass and inversely proportional to the square of the distance between their centers of gravity.

62. Laws of Weight.—

1. *Bodies weigh most at the surface of the earth. Below the surface, the weight decreases directly as the distance to the center of the earth decreases.*

2. *Above the surface, the weight decreases inversely as the square of the distance.*

63. Change in Motion of a Body.—A change in the motion of a body cannot take place without the action of an accelerating or retarding force. The force required to

produce a given acceleration or retardation in a body is given by the following rule, where

f = force in pounds;
a = acceleration or retardation in feet per second.

Rule 5.—*Multiply the mass of the body by the acceleration, or retardation, in feet per second.*

Or, $$f = m\,a.$$

Since $m = \frac{W}{g}$ (see rule 4), this may also be written $f = \frac{W}{g}a$.

EXAMPLE.—What force will be required to give a body weighing 90 pounds an acceleration of 5 feet per second ?

SOLUTION.—Applying rule 5, we get

$$f = \frac{90}{32.16} \times 5 = 13.99+ \text{ lb., say } 14 \text{ lb. Ans.}$$

64. According to the first law of motion, a body in motion not acted upon by any external force will continue its motion without any further application of a force. In practice, however, the motion of a body is always opposed by some resisting force or forces. According to the third law of motion, the force required to overcome the resistance is equal to the resistance.

The opposing forces are usually constant, or nearly so. Taking the opposing forces into account, the actual force required to accelerate a body meeting with resistance will be the sum of the accelerating force and the opposing forces.

ILLUSTRATION.—Imagine a weight of 321.6 pounds to be lying on a smooth plane surface. Assume that it has been determined experimentally that a force of 100 pounds is required to be exerted continually to overcome the friction between the weight and the surface. What force will be required to produce an acceleration of 2 feet per second ?

By rule 5, the accelerating force is $\frac{321.6}{32.16} \times 2 = 20$ pounds. As a force of 100 pounds must be exerted continually to overcome the resistance due to friction, a force of $100 + 20 = 120$ pounds will be required to produce the required acceleration.

65. The question is often asked, what force is required to start a flywheel and keep it going at a stated number of

revolutions per minute? This question, or similar questions, cannot be answered without knowing the time in which the flywheel is to attain the given speed. An accelerating force depending on the mass of the wheel, its diameter, the distribution of the material of which it is made, the time of acceleration, and the constant resistances, is required to bring the wheel up to its speed. When this speed has been attained, the force required to keep it going will be that required to overcome the frictional and air resistances.

MOMENTUM.

66. Experience teaches us that the same force acting upon bodies of different weights produces different effects. For example, if a given force imparts a velocity of 10 feet per second in a certain time to a body weighing 1 pound, we know from experience and observation that it cannot impart the same velocity in the same time when acting upon a body weighing 1,000 pounds.

Scientists have shown that the velocity imparted to a body in a given time by a force *varies directly as the force* and *inversely as the mass* of the body. Hence, forces may be compared with one another by comparing their effects in imparting velocities to bodies whose masses are known.

67. The product obtained by multiplying the mass of a body by its velocity in feet per second is called the **momentum** of the body; it represents the magnitude of the force that will produce the given velocity in the body in 1 second. Hence, we may call momentum the **time effect** of a force.

68. According to the third law of motion, action and reaction are equal to each other. Consequently, if the force required to produce a stated momentum in a given time is known, it is likewise known what force is required to *destroy* this momentum, or to bring the body to rest, in an equal period of time.

When a liquid body is flowing in a stream, as from a nozzle, the weight to be considered in problems involving

momentum is the weight of the liquid discharged in 1 second. For example, let it be required to estimate the force with which a man must hold the nozzle of a fire hose to prevent its slipping through his hands when a stream of water issues from it with a velocity of 20 feet per second, the area of the opening in the nozzle being 1 square inch and the weight of a cubic inch of water .0361 pound. The volume of water discharged in 1 second is $1 \times 12 \times 20 = 240$ cubic inches, and since the weight of 1 cubic inch is .0361 pound, the weight discharged per second is $240 \times .0361 = 8.664$ pounds. The momentum of the stream is $\frac{8.664}{32.16} \times 20 = 5.38$ pounds. This is the constant force required to give a body of water weighing 8.664 pounds a velocity of 20 feet per second, in 1 second; it also represents the magnitude of the reaction on the nozzle, and the man must hold the nozzle with a force equal to the reaction, or 5.38 pounds, in order to prevent its slipping through his hands.

WORK, POWER, AND ENERGY.

WORK.

69. Work is the overcoming of resistance continually occurring along the path of motion.

Motion in itself is not work; a force must overcome a resistance in order that work may be done.

70. Unit of Work.—The unit by which the work done by a force is measured is the work done in overcoming a resistance of 1 pound through a space of 1 foot; this unit is called a **foot-pound.** According to the definition, it may be considered as the force required to raise 1 pound 1 foot vertically. All work is measured by this standard.

A horse going up a hill does an amount of work equal to its own weight, plus the weight of the wagon and its contents, plus the frictional resistances reduced to an equivalent

weight, multiplied by the vertical height of the hill. Thus, if the horse weighs 1,200 pounds, the wagon and contents 1,200 pounds, and the frictional resistances equal 400 pounds, then if the vertical height of the hill is 100 feet, the work done is equal to $(1,200 + 1,200 + 400) \times 100 = 280,000$ foot-pounds.

Rule 6.—*In all cases the force (or resistance) multiplied by the distance through which it acts equals the work. If a weight is raised, the weight multiplied by the vertical height of the lift equals the work.*

71. The total amount of work done in overcoming a given resistance through a given distance is independent of time; that is, it is immaterial whether it takes 1 minute or 1 year in which to do it; but in order to compare the *rate* at which work is done by different machines with a common standard, time must be considered. If one machine does a certain amount of work in 10 minutes and another machine does exactly the same amount of work in 5 minutes, the second machine can do twice as much work as the first in an equal period of time.

POWER.

72. **Power** is a term used to denote the rate at which work is done.

73. The common unit used for expressing the rate at which work is done is the **horsepower.**

One horsepower is 33,000 foot-pounds of work per minute; in other words, it is 33,000 pounds raised vertically 1 foot in 1 minute, or 1 pound raised vertically 33,000 feet in 1 minute, or any combination that will, when multiplied together, give 33,000 foot-pounds in 1 minute. Thus, the work done in raising 110 pounds vertically 5 feet in 1 second is a horsepower; for, since in 1 minute there are 60 seconds, $110 \times 5 \times 60 = 33,000$ foot-pounds in 1 minute.

EXAMPLE.—In a steam engine the force impelling the piston forwards and backwards is 10,000 pounds. This force overcomes the

At a point equal to one-half the height of the fall, the potential and kinetic energies are equal. At the end of the first quarter the potential energy is three-fourths and the kinetic energy one-fourth; at the end of the third quarter the potential energy is one-fourth and the kinetic energy three-fourths.

A pound of coal has a certain amount of potential energy. When the coal is burned, the potential energy is liberated and changed into kinetic energy in the form of heat. The kinetic energy of the heat changes water into steam, which thus has a certain amount of potential energy. The steam acting on the piston of an engine causes it to move through a certain space, thus overcoming a resistance, changing the potential energy of the steam into kinetic energy, and thus doing work.

Potential energy, *then, is the energy stored within a body that may be liberated and produce motion, thus generating kinetic energy and enabling work to be done.*

78. The principle of **conservation of energy** teaches that energy, like matter, can never be destroyed. If a clock is put in motion, the potential energy of the spring is changed into kinetic energy of motion, which turns the wheels, thus producing friction.

The friction produces heat, which dissipates into the surrounding air, but still the energy is not destroyed—it merely exists in another form.

79. Work of Acceleration and Retardation. — The theoretical amount of work that must be done in order to start a body from a state of rest and accelerate it until it reaches a given velocity is equal to the kinetic energy of the body at the given velocity. Likewise, the theoretical amount of work that must be done on a moving body to retard it and finally bring it to rest is equal to the kinetic energy the moving body possessed at the moment retardation began. The work that must be done in changing the velocity of a body is equal to the difference in the kinetic energies at the initial and final velocities. Since the motion

of all bodies is opposed by some resisting force or forces, the actual amount of work required to give a body the given velocity will be the sum of the work of acceleration and the work required to overcome the outside resisting forces.

EXAMPLE 1.—A body weighing 1,000 pounds is started from rest and is to attain a velocity of 88 feet per second in 2 minutes, passing over a distance of 5,280 feet in that time. If a constant force of 120 pounds must be exerted to overcome the frictional resistances, what work must be done?

SOLUTION.—According to this article, the work required to accelerate the body is $\frac{1{,}000 \times 88^2}{64.32} = 120{,}398$ foot-pounds. As a constant force of 120 pounds must act through a distance of 5,280 feet to overcome the frictional resistances, the work done in overcoming friction is $5{,}280 \times 120 = 633{,}600$ foot-pounds. Then, the total amount of work done is $120{,}398 + 633{,}600 = 753{,}998$ ft.-lb. Ans.

EXAMPLE 2.—What horsepower will be required to do the work calculated in the last example?

SOLUTION.—As the work is done in 2 minutes, the horsepower is $\frac{753{,}998}{2 \times 33{,}000} = 11.424$ H. P., nearly. Ans.

FORCE OF A BLOW.

80. The questions are frequently asked, with what force will a falling hammer strike, or with what force will a projectile fired from a gun strike an object? These questions cannot be answered directly, as they are based on a misconception. A moving body possesses kinetic energy, or ability to do work, which ability can only be expressed in foot-pounds, but not in pounds of force, since the work done by the hammer or projectile in coming to rest is not a manifestation of force, but of energy.

Work is the product of force into distance; hence, if the amount of work a body has done or is capable of doing is known, the force can be determined for each case if, by some means, it is possible to determine exactly the distance in which the work is done. This distance depends on various resistances, such as that due to moving the object struck, the resistance to penetration, friction, the resistance

to shearing or deformation of the body, etc. The distance through which these resisting forces act is generally indeterminate, and since the average of the resisting forces varies generally with the distance, this average resisting force is also indeterminate; hence, the force that, acting through a distance, will absorb all the kinetic energy of the hammer or projectile cannot be determined for the practical reasons given.

COMPOSITION AND RESOLUTION OF FORCES.

81. According to Art. **44,** in order that forces may be compared with one another, three conditions must be fulfilled. These conditions may all be represented by a line; hence, we may represent forces by lines. Thus, in Fig. 2, let A be the point of application of the force, let the length of the line AB represent its magnitude, and let the arrowhead indicate the direction in which the force acts; then, the line AB fulfils the three conditions and the force is fully represented.

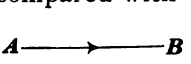

FIG. 2.

COMPOSITION OF FORCES.

82. When two or more forces act upon a body at the same time along lines that meet in a common point, their combined effect on the body may be obtained by an application of the principle of the **triangle of forces.**

In Fig. 3 (*a*), let A and B be two forces having the magnitudes and directions represented by the two lines. To find

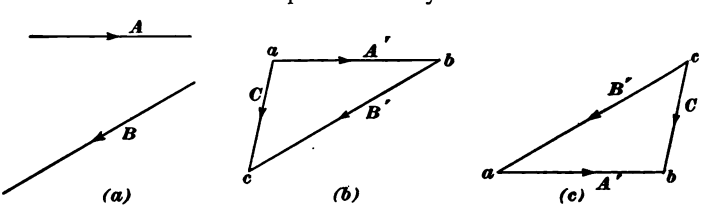

FIG. 3.

the effect due to the combined action of these two forces, draw in any convenient location a line parallel to either of the

lines representing the two forces, making it equal in length, to some scale, to the magnitude of the force. Mark upon it an arrowhead pointing in the same direction as the arrowhead on the line representing the force. Then, from one extremity of the line just drawn draw a line parallel to the line representing the second force and equal in length, to the same scale, to the magnitude of the second force, and mark the arrowhead upon it. It is essential that the second line be so drawn that when passing over the two lines with a pencil, commencing at the beginning of either force, the arrowheads will both point in the same direction in reference to the direction of motion of the pencil.

The second line may be drawn from either end of the first line, but its direction must be made to fulfil the above absolutely essential condition. Thus, in Fig. 3 (*b*), the line B' has been drawn from the right-hand extremity of A'. Starting at *a* with a pencil and moving toward *b*, the pencil will move in the direction in which the arrowhead on A' points. Passing over B' from *b* to *c*, the pencil will move in the direction in which the arrowhead on B' points; we thus see that the lines are drawn correctly in reference to each other.

In Fig. 3 (*c*), the line B' has been drawn from the left-hand extremity of A'. Starting at *c* and following up the lines, it is seen that in this case the arrowheads both point in the same direction relative to the direction of motion of the pencil, thus showing the lines to be located correctly.

The two lines having been drawn, complete the triangle by drawing the line C. This line, called the **resultant,** represents the combined effect of the two forces; it gives the direction along which the two forces will act when combined. The magnitude of their combined effect is found by measuring this line by the scale with which A' and B' were laid off. The resultant will always have a direction opposite in sense to that of the forces; that is, if we pass a pencil around the triangle in the direction in which the arrowheads on the lines A' and B' point, the arrowhead on C, representing the direction of action of the resultant, must point in a direction opposite to that in which the pencil moves.

83. In practice it is often desired to find not only the magnitude and direction, but also the *actual location of the resultant*, the magnitudes and lines of action of the two forces being known. (By location of the forces and resultant is meant the location of the lines along which the forces actually act.) This can readily be done by producing the lines giving the location of the forces until they meet and then drawing the triangle of forces with their point of intersection as the starting point.

ILLUSTRATION.—In Fig. 4 is shown a head-frame erected at the mouth of a deep, vertical shaft. The hoisting rope that leads to the

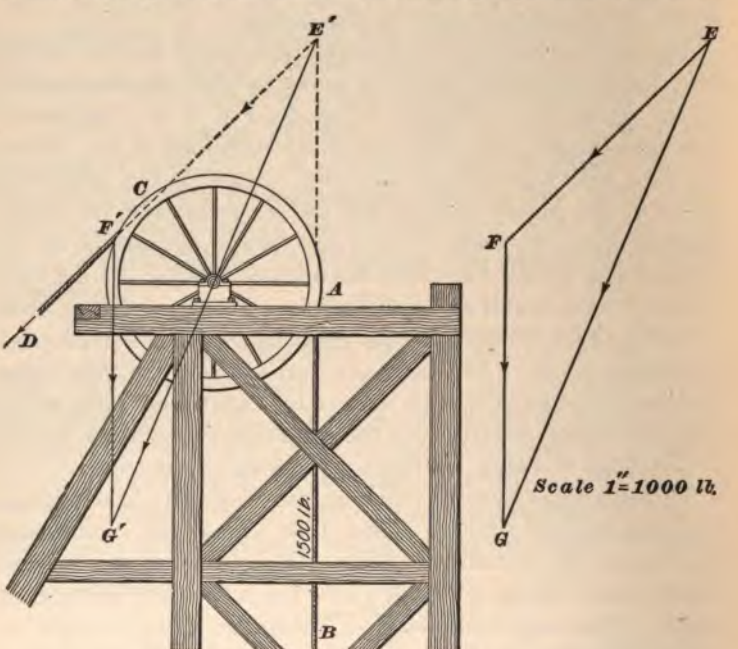

FIG. 4.

hoisting engine passes over a sheave at the top of the head-frame. A weight of 1,500 pounds hangs at the end of the rope. Neglecting the weight of the rope and sheave, what is the total pressure on the bearings of the sheave?

According to Art. **51**, the stress in both parts of the hoisting rope is 1,500 pounds. The part $A\,B$ of the rope supports the weight;

consequently, the force acting along $A\,B$ is downwards. The force acting along $C\,D$ is the pull exerted by the engine and is toward the engine. To find the resultant of these two forces, draw the triangle of forces. Choosing a scale of 1 inch per 1,000 pounds, draw the line $E\,F$ parallel to $C\,D$, making it $\frac{1500}{1000} = 1.5$ inches long. From F draw $F\,G$ parallel to $A\,B$ and $\frac{1500}{1000} = 1.5$ inches long. Join E and G; then $E\,G$ will be the resultant whose magnitude is measured by the same scale. Measuring $E\,G$, it is found to be 2.75 inches long. As each inch represents 1,000 pounds, the magnitude of the resultant is $2.75 \times 1{,}000 = 2{,}750$ pounds.

To find the actual location of the resultant in reference to $A\,B$ and $C\,D$, produce both lines, as shown in dotted lines, until they intersect at E'. Starting the triangle of forces at E', lay off $E'\,F' = 1.5$ inches. From F' draw $F'\,G' = 1.5$ inches and parallel to $A\,B$. Join $E'\,G'$. Upon measurement it is found to be 2,750 pounds. As inspection shows that the resultant passes through the center of the bearings, the total pressure on the bearings is 2,750 pounds. Ans.

RESOLUTION OF FORCES.

84. Since two forces can be combined to form a single resultant force, we may also treat a single force as if it were the resultant of two forces whose action upon a body will be the same as that of a single force. Thus, in Fig. 5, the force $O\,A$ may be resolved into two forces $O\,B$ and $B\,A$. If the force $O\,A$ acts upon a body moving or at rest upon a horizontal plane, and the resolved force $O\,B$ is vertical and $B\,A$ horizontal, $O\,B$, measured to the same scale as $O\,A$, is the magnitude of that part of $O\,A$ that pushes the body *downwards*, while $B\,A$ is the magnitude of that part of the force $O\,A$ that is exerted in pushing the body in a *horizontal* direction. $O\,B$ and $B\,A$ are called the **components** of the force $O\,A$, and when these components are vertical and horizontal, as in the present case, they are called the *vertical component* and the *horizontal component* of the force $O\,A$.

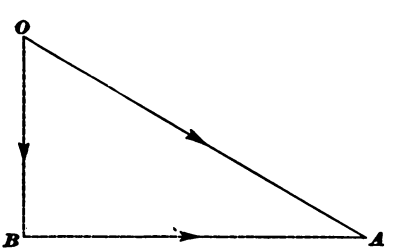

FIG. 5.

85. It frequently happens that the position, magnitude, and direction of a certain force is known, and that it is desired to know the effect of the force in some direction other than that in which it acts. Thus, in Fig. 5, suppose that OA represents, to some scale, the magnitude, direction, and line of action of a force acting upon a body at A, and that it is desired to know what effect OA produces in the direction BA. Now BA, instead of being horizontal, as in the figure, may have any direction. To find the value of the component of OA which acts in the direction BA, we use the following rule:

Rule 8.—*From one extremity of the line representing the given force, draw a line parallel to the direction in which it is desired that the component shall act; from the other extremity of the given force, draw a line perpendicular to the component first drawn, and intersecting it. The length of the component, measured from the point of intersection to the intersection of the component with the given force, will be the magnitude of the effect produced by the given force in the required direction.*

Thus, suppose OA, Fig. 5, represents a force acting upon a body resting upon a horizontal plane, and it is desired to know what *vertical pressure* OA produces on the body. Here the desired direction is vertical; hence, from one extremity, as O, draw OB parallel to the desired direction (vertical in this case), and from the other extremity draw AB perpendicular to OB and intersecting OB at B. Then OB, when measured to the same scale as OA, will be the value of the vertical pressure produced by OA.

86. Tangential Pressure.—One of the most familiar applications of the principle of the resolution of forces occurring in steam engineering is the case of the connecting-rod and crank. When the piston is at the end of its stroke and the crankpin is in a line drawn through the center of the cylinder and crank-shaft, a position that is expressed by saying the engine is "on the center," the pressure of the connecting-rod on the crankpin acts directly against the

bearings of the shaft and there is no turning effect on the pin. After the pin leaves the center, the pressure exerted on it by the connecting-rod may be resolved into two components: One of these components acts in the direction of the center line PO of the crank (see Fig. 6) and merely

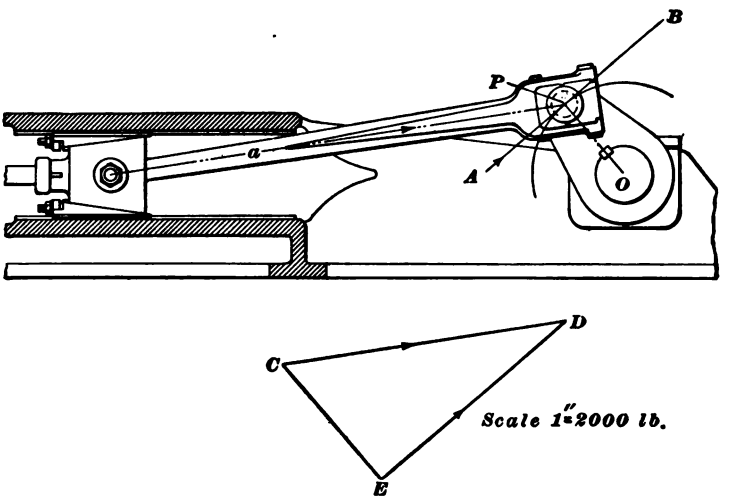

FIG. 6.

exerts a pressure on the bearings of the shaft; the other component acts along the line AB at right angles to the center line PO and *tangent* to the circle described by the crankpin. This is the force that tends to turn the crank and is called the **tangential pressure** on the crankpin.

When the crank is on the center, there is no tangential pressure on the pin and no tendency to turn the crank. The tangential pressure gradually increases as the pin leaves the center and becomes greatest at the point where the connecting-rod and crank are at right angles to each other; it then decreases until the next center is reached. At the position where the connecting-rod and crank are at right angles to each other, the tangential pressure on the pin is equal to the total pressure exerted on it by the connecting-rod, and there is no component in the direction of the center line PO of the crank.

EXAMPLE.—If a force of 3,000 pounds acts along the connecting-rod *a* in the direction of the arrow (see Fig. 6), what is the tangential pressure on the crankpin?

SOLUTION.—As the tangential pressure is the pressure perpendicular to the crank, draw *A B* through the crankpin center at right angles to the center line of the crank; *A B* then represents the line along which the tangential pressure acts. Then, in any convenient location, draw *C D* parallel to the connecting-rod. Choosing a scale of 2,000 pounds = 1 inch, make the line *C D* 3,000 ÷ 2,000 = 1½ inches long. From *D* draw an indefinite line *D E* parallel to *A B*, and draw *C E* perpendicular to *D E*. Now *E D*, measured to the scale adopted, will be the magnitude of the tangential pressure for the position of the crank and connecting-rod shown in the figure. Upon measurement, it is found to be 1.3 inches long. Then 1.3 × 2,000 = 2,600 lb., the tangential pressure. Ans.

87. When the total pressure on the piston rod of a steam engine is known, the force acting along the connecting-rod and the force acting upon the guides can be determined by the following application of the principle of the resolution of force:

Draw a line, as *A B*, Fig. 7, parallel to the line of motion of the crosshead *a*. Make its length, to some scale, equal in

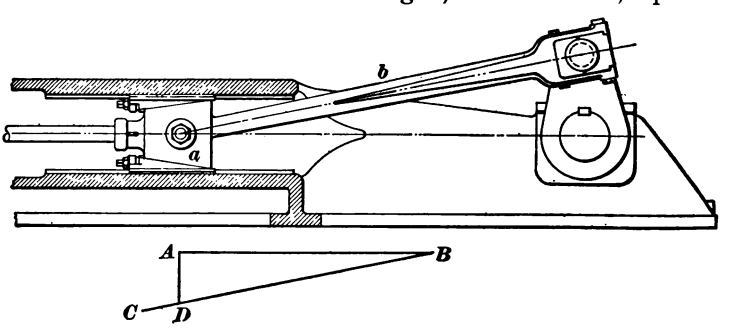

FIG. 7.

magnitude to the force impelling the piston. From one extremity of *A B* draw a line *B C* parallel to the center line of the connecting-rod *b*. From the other extremity of *A B* draw a line at a right angle to *A B*, producing it until it intersects the line *B C* at *D*. Then, *B D* represents the force acting along the connecting-rod and *A D* represents the force acting upon the guides.

FRICTION.

88. Friction is the resistance that a body meets with from the surface upon which it moves.

89. The ratio between the *resistance* to the motion of a body due to friction and the *perpendicular* pressure between the surfaces is called the **coefficient of friction.**

If a weight W, as in Fig. 8, rests upon a horizontal plane, and has a cord fastened to it passing over a pulley a, from

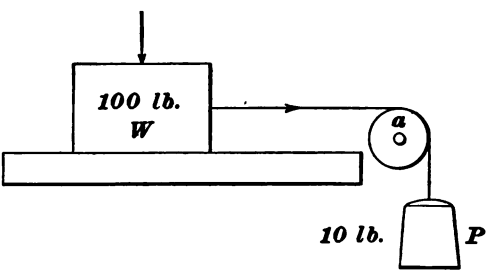

FIG. 8.

which a weight P is suspended, then, if P is just sufficient to start W, the ratio of P to W, or $\frac{P}{W}$, is the coefficient of friction between W and the surface it slides upon. The weight W is the perpendicular pressure, and P is the force necessary to overcome the resistance to the motion of W due to friction. If $W = 100$ pounds and $P = 10$ pounds, the coefficient of friction for this particular case would be $\frac{P}{W} = \frac{10}{100} = .1$.

90. Laws of Friction.—

1. *Friction is directly proportional to the perpendicular pressure between the two surfaces in contact.*

2. *Friction is independent of the extent of the surfaces in contact when the total perpendicular pressure remains the same.*

3. *Friction increases with the roughness of the surfaces.*

4. *Friction is greater between surfaces of the same material than between those of different materials.*

5. *Friction is greatest at the beginning of motion.*

6. *Friction is greater between soft bodies than between hard ones.*

7. *Rolling friction is less than sliding friction.*

8. *Friction is diminished by polishing or lubricating the surfaces.*

91. Law 1 shows why the friction is so much greater on journals after they begin to heat than before. The heat causes the journal to expand, thus increasing the pressure between the journal and its bearing, and, consequently, increasing the friction.

Law 2 states that, no matter how small may be the surface that presses against another, if the perpendicular pressure is the same the friction will be the same. Therefore, large surfaces are used where possible, not to reduce the friction, but to reduce the wear and diminish the liability of heating.

For instance, if the perpendicular pressure between a journal and its bearing is 10,000 pounds, and the coefficient of friction is .2, the amount of friction is $10{,}000 \times .2 = 2{,}000$ pounds. Suppose that the area receiving the pressure is 80 square inches, then the amount of friction for each square inch is $2{,}000 \div 80 = 25$ pounds.

If the area receiving the pressure had been 160 square inches, the friction would have been the same, that is, 2,000 pounds; but the friction per square inch would have been $2{,}000 \div 160 = 12\frac{1}{2}$ pounds, just one-half as much as before, and the wear and liability to heat would be one-half as great also.

92. The values of the coefficient of friction given in the following tables are average values determined by General Morin, many years ago. Under certain conditions the coefficient may be considerably less than is given in the tables; it varies greatly, but the variation depends on so many conditions and the numerous experiments that have been made have given such contradictory results that no definite rules have yet been derived for determining the

exact values under any condition. The student is, therefore, advised to use the values given in the tables, except where careful experiments have been made that give reliable values for the particular case under consideration. To find the force, in pounds, necessary to overcome friction, the coefficient taken from the table is multiplied by the perpendicular pressure, in pounds, on the surface considered. If the force acts at an angle to the surface, the perpendicular force can be found by resolving the given force into two components, one perpendicular and the other parallel to the surface.

TABLE I.

COEFFICIENTS OF FRICTION FOR PLANE SURFACES.

(Reduced from M. Morin's Data.)

Description of Surfaces in Contact.	State of the Surfaces.	Coefficient of Friction.
Wrought iron on cast iron.	Slightly greasy	.18
Wrought iron on bronze..	Slightly greasy	.18
Cast iron on cast iron....	Slightly greasy	.15
Cast iron on bronze.....	Slightly greasy	.15
Bronze on bronze.....	Dry	.20
Bronze on cast iron.....	Dry	.22
Bronze on wrought iron..	Slightly greasy	.16
Cast iron, wrought iron, steel, and bronze sliding on one another or sliding on themselves.	Ordinary lubrication with lard, tallow, and oil	.07–.08
Cast iron, wrought iron, steel, and bronze sliding on one another or sliding on themselves.	Continuous lubrication	.05

TABLE II.

COEFFICIENTS OF FRICTION FOR JOURNAL FRICTION.

(Reduced from M. Morin's Data.)

Journals.	Bearings.	Lubricant.	Coefficient of Friction.	
			Ordinary Lubrication.	Continuous Lubrication.
Cast iron	Cast iron	Lard, olive oil, tallow	.07–.08	.03–.054
Cast iron	Bronze	Lard, olive oil, tallow	.07–.08	.03–.054
Wrought iron	Cast iron	Lard, olive oil, tallow	.07–.08	.03.–054
Wrought iron	Bronze	Lard, olive oil, tallow	.07–.08	.03–.054
Wrought iron	Lignum vitæ	Lard, olive oil	.11	
Bronze	Bronze	Oil	.10	
Bronze	Bronze	Lard	.09	

93. In the case of a weight sliding along a horizontal plane surface, the pressure is equal to the weight. When the surface is inclined, the weight acts vertically downwards, and the pressure perpendicular to the surface can be found by the principle of the resolution of forces. In many cases the pressure on the surfaces is due to the combined action of several forces that must be combined into one common resultant force.

94. The *work* that must be done in overcoming the resistance of friction depends on the distance through which the resistance is overcome. It may be calculated by the following rule:

Rule 9.—*Multiply the total pressure in pounds by the distance in feet and by the coefficient of friction.*

Let W = work in foot-pounds;
f = coefficient of friction;
p = total pressure in pounds;
d = distance in feet.

Then, $$W = p\,d\,f.$$

EXAMPLE.—The average perpendicular pressure on the guide of a steam engine due to the force impelling the piston is 2,500 pounds. The pressure due to the weight of the crosshead and connecting-rod is 400 pounds. The crosshead moves at the rate of 500 feet per minute; what horsepower is required to overcome the friction on the guides?

SOLUTION.—The total perpendicular pressure is $2{,}500 + 400 = 2{,}900$ pounds. Since the lubrication is usually intermittent, the coefficient of friction, for a brass slipper working on a cast-iron guide, may be taken as .08. The resistance being overcome through a distance of 500 feet each minute, the work done in overcoming friction is $2{,}900 \times 500 \times .08 = 116{,}000$ foot-pounds per minute. Then, the horsepower is $116{,}000 \div 33{,}000 = 3.52$ H. P., nearly. Ans.

95. In the case of a shaft rotating in a bearing, the distance through which friction is overcome each minute is found by multiplying the circumference of the journal by the number of revolutions per minute. For a shaft, or any other body rotating in a bearing, the force required to overcome friction, as calculated by multiplying the pressure by the coefficient of friction, is the force that must be applied *at the surface of the journal.*

96. Allowable Pressures.—It has been found by experience that when the pressure per unit of area exceeds a certain amount, the lubricant will be forced out and the bodies rubbing on each other will heat and, finally, seize.

The pressures that can safely be allowed on the bearings of steam engines, on guides, thrust bearings, crankpins, crosshead pins, etc. vary considerably, being dependent to a large extent on the character of the workmanship, the degree of finish, the variation of the pressure, the character of the lubrication, and the quality of the lubricant. With fair workmanship, the following pressures per square inch represent average practice in steam-engine work:

TABLE III.

PRESSURES PER SQUARE INCH ALLOWABLE IN STEAM-ENGINE WORK.

Engine Bearing.	Slow-Speed Stationary Engines. Pounds.	High-Speed Stationary and Marine Engines. Pounds.
Crankpins, iron.....	800	400 to 600
Crankpins, steel.....	1,200	
Wristpin.....	1,200	600 to 800
Main bearings.....	200	200
Guides	100	100
Thrust bearings.....		60

97. For crankpins, wristpins, and guides, the allowable pressures given represent the pressures corresponding to the maximum load, which in the case of a wristpin and crankpin occurs when the crank, connecting-rod, and piston rod are in a straight line, and in the case of guides, when the connecting-rod and crank are at right angles to each other. In the case of pins and journals, the area to be considered in calculating the pressure on the bearing is the **projected area,** which is found by multiplying the length of the journal by its diameter.

EXAMPLES FOR PRACTICE.

1. A body weighs 90 pounds; what is its mass? Ans. 2.799, nearly.

2. What force will be required to accelerate a body at the rate of 2 feet per second, the body weighing 450 pounds, and the frictional resistances being equal to 10 per cent. of the weight of the body?

Ans. 72.99 lb., nearly.

3. What work is done in raising 950 pounds 17 feet?

Ans. 16,150 ft.-lb.

4. If an engine does 205,000 foot-pounds of work per minute, what is its horsepower? Ans. 6.21 H. P., nearly.

5. What is the kinetic energy of a shell fired from a cannon with a velocity of 1,800 feet per second, the shell weighing 1,000 pounds?

Ans. 50,373,135 ft.-lb., nearly.

6. Taking the coefficient of friction at .15, what horsepower will be required to pull 100 pounds at a uniform speed of 5 feet per second along a level surface? Ans. .136 H. P.

CENTER OF GRAVITY.

98. *The center of gravity of a body is that point at which the body may be balanced, or it is the point at which the whole weight of a body may be considered as concentrated.*

This point is not always *in* the body; in the case of a horseshoe or a ring it lies outside of the substance of, but within the space enclosed by, the body.

In a moving body, the line described by its center of gravity is always taken as the path of the body. In finding the distance that a body has moved, the distance that the center of gravity has moved is taken.

The definition of the center of gravity of a body may be applied to a system of bodies if they are considered as being connected at their centers of gravity.

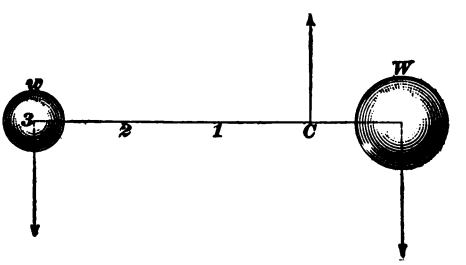

FIG. 9.

If w and W, Fig. 9, are two bodies of known weight, their center of gravity will be at C. This point C may be readily determined as follows:

Rule 10.—*The distance of the common center of gravity from the center of gravity of the large weight is equal to the weight of the smaller body multiplied by the distance between the centers of gravity of the two bodies, and this product divided by the sum of the weights of the two bodies.*

EXAMPLE.—In Fig. 9, $w = 10$ pounds, $W = 30$ pounds, and the distance between their centers of gravity is 36 inches; where is the center of gravity of both bodies situated?

SOLUTION.—Applying the rule, $10 \times 36 = 360$. $10 + 30 = 40$. $360 + 40 = 9$ in., distance of center of gravity from center of large weight. Ans.

99. It is now very easy to extend this principle to find the center of gravity of any number of bodies, when their weights and the distances apart of their centers of gravity are known, by the following rule:

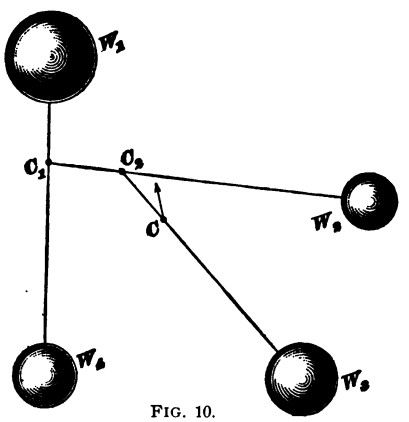

FIG. 10.

Rule 11.—*Find the center of gravity of two of the bodies, as* W_1 *and* W_4 *in Fig. 10. Assume that the weight of both bodies is concentrated at* C_1, *and find the center of gravity of this combined weight at* C_1 *and the weight of* W_2. *Let it be at* C_2; *then find the center of gravity of the combined weights of* W_1, W_4, W_2 (*concentrated at* C_2), *and* W_3. *Let it be at* C; *then* C *will be the center of gravity of the four bodies.*

100. To find the center of gravity of **any parallelogram:**

Rule 12.—*Draw the two diagonals, Fig. 11, and their point of intersection* C *will be the center of gravity.*

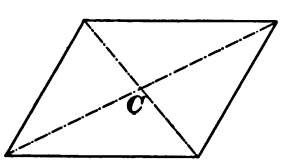

FIG. 11.

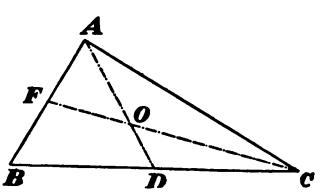

FIG. 12.

101. To find the center of gravity of a **triangle,** as $A\,B\,C$, Fig. 12:

center of gravity of homogeneous solids may often be determined without analysis, or it may be limited to a certain plane, line, or point. Thus, the center of gravity of a sphere, or any other regular body, is situated at its center; of a cylinder, in the middle of its axis; of a thin plate having the form of a circle or regular polygon, in the center of the figure; of a straight wire of uniform cross-section, in the middle of its length.

EXAMPLES FOR PRACTICE.

1. A spherical shell has a wrought-iron handle attached to it. The shell is 10 inches in diameter and weighs 20 pounds. The handle is $1\frac{1}{2}$ inches in diameter, and the distance from the center of the shell to the end of the handle is 4 feet. Where is the center of gravity? Take the weight of a cubic inch of wrought iron as .278 pound.

Ans. 13.612 in. from center of shell.

2. The distance between the centers of two bodies is 51 inches. The weights of the bodies being 20 and 73 pounds, where is the center of gravity? Ans. 10.968 in. from the center of the larger weight.

3. Weights of 5, 9, and 12 pounds lie in one straight line in the order named. Distance from the 5-pound weight to the 9-pound weight is 22 inches, and from the 9-pound weight to the 12-pound weight is 18 inches. Where is the center of gravity?

Ans. 13.923 in. from 12-lb. weight.

CENTRIFUGAL FORCE.

104. If a body is fastened to a string and whirled, so as to give it a circular motion, there will be a pull on the string that will be greater or less, according as the velocity increases or decreases. The cause of this pull on the string will now be explained.

Suppose that the body is revolved horizontally, so that the action of gravity upon it will always be the same. According to the first law of motion, a body put in motion tends to move in a straight line unless acted upon by some other force, causing a change in the direction. When the body moves in a circle, the force that causes it to move

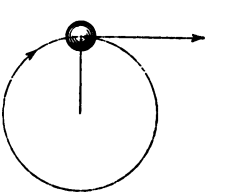

FIG. 15.

in a circle instead of in a straight line is exactly equal to the tension of the string. If the string were cut, the pulling force that drew it away from the straight line would be removed, and the body would then "fly off at a tangent"; that is, it would move in a straight line tangent to the circle, as shown in Fig. 15.

Since, according to the third law of motion, every action has an equal and opposite reaction, we call the force that acts as an equal and opposite force to the pull of the string the **centrifugal force,** and it acts *away* from the center of motion.

105. The other force, or tension, of the string is called the **centripetal force,** and it acts *toward* the center of motion. It is evident that these two forces, acting in opposite directions, tend to pull the string apart, and, if the velocity be increased sufficiently, the string will break. It is also evident that no body can revolve without generating centrifugal force.

106. To Find the Centrifugal Force of Any Revolving Body.—The value of the centrifugal force, expressed in pounds, of any revolving body is calculated by the following rule:

Rule 14.—*The centrifugal force equals the continued product of .00034, the weight of the body in pounds, the radius in feet (taken as the distance between the center of gravity of the body and the center about which it revolves), and the square of the number of revolutions per minute.*

Let F = centrifugal force in pounds;
W = weight of revolving body in pounds;
R = radius in feet of circle described by center of gravity of revolving body;
N = revolutions per minute of revolving body.

Then, $$F = .00034\ WRN^2.$$

EXAMPLE.—What is the tension in the string of Fig. 15, if the ball weighs 2 pounds and is revolved around at the rate of 500 revolutions per minute? The string is 2 feet long.

SOLUTION.—Applying the rule just given, we get

$$F = .00034 \times 2 \times 2 \times 500^2 = 340 \text{ lb.} \quad \text{Ans.}$$

107. In flywheels, belt wheels, and pulleys the centrifugal force tends to tear the rim asunder; this tendency is resisted by the tenacity of the material of which the wheel is composed. Since the centrifugal force increases as the square of the number of revolutions, it will be seen that an apparently slight increase in the number of revolutions per minute may be sufficient to burst the wheel.

108. For solid cast-iron wheels and for built-up wheels of cast iron where the strength of the joint is equal to the strength of the rim, the greatest number of revolutions per minute that practice has indicated to be safe may be found by the following rule:

Rule 15.—*Divide 1,930 by the diameter of the wheel.*

Or.
$$N = \frac{1{,}930}{d};$$

where d = diameter of the wheel in feet;
N = number of revolutions per minute.

EXAMPLE.—What is the maximum number of revolutions allowable for a cast-iron flywheel 27 feet 6 inches in diameter?

SOLUTION.—Applying rule **15**, we get

$$N = \frac{1{,}930}{27.5} = 70 \text{ rev. per min.} \quad \text{Ans.}$$

EQUILIBRIUM.

109. When a body is at rest, all the forces that act upon it balance one another and are said to be in **equilibrium.** The most important force to be considered is the attraction of the earth, which acts upon every particle of a body. There are three states of equilibrium: *stable*, *unstable*, and *neutral.*

110. A body is in **stable equilibrium** when, if slightly rotated about its support in such manner as to change the

elevation of its center of gravity, it tends to return to its position of rest. Examples of bodies in a state of stable equilibrium are a cube resting on one of its sides, a cone resting on its base, a pendulum, etc. A body can only be in stable equilibrium when a rotation about its support raises the center of gravity.

111. A body is in **unstable equilibrium** when a rotation about its support, so as to change the elevation of its center of gravity, tends to make it fall farther from its position of rest. Examples of bodies in unstable equilibrium are a cube balanced on one of its edges, a cone standing upon its point, an egg balanced upon its end, etc. When a body is in unstable equilibrium, any rotation, no matter how slight, tends to lower its center of gravity.

112. A body is in **neutral equilibrium** when a rotation about its support does not change the elevation of its center of gravity. Examples of bodies in neutral equilibrium are a sphere of uniform density and a cone resting on its side.

113. A vertical line drawn through the center of gravity of a body is called the line of direction. So long as the line of direction falls within the base, the body will stand; when the line of direction falls outside of the base, the body will fall.

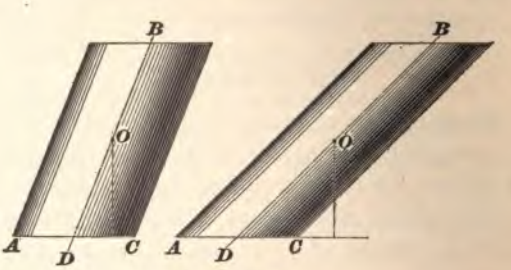

FIG. 16.

Let $A\,C\,B$, Fig. 16, be a cylinder whose base is oblique to the center line $B\,O\,D$, and let O be the center of gravity of

the cylinder. So long as the vertical line drawn through O falls between A and C, the cylinder will stand, but the instant it falls without the base, the cylinder will fall.

114. The center of gravity of a body has a tendency to always seek the lowest possible position.

MACHINE ELEMENTS.

THE LEVER, WHEEL, AND AXLE.

FUNDAMENTAL PRINCIPLES.

1. A **lever** is a bar capable of being turned about a pivot, or point, as in Figs. 1, 2, and 3.

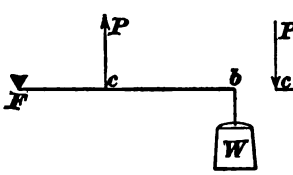

FIG. 1.

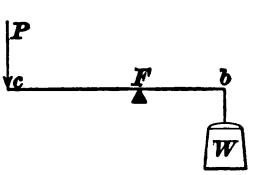

FIG. 2.

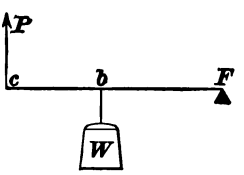

FIG. 3.

The object W to be lifted is called the **weight**; the force is represented by P*; and the point, or pivot F, is called the **fulcrum.**

* The force applied to a lever, screw, wheel and axle, or any similar machine element in order to raise a given weight, was formerly called the *power*, and the arm of a lever to which the force is applied was called the *power arm;* at present, however, the term power is universally used to represent the rate at which work is done, and, hence, its application to those cases where a simple force is meant often leads to a serious confusion of ideas regarding the relation between force, work, and power. To prevent this confusion, we will use the word power only in accordance with the definition given.

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2. That part of the lever $F\,b$ between the fulcrum and the weight is called the **weight arm,** and the part $F\,c$ between the fulcrum and the force is called the **force arm.**

In order that the lever will be in equilibrium (balance), *the force multiplied by the force arm must equal the weight multiplied by the weight arm;* that is, $P \times F\,c$ must equal $W \times F\,b$.

3. If F is taken as the center of a circle, and arcs are described through b and c, it will be seen that if the weight arm is moved through a certain angle, the force arm will move through the same angle. Since in the same or equal angles the lengths of the arcs are proportional to the radii with which they were described, it is seen that the force arm is proportional to the distance through which the force acts, and the weight arm is proportional to the distance through which the weight moves. Hence, instead of writing $P \times F\,c = W \times F\,b$, we might have written $P \times$ (distance through which P acts) $= W \times$ (distance through which W moves). This is the general law of all machines, and can be applied to any mechanism from the simple lever up to the most complicated arrangement. When stated in the form of a rule it is as follows:

Rule 1.—*The force multiplied by the distance through which it acts equals the weight multiplied by the distance through which it moves.*

4. In the above rule, it will be noticed that there are four requirements necessary for a complete knowledge of the lever, viz.: the force, the weight, the force arm, or distance through which the force acts, and the weight arm, or distance through which the weight moves. If any three are given, the fourth may be found by letting x represent the requirement that is to be found, and multiplying the force by the force arm and the weight by the weight arm; then, dividing the product of the two known numbers by the number by which x is multiplied, the result will be the requirement that is to be found.

EXAMPLE.—If the weight arm of a lever is 6 inches long and the force arm is 4 feet long, how great a weight can be raised by a force of 20 pounds at the end of the force arm?

SOLUTION.—In this example, the weight is unknown; hence, representing it by x, we have, after reducing the 4 feet to inches, $20 \times 48 = 960$ = force multiplied by the force arm, and $x \times 6$ = weight multiplied by the weight arm. Dividing the 960 by 6, the result is 160 pounds, the weight. Ans.

5. If the distance through which the force acted or the weight had moved had been given instead of the force arm or weight arm, and it were required to find the force or weight, the process would have been exactly the same, using the given distance instead of the force arm or weight arm.

EXAMPLE.—If, in the above example, the weight had moved $2\frac{1}{2}$ inches, through what distance would the force have acted?

SOLUTION.—In this example, the distance through which the force acts is required. Let x represent the distance. Then, $20 \times x$ = distance multiplied by force, and $2\frac{1}{2} \times 160 = 400$ = distance multiplied by the weight. Hence, $x = \frac{400}{20} = 20$ inches = distance through which the force arm moves. Ans.

The ratio between the weight and the force is $160 \div 20 = 8$. The ratio between the distance through which the weight moves and the distance through which the force acts is $2\frac{1}{2} \div 20 = \frac{1}{8}$. This shows that while a force of 1 pound can raise a weight of 8 pounds, the 1-pound weight must move through 8 times the distance that the 8-pound weight does. It will also be noticed that the ratio of the lengths of the two arms of the lever is also 8, since $48 \div 6 = 8$.

6. The law that governs the straight lever also governs the bent lever, but care must be taken to determine the true lengths of the lever arms, which are, in every case, *the perpendicular distances from the fulcrum to the line of direction of the weight or force.*

Thus, in Figs. 4, 5, 6, and 7, $F\,c$ in each case represents the force arm and $F\,b$ the weight arm.

7. A **compound lever** is a series of single levers arranged in such a manner that when a force is applied to the first it is communicated to the second, and from that to the third, and so on.

Fig. 8 shows a compound lever. It will be seen that when a force is applied to the first lever at P it will be communicated to the second lever at P, from this to the third lever at P, and thus raise the weight W.

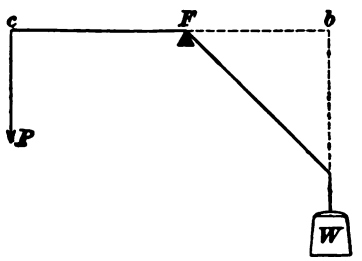

FIG. 4.

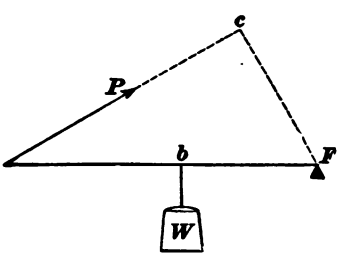

FIG. 5.

The weight that the force applied to the first lever could raise acts as the force of the second, and the weight that this force could raise through the second lever acts as the force

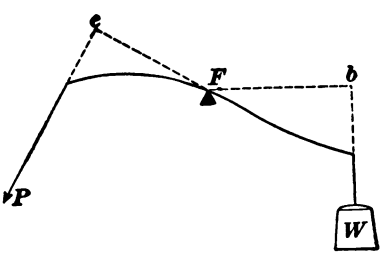

FIG. 6.

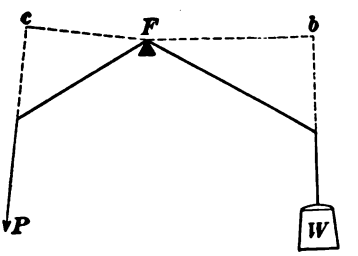

FIG. 7.

of the third lever, and so on, no matter how many single levers make up the compound lever.

In this case, as in every other, the force multiplied by the

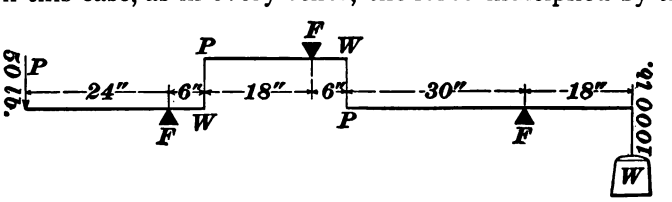

FIG. 8.

distance through which it acts equals the weight multiplied by the distance through which it moves.

Hence, if we move the P end of the lever, say 4 inches, and the end carrying the weight W moves $\frac{1}{5}$ inch, we know that the ratio between P and W is the same as the ratio between $\frac{1}{5}$ and 4; that is, 1 to 20, and, hence, that 10 pounds at P will balance 200 pounds at W, without measuring the lengths of the different lever arms. If the lengths of the lever arms are known, the ratio between P and W may be readily obtained from the following rule:

Rule 2.—*The continued product of the force and each force arm equals the continued product of the weight and each weight arm.*

EXAMPLE.—If, in Fig. 8, $PF = 24$ inches, 18 inches, and 30 inches, respectively, and $WF = 6$ inches, 6 inches, and 18 inches, respectively, how great a force at P will it require to raise 1,000 pounds at W? What is the ratio between W and P?

SOLUTION. — Let x represent the force; then, $x \times 24 \times 18 \times 30 = 12{,}960 \times x =$ continued product of the force and each force arm. $1{,}000 \times 6 \times 6 \times 18 = 648{,}000 =$ continued product of the weight and each weight arm; and, since $12{,}960 \times x = 648{,}000$,

$$x = \frac{648{,}000}{12{,}960} = 50 \text{ lb.} = \text{the force.} \quad \text{Ans.}$$

$1{,}000 \div 50 = 20 =$ ratio between W and P. Ans.

8. The **wheel and axle** consists of *two cylinders of different diameters rigidly connected*, so that they turn

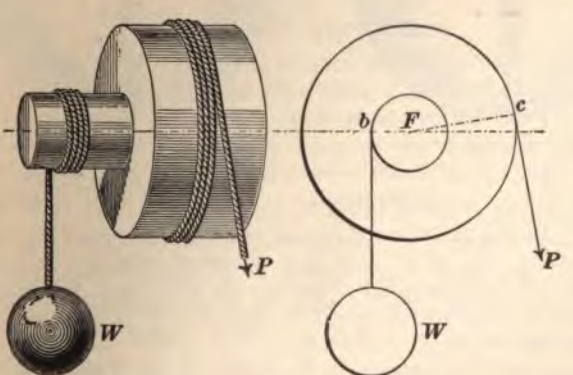

FIG. 9.

together about a common axis, as in Fig. 9. Then, as before, $P \times$ distance through which it acts $= W \times$ distance through which it moves; and, since these distances are proportional to the radii of the force cylinder and weight cylinder, $P \times Fc = W \times Fb$.

It is not necessary that an entire wheel be used; an arm, projection, radius, or anything that the force causes to revolve in a circle, may be considered as the wheel. Consequently, if it is desired to hoist a weight with a windlass, Fig. 10, the force is applied to the handle of the crank, and the distance between the center line of the crank-handle and the axis of the drum corresponds to the radius of the wheel.

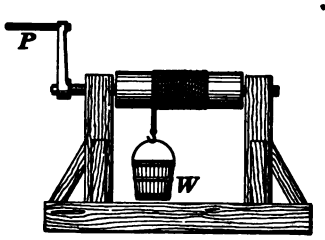

FIG. 10.

EXAMPLE.—If the distance between the center line of the handle and the axis of the drum in Fig. 10 is 18 inches and the diameter of the drum is 6 inches, what force will be required at P to raise a load of 300 pounds?

SOLUTION.— $P \times (18 \times 2) = 300 \times 6$, or $P = 50$. Ans.

EXAMPLES FOR PRACTICE.

1. The lever of a safety valve is of the form shown in Fig. 1, where the force is applied at a point between the fulcrum and the weight lifted. If the distance from the fulcrum to the valve is $5\frac{1}{2}$ inches and from the fulcrum to the weight is 42 inches, what total force is necessary to raise the valve, the weight being 78 pounds and the weight of valve and lever being neglected? Ans. 595.64 lb.

2. If, in Fig. 8, $PF = 10$ inches, 12 inches, 14 inches, and 16 inches, respectively, and $WF = 2$ inches, 3 inches, 4 inches, and 5 inches, respectively, (*a*) how great a weight can a force of 20 pounds raise? (*b*) What is the ratio between W and P? (*c*) If P moves 4 inches, how far will W move?

Ans. (*a*) 4,480 lb. (*b*) 224. (*c*) $\frac{1}{56}$ in.

3. A windlass is used to hoist a weight. If the diameter of the drum on which the rope winds is 4 inches, and the distance from the center of the handle to the axis of the drum is 14 inches, how great a weight can a force of 32 pounds applied to the handle raise?

Ans. 224 lb.

PULLEYS.

9. Pulleys for the transmission of power by belts may be divided into two principal classes: (1) The *solid pulley* shown in Fig. 11, in which the hub, arms, and rim are one entire casting. (2) The *split pulley* shown in Fig. 12, which is cast in halves.

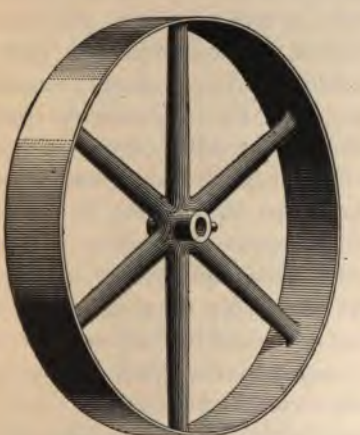

FIG. 11.

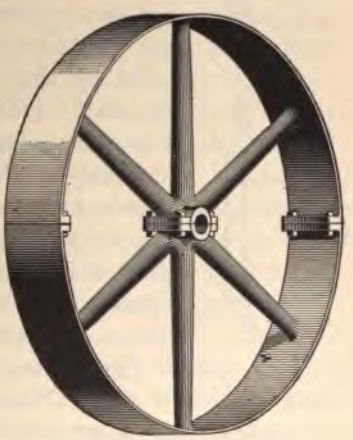

FIG. 12.

The latter style of pulley is more readily placed on and removed from the shaft than the solid pulley. Pulleys are generally cast in halves or parts when they are more than 6 feet in diameter; this is done on account of the shrinkage strain in large pulley castings, which renders them liable to crack as a result of the unequal cooling of the metal.

10. Of late years, wooden pulleys have come into extensive use. They are built of segments securely glued together, maple being the wood used. Wooden split pulleys can be procured that are fitted with removable bushings, thus allowing the same pulley to be readily adapted to various diameters of shafting. They are somewhat lighter than cast-iron pulleys.

11. Crowning Pulley Faces.—In Fig. 13, suppose the shafts *a* and *b* to be parallel. Let the pulley on the shaft *a* be cone-shaped, as shown. The right-hand side of the belt will be pulled ahead more rapidly than the left-hand side, because of the greater diameter and, consequently, greater speed of the right-hand end of the pulley. It has been observed that in this case the belt will leave its normal position, which is indicated by the dotted lines, and climb toward the part of the pulley that has the largest diameter, as shown in the illustration. This tendency of the belt to climb toward the high side is taken advantage of to make the belt stay on a pulley. Suppose the pulley on the shaft *a* is replaced with one formed of two equal cones, with the large diameter in the center. Then, each side of the belt will tend to climb toward the highest point, and the consequence is that the belt will stay in the center of the pulley.

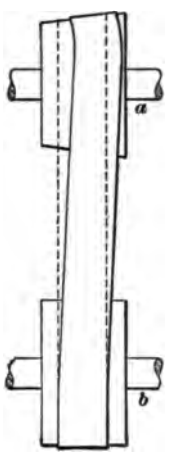

FIG. 13.

A pulley with its surface formed in this way is said to be **crowned.** The surface need not necessarily represent the frustums of two cones; it may simply be curved, as shown in Fig. 14. It is only required that the pulley be larger in diameter in the center.

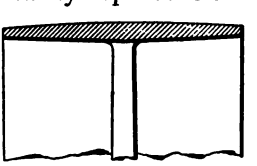
FIG. 14.

As to the proper amount of crowning necessary, the practice of the makers of pulleys differs considerably; usually, though, it is from $\frac{3}{16}$ to $\frac{1}{2}$ inch per foot of width of the pulley face.

12. Balancing Pulleys.—All pulleys that rotate at high speeds should be balanced. If they are not, the centrifugal force that is generated by the rotation of the pulley, is greater on one side than on the other and will cause the pulley shaft to vibrate and shake. Pulleys are usually balanced in the manner shown in Fig. 15. A closely fitting arbor, which is simply a truly cylindrical piece of iron or steel, is driven into the bore of the pulley, which is then placed on the so-called "balancing ways." These are two planed iron or steel bars, preferably planed to a knife edge. These bars are placed on convenient supports far enough apart to allow the pulley to go between them. The bars should be carefully leveled with a spirit level, so that both bars are in the same horizontal plane. When the arbor rests on the ways and the pulley is slightly rotated, the latter will quickly come to rest with the heavy part downwards. Now, either some metal must be removed from the heavy part or some weight added to the light part. For small pulleys, a convenient substance to use for finding the proper location and weight of the counterweight is ordinary glazier's putty. By repeated trials the proper weight of the counterweight is found, and then a mass of metal of convenient shape equal in weight to the putty is fastened to the light part of the pulley in whatever way is safe and convenient. The proper weight and location of the counterweight will have been obtained when the pulley will be at rest on the balancing ways in any position in which it is put. In other words, the pulley is balanced when it is in neutral equilibrium. This balance is called a **standing balance.** The method just explained answers very well for pulleys that are narrow in comparison

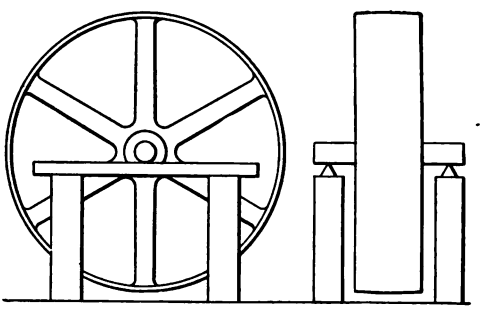

FIG. 15.

to their diameter. When pulleys with a wide face are run at a high speed, it is often found that serious vibrations are set up even when they are in perfect standing balance, thus showing that they do not possess the so-called **running balance.** No method has yet been found that will insure a running balance at all speeds, nor has any method become known by which it can be discovered directly where to apply the counterweight. The usual remedy for pulleys not possessing a running balance is to turn carefully the inside of the rim to run true with the outside of the pulley. Absence of running balance is, in all cases, due to an unequal distribution of metal in reference to the axis of the shaft, this unequal distribution being due to lack of homogeneity of the metal, to poor foundry work, or to poor lathe work.

13. Pulleys should run true in order that the strain, or tension, of the belt will be equal at all parts of the revolution, thus making the transmitting power equal. The smoother the surface of a pulley, the greater is its driving power.

The transmitting power of a pulley can be increased by covering its face with a leather or rubber band; this increases the driving power about one-quarter.

14. Relation Between Speeds of Drivers and Driven Pulleys.—The pulley that imparts motion to the belt is called the **driver**; that which receives the motion is called the **driven.**

The revolutions of any two pulleys over which a belt is run vary in an inverse proportion to their diameters; consequently, if a pulley 20 inches in diameter is driven by one 10 inches in diameter, the 20-inch pulley will make 1 revolution while the 10-inch pulley makes 2 revolutions, or they are in the ratio of 2 to 1. From this fact, the following formulas have been deduced:

Let D = diameter of the driver;
d = diameter of the driven;
N = number of revolutions of the driver;
n = number of revolutions of the driven.

NOTE.—The words revolutions per minute are frequently abbreviated to R. P. M.

15. To find the diameter of the driving pulley when the diameter of the driven pulley and the number of revolutions per minute of each is given:

Rule 3.—*The diameter of the driving pulley equals the product of the diameter and the number of revolutions of the driven pulley divided by the number of revolutions of the driving pulley.*

Or, $$D = \frac{d\,n}{N}.$$

EXAMPLE.—The driving pulley makes 100 revolutions per minute, the driven pulley makes 75 revolutions per minute and is 18 inches in diameter; what is the diameter of the driving pulley?

SOLUTION.—Applying the rule just given and substituting, we have $D = \frac{18 \times 75}{100} = 13\frac{1}{2}$ in. Ans.

16. The diameter and number of revolutions per minute of the driving pulley being given, to find the diameter of the driven pulley, which must make a given number of revolutions per minute:

Rule 4.—*The diameter of the driven pulley equals the product of the diameter and the number of revolutions of the driving pulley divided by the number of revolutions of the driven pulley.*

Or, $$d = \frac{D\,N}{n}.$$

EXAMPLE.—The diameter of the driving pulley is $13\frac{1}{2}$ inches and it makes 100 revolutions per minute; what must be the diameter of the driven pulley to make 75 revolutions per minute?

SOLUTION.—Applying rule 4 we have $d = \frac{13\frac{1}{2} \times 100}{75} = 18$ in. Ans.

17. To find the number of revolutions per minute of the driven pulley, its diameter and the diameter and the number of revolutions per minute of the driving pulley being given:

Rule 5.—*The number of revolutions of the driven pulley equals the product of the diameter and the number of revolutions of the driver divided by the diameter of the driven pulley.*

Or, $$n = \frac{DN}{d}.$$

EXAMPLE.—The driving pulley is 13½ inches in diameter and makes 100 revolutions per minute; how many revolutions will the driven pulley make in 1 minute if it is 18 inches in diameter?

SOLUTION.—Formula, $n = \frac{DN}{d}$.

Substituting, we have $n = \frac{13\frac{1}{2} \times 100}{18} = 75$ R. P. M. Ans.

18. To find the number of revolutions per minute of the driving pulley, its diameter and the diameter and the number of revolutions per minute of the driven pulley being given:

Rule 6.—*The number of revolutions of the driving pulley equals the product of the diameter and the number of revolutions of the driven pulley divided by the diameter of the driving pulley.*

Or, $$N = \frac{dn}{D}.$$

EXAMPLE.—The driven pulley is 18 inches in diameter and makes 75 revolutions per minute; how many revolutions will the driving pulley make in 1 minute if it is 13½ inches in diameter?

SOLUTION.—Formula, $N = \frac{dn}{D}$.

Substituting, we have $N = \frac{18 \times 75}{13\frac{1}{2}} = 100$ R. P. M. Ans.

BELTS.

19. A **belt** is a flexible connecting band that drives a pulley by its frictional resistance to slipping at the surface of the pulley. Belts are most commonly made of leather, cotton, or rubber, and are united in long lengths by cementing, riveting, or lacing.

20. Leather belts are made single and double. A single belt is one composed of a single thickness of leather; a double belt is one composed of two thicknesses of leather cemented and riveted together the whole length of the belt.

21. Cotton belts are in use to some extent, as are also belts made of a number of layers of duck sewed together and impregnated with a preparation rendering them waterproof. These belts, in accordance with the number of layers or "plys," are called two-ply, three-ply, etc. Four-ply cotton and duck belting is about equal to single leather belting, and eight-ply to double leather belting.

22. Rubber belts are especially adapted for use in damp or wet places; they will endure a great degree of heat or cold without injury, are quite durable, and are claimed to be less liable to slip than leather belts.

CALCULATIONS FOR BELTS.

23. To Find the Length of a Belt.—In practice, the necessary length for a belt to pass around pulleys that are already in their position on a shaft is usually obtained by passing a tape line around the pulleys, the stretch of the tape line being allowed as that necessary for the belt. The lengths of open-running belts for pulleys not in position can be obtained approximately as follows:

Rule 7.—*The length of a belt for open-running pulleys equals* $3\frac{1}{4}$ *times one-half the sum of the diameters of the pulleys plus 2 times the distance between the centers of the shaft.*

Let D = diameter of one pulley in inches;
d = diameter of the other pulley in inches;
L = distance between the centers of the shafts in inches;
B = length of the belt in inches.

Then, $$B = 3\tfrac{1}{4}\left(\frac{D+d}{2}\right) + 2L.$$

EXAMPLE.—The distance between the centers of two shafts is 9 feet 7 inches; the diameter of the large pulley is 36 inches and the diameter of the small one is 14 inches; what is the necessary length of the belt?

SOLUTION.—Applying the rule just given and substituting the values given, we have, since 9 feet 7 inches = 115 inches,

$$B = 3\tfrac{1}{4}\left(\frac{36+14}{2}\right) + 2 \times 115 = 311\tfrac{1}{4} \text{ in., or 25 ft. } 11\tfrac{1}{4} \text{ in. Ans.}$$

The length of crossed belts cannot be determined by any simple calculation, it being a rather difficult mathematical problem.

24. To Find the Width of Belts.—A belt should be wide enough to bear safely and for a reasonable length of time the greatest tension that will be put upon it. This will be the tension of the driving side. The safe tension for single belts may be taken as 60 pounds per inch of width; single belts average $\frac{3}{16}$ inch in thickness. The tension on the driving side, however, does not represent the force tending to turn the pulley. The force tending to turn the pulley, or the **effective pull,** is the difference in tension between the driving side and the slack side of the belt. The tension on the driving side depends on three factors: the *effective pull* of the belt, the *coefficient of friction* between the belt and pulley, and the size of the *arc of contact* of the belt on the smaller pulley.

25. The effective pull that may be allowed per inch of width for single leather belts with different arcs of contact (the arc in which the belt touches the smaller pulley), is given in Table I.

26. To Find the Arc of Contact.—The arc of contact in degrees, or as a fraction of the circumference, can be determined, practically, as follows: Stretch a string over the two pulleys to represent the belt. Then, take another string, wrap it around the small pulley and cut it off so that the ends meet. This represents the circumference of the small pulley. Now take a third string, hold one end at the

TABLE I.

ALLOWABLE BELT PULLS.

Arc Covered by Belt.		Allowable Effective Pull Per Inch of Width in Pounds.
Degrees.	Fraction of Circumference.	
90	.250	23.0
112½	.312	27.4
120	.333	28.8
135	.375	31.3
150	.417	33.8
157½	.437	34.9
180 or over	.500	38.1

beginning of the arc of contact, as shown by the string stretched around both pulleys, wrap it around the smaller pulley, and cut it off at the end of the arc of contact. The length of this last string represents the length of the arc of contact. We now have the proportion: *the length of the string representing the circumference* : *the length of the string representing the arc of contact* :: 360 (*the number of degrees in a circle*) : *the number of degrees in the arc of contact.* Whence, the number of degrees in the arc of contact equals the quotient obtained by dividing the product of the length of the arc of contact and 360 by the circumference of the pulley.

To obtain the fraction of the circumference, divide the length of the string representing the arc of contact by the circumference.

27. To use the table for finding the width of a single leather belt for transmitting a given number of horsepower, we have the following rule,

where C = allowable effective pull, from table;
H = horsepower to be transmitted;
W = width of single belt in inches;
V = velocity of belt in feet per minute.

Rule 8.—*Multiply the horsepower to be transmitted by 33,000, and divide this product by the product of the velocity of the belt and the allowable effective pull, as taken from the table. The quotient will be the width of the belt.*

Or, $$W = \frac{33{,}000\,H}{VC}.$$

EXAMPLE.—What width of single belt is needed to transmit 20 horsepower, the arc of contact on the small pulley being 135° and the speed of the belt 1,500 feet per minute?

SOLUTION.—According to Table I, the allowable effective pull for 135° is 31.3 pounds. Then, applying rule **8**, we have

$$W = \frac{33{,}000 \times 20}{1{,}500 \times 31.3} = 14 \text{ in., nearly.} \quad \text{Ans.}$$

28. To Find the Horsepower of a Belt.—The horsepower that a single belt will transmit is given by the following rule:

Rule 9.—*Multiply together the effective pull taken from the table, the width of the belt in inches, and the speed of the belt in feet per minute. Divide the product by 33,000.*

Or, $$H = \frac{C\,W\,V}{33{,}000}.$$

29. Speed of Belts.—By applying rule **8** to the same belt running at different velocities, it will be seen that the higher the velocity, the greater is the horsepower that the same belt can transmit, and from rule **9** it will be seen that the higher the speed of the belt, the less may be its width to transmit a given horsepower. From this it follows that a belt should be run at as high a velocity as conditions will permit, the maximum velocity allowable for a laced belt being about 3,500 feet per minute for ordinary single leather and double leather belts. For belts spliced by

cementing, where the splice is practically as strong as the belt itself, the velocity may be as high as 5,000 feet per minute. Cases are on record where wide main belts have been run at as high a velocity as 6,000 feet per minute.

30. In choosing a proper belt speed, due regard must be paid to commercial conditions. While a high speed of the belt means a narrow and, consequently, a cheaper belt, the increased cost of the larger pulleys that may be required may offset the gain due to the high belt speed, at least as far as first cost is concerned.

For illustration, let the problem be to transmit 10 horsepower from one shaft to another. Let the revolutions of both the driven and the driving shaft be equal, and let the shafts make 200 revolutions per minute. Choosing a belt speed of 2,000 feet per minute, the width of a single belt to transmit the given horsepower will be, by rule **8**, say, $4\frac{1}{2}$ inches. The diameter of the pulley that at 200 revolutions per minute will give a belt speed of 2,000 feet per minute is $\frac{2{,}000 \times 12}{200 \times 3.1416} = 38\frac{1}{4}$ inches, nearly. Taking the price of a 38-inch cast-iron pulley, 5-inch face, as \$15, we have the price of two pulleys as \$30. Let the distance between the pulleys be 20 feet. Then, the length of belt, according to rule **7**, is 50 feet 4 inches. Taking 51 feet as the length of the belt, and the price of a single leather belt $4\frac{1}{2}$ inches wide at 50 cents per foot, the price of the belt will be \$25.50. Then, the cost of belt and pulleys, not counting freight or express charges, etc., will be \$25.50 + \$30 = \$55.50.

Choosing a belt speed of 3,500 feet per minute, the width of belt will be $2\frac{1}{2}$ inches, nearly. The proper diameter of the pulley is $\frac{3{,}500 \times 12}{200 \times 3.1416} = 66\frac{7}{8}$ inches, say, 67 inches. The length of the belt will be 59 feet, about. The price of the belt at 30 cents per foot is \$17.70. The price of two 67-inch pulleys $3\frac{1}{2}$-inch face is, say, \$80. Then, the total first cost is \$80 + \$17.70 = \$97.70, showing that in this

particular case the use of a low belt speed reduces the first cost by $97.70 - $55.50 = $42.20.

The above illustration is not intended, and must not be construed, to be an argument against high belt speed; it simply shows the advisability of considering the commercial features in each and every case. In many cases it will be found that the narrow, high-speed belt is by far the more economical one to use.

31. Double belts are made of two single belts cemented and, usually, riveted together their whole length, and are used where much power is to be transmitted. As the effective pull for single belts, as given in Table I, is based primarily on the strength through the lace holes, a double belt, which is twice as thick, should be able to transmit twice as much power as a single belt, and in fact more than this, where, as is quite common, the ends of the belt are cemented instead of laced.

Where double belts are used on small pulleys, however, the contact with the pulley face is less perfect than it would be if a single belt were used, owing to the greater rigidity of the former. More work is, also, required to bend the belt as it runs over the pulley than in the case of the more pliable single belt, and the centrifugal force tending to throw the belt from the pulley also increases with the thickness. Moreover, in practice, it is seldom that a double belt is put on with twice the tension of a single belt. For these reasons, the width of a double belt required to transmit a given horsepower is generally assumed to be seven-tenths the width of a single belt to transmit the same power. On this basis, rules **8** and **9** become for double belts, by multiplying rule **8** by $\frac{7}{10}$ and dividing rule **9** by $\frac{7}{10}$, as follows:

Rule 10.—*To find the width of a double belt, multiply the horsepower to be transmitted by 23,100. Divide this product by the product of the velocity of the belt and the allowable effective pull, as taken from the table.*

Or,
$$W = \frac{23{,}100\,H}{V\,C}.$$

Rule 11.—*To find the horsepower that a double belt can transmit, multiply together the effective pull taken from the table, the width of the belt, and its velocity. Divide the product by 23,100.*

Or, $$H = \frac{C\,W\,V}{23{,}100}.$$

EXAMPLE 1.—What width of double belt is required to transmit 20 horsepower, the arc of contact on the smaller pulley being 135° and the speed of the belt 1,500 feet per minute?

SOLUTION.—According to Table I, the effective pull is 31.3 pounds. Then, by rule **10,**

$$W = \frac{23{,}100 \times 20}{1{,}500 \times 31.3} = 10 \text{ in., nearly.} \quad \text{Ans.}$$

EXAMPLE 2.—What horsepower can be transmitted by a 6-inch double belt running at 400 feet per minute with an arc of contact of 180°?

SOLUTION.—According to Table I, the effective pull is 38.1 pounds. Applying rule **11,** we have

$$H = \frac{38.1 \times 6 \times 400}{23{,}100} = 4 \text{ H. P., nearly.} \quad \text{Ans.}$$

THE CARE AND USE OF BELTS.

32. Leather Belts.—It is a much disputed question as to which side of the belt should be run next to the pulley. The more common practice, it is believed, is to run the belt with the hair or grain side nearest the pulley. This side is harder and more liable to crack than the flesh side. By running it on the inside the tendency is to cramp or compress it as it passes over the pulley, while if it ran on the outside, the tendency would be for it to stretch and crack. The flesh side is the tougher side, but for the reason given above the life of the belt will be longer if the wear comes upon the grain side. The lower side of the belt should be the driving side, the slack side running from the top of the driving pulley. The sag of the belt will then cause it to encompass a greater length of the circumference. Long belts, running in any other direction than vertical, work

better than short ones, as their weight holds them more firmly to their work.

It is bad practice to use rosin to prevent slipping. It gums the belt, causes it to crack, and prevents slipping for only a short time. If a belt, in good condition, persists in slipping, a wider belt should be used. Sometimes larger pulleys on the driving and driven shafts are of advantage, as they increase the belt speed and reduce the stress on the belt. Belts may be kept soft and pliable by oiling them once a month with castor oil or neatsfoot oil.

33. The Flapping of Belts.—One of the most annoying troubles experienced with belting of all kinds is the violent flapping of the slack side. Flapping may be due to any one of several causes, or to a combination of them. The most usual cause is that one or both of the pulleys run out of true. The belt is then alternately stretched and released, and while this may not cause flapping at one speed, it will usually do so at a higher speed. If the belt is rather slack, tightening it somewhat may cure or alleviate the flapping. The most obvious and best remedy, but the most expensive, is to turn the pulleys to run true.

Pulleys being out of line with each other are another prolific source of flapping, especially when one or both run out of true. First, bring the pulleys in line; if this fails, tighten the belt if it is rather loose. If no improvement is noticed and it is not possible to turn the pulleys, try to lower the belt speed a little, either by the substitution of smaller pulleys or by changing the speed of the driven shaft, according to circumstances.

With belts running at speeds above 4,000 feet per minute, flapping may occur when the pulleys are perfectly true and in line with each other, even when the belt has the proper tension. This is believed to be due to air becoming entrapped between the face of the pulley and the belt. At any rate, it has been observed that perforating the belt with a series of small holes will cure this trouble. Perforated belts may now be bought in the market.

Lack of steadiness in running, due either to sudden variations in the speed of the engine, or sudden changes in the load of the machines driven by the belt, will produce a flapping that it is almost impossible to cure. The only known cure is to take such steps as will insure steady running, as for instance, increasing the weight of the flywheel on the engine, or placing a flywheel instead of a pulley on the driven machine.

The belt not being joined square will also cause flapping, especially when the belt is running at a rather high speed. The remedy is to unlace or unfasten the joint and make it square.

Too great a distance between the pulleys may also cause flapping. In general, the distance between the pulleys should not exceed 15 feet for belts up to 4 inches in width; 20 feet for belts above 4 and below 12 inches; 25 feet for belts above 12 inches and below 18 inches; and 30 feet for larger belts. The distances here given are occasionally exceeded considerably, but as no experiments have ever been made public that would enable a fairly correct formula to be deduced for the distance between pulleys when the belt speed, width of belt, and effective pull are known, they must be taken as representing average practice.

A horizontal belt is, by many, considered to have the proper tension when it has about 1 inch of sag, while in motion, for every 8 feet between the pulleys. For belts other than horizontal, this should be less, there being no sag at all for vertical belts.

JOINING THE ENDS OF BELTS.

34. Lacing.—The ends of a belt may be joined by *lacing*, *sewing*, *riveting*, or *cementing*. Many ways of **lacing** belts are used. A very satisfactory method for belts up to 3 inches in width is shown in Fig. 16. Cut the ends of the belt square, using a sharp knife and a try square. Punch a row of holes according to the width of the belt, punching

corresponding holes exactly opposite each other in each end of the belt, using 3 holes in belts up to 2 inches wide, and 5 holes in belts between 2 and 3 inches wide. The number of holes in the row should always be uneven for the style of lacing shown. In the figure, *A* is the outside of the belt and *B* the side running nearest the pulley. The lacing should be drawn halfway through one of the middle holes from the under side, as at *1*; before going any further, it is well to see to it that the belt has no twists in it, or, in the case of a crossed belt, that it has not been given a wrong twist. The same side of the belt should run over both pulleys, which will be the case with a crossed belt if it has been twisted correctly. Having made sure that the belt is fair, pass the end of the lace on the upper side of the belt through *2* under the belt and up through *3*, back again through *2* and *3*, through *4* and up through *5*, where an incision is made in one side of the lacing, which forms a barb that will prevent the end from pulling through. Lace the right-hand side in the same manner. The lacing may advantageously be carried on at once to the right and left alternately.

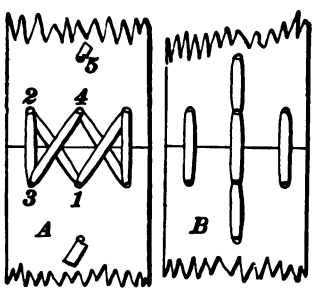

FIG. 16.

35. For belts wider than 3 inches, the lacing shown in Fig. 17 is a good one. As will be observed, there are two rows of holes. The number of holes in the row nearest the joint should exceed by one the number of holes in the second row. For belts up to 4½ inches wide, use 3 holes in the row nearest the joint and 2 holes in the second row. For belts up to 6 inches wide, use 4 and 3 holes, respectively. For larger belts, make the total number of holes in each end either one or two more than the number of inches of width, with the object of getting an odd total number of holes. For example, for a 10-inch belt, the total number of

holes should be $10 + 1 = 11$. For a 13-inch belt, it should be $13 + 2 = 15$ holes. The outside holes of the first row should not be nearer the edges of the belt than $\frac{3}{4}$ inch, nor should the first row be nearer the joint than $\frac{1}{8}$ inch. The second row should be at least $1\frac{3}{4}$ inches from the end. In the figure, *A* is the outside and *B* the side nearest the pulley. Begin at one of the center holes in the outside row, as *1*, and continue through *2*, *3*, *4*, *5*, *6*, *7*, *4*, *5*, *2*, *3*, etc.

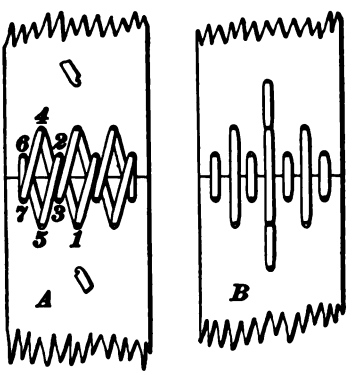

FIG. 17.

Another method is to begin the lacing on one side instead of in the middle. This method will give the rows of lacing on the under side of the belt the same thickness all the way across. The lacing should not be crossed on the side of the belt that runs next to the pulley.

Lacing affords convenient means of *shortening belts* when they stretch and of *increasing their tension*. If the belts are at all large, the ends of the belt should be drawn together by belt clamps.

36. Cementing.—Cementing makes probably the best kind of a joint ever devised. It has the serious disadvantage, however, that the stretch of the belt cannot be readily taken up, and, hence, tightening pulleys must be used when the center-to-center distance of the pulleys is not adjustable.

FIG. 18.

In dynamo driving, where endless belts are used to the exclusion of all others, the dynamo is usually mounted on a slide, so that the tension of the belt can be adjusted. For a cemented joint, the ends of the belt should be pared down with a very sharp knife to the form shown in Fig. 18, which shows a form that is recommended by belt manufacturers.

Warm the belt ends near a fire, apply the belt cement while hot, and press the joint together by two boards, one on the top and one on the bottom. Belt cement can be obtained of any dealer in engineer's supplies and full directions for using it will always be found on the can. These directions should be implicitly followed.

If belt cement cannot be obtained, a good cement may be made by melting together over a slow fire 16 parts of gutta percha, 4 parts of india rubber, 2 parts of pitch, 1 part of shellac, and 2 parts of linseed oil, by weight. Cut all ingredients very small, mix well, and use while hot.

37. Stretch of Belts.—New leather belts will stretch from one-fourth to one-half of an inch per foot of length, and hence must be taken up until the limit of stretch has been reached. Rubber belts are said to stretch continuously. Cotton and duck belts are said not to stretch with use.

38. Precautions to be Observed When Using Rubber Belts.—When rubber belts are used, animal oils or animal grease should never be used on them. If the belt should slip, it should be lightly moistened on the side nearest the pulley with boiled linseed oil.

EXAMPLES FOR PRACTICE.

1. The main line shaft is driven at 90 revolutions per minute and is to drive another shaft at 120 revolutions per minute; the latter shaft carries a pulley 20 inches in diameter. How large a pulley should be used on the main line shaft? Ans. 26⅔ in.

2. The belt wheel of an engine is 10 feet in diameter and makes 65 revolutions per minute; the line shaft is to run at 150 revolutions per minute. What size pulley should be used? Ans. 52 in.

3. The driving pulley is 48 inches in diameter and makes 90 revolutions per minute. The driven pulley being 20 inches in diameter, how many revolutions per minute will the driven shaft make?

Ans. 216 R. P. M.

4. The belt wheel of an engine being 5 feet in diameter, the driven pulley 45 inches, and the driven shaft to make 90 revolutions per minute, what should be the number of revolutions of the belt wheel?

Ans. 67½ R. P. M.

5. Two pulleys, 48 and 32 inches in diameter, respectively, are 10 feet 6 inches from center to center. What is the length of an open belt for these pulleys? Ans. 31 ft. 10 in.

6. A belt speed of 2,400 feet per minute having been chosen, what width of single belt will be needed to transmit 30 horsepower? The pulleys over which the belt runs are equal in size. Ans. 11 in., nearly.

7. What horsepower can be transmitted by a single belt 11 inches wide traveling at the rate of 1,000 feet per minute over a small pulley on which its arc of contact is 150°? Ans. 11¼ H. P., nearly.

8. With a belt speed of 1,300 feet per minute, what width of double belt will be required to transmit 5 horsepower over pulleys equal in diameter? Ans. 2½ in., nearly.

WHEEL WORK.

DRIVER AND FOLLOWER.

39. A combination of wheels and axles, as in Fig. 19, is called a train. The wheel in a train to which motion is imparted from a wheel on another shaft, by such means as a belt or gearing, is called the **driven wheel** or **follower**; the wheel that imparts the motion is called the **driver.**

It will be seen that the wheel and axle bears the same relation to the train that a simple lever does to the compound lever. Letting D_1, D_2, D_3, etc. represent the diameters of the driven wheels, and d_1, d_2, d_3, etc. the diameters of the different drivers, we have the following rule:

Rule 12.—*The continued product of the force and the diameters of the driven wheels equals the continued product of the weight, the diameter of the drum that moves the weight, and the diameters of the drivers.*

Or, $P \times D_1 \times D_2 \times D_3$, etc. $= W \times d_1 \times d_2 \times d_3$, etc.

Since the radii of several circles are in the same proportion to one another as the diameters of the same circles, it follows that the radii may be used instead of the diameters in the above rule.

EXAMPLE.—If the radius of the pulley A, Fig. 19, is 20 inches, of C 15 inches, and of E 24 inches, and the radius of the drum F is 4 inches, of the pinion D 5 inches, and of the pinion B 4 inches, how great a weight will a force of 1 pound applied at P raise?

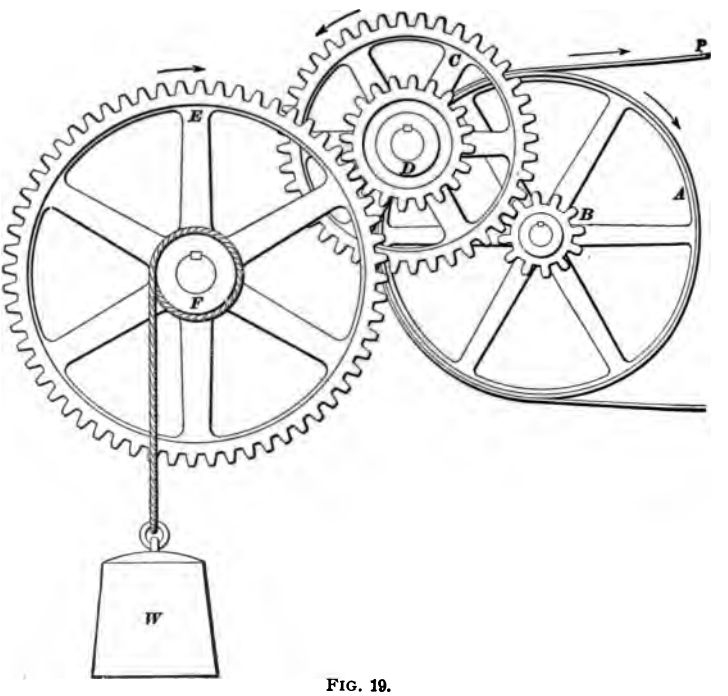

FIG. 19.

SOLUTION.—Applying rule **12**, we have

$$1 \times 20 \times 15 \times 24 = W \times 4 \times 5 \times 4,$$

or

$$W = \frac{7{,}200}{80} = 90 \text{ lb.} \quad \textbf{Ans.}$$

40. Although the combination of wheels in this example enables the lifting of a weight 90 times as great as the force applied, it has been necessary to exert the force through a distance 90 times the height through which the weight was raised. It is a universal law in the application of machines that *whenever there is a gain in power without a corresponding increase in the initial force, there is a loss in speed;* this is true of any machine.

In the example, if P were to move the entire 90 inches in 1 second, W would move only 1 inch in the same period of time.

41. Instead of using the diameter or radius of a gear, as in the last example, the number of teeth may be used when computing the weight that can be raised, or the velocity.

EXAMPLE.—The radius of the pulley A, Fig. 19, is 40 inches and that of F is 12 inches. The number of teeth in B is 9, in C 27, in D 12, and in E 36. If the weight to be lifted is 1,800 pounds, how great a force at P is it necessary to apply to the belt?

SOLUTION.—Let P represent the force; then, by the rule

$$P \times 40 \times 27 \times 36 = 1{,}800 \times 12 \times 9 \times 12,$$

or

$$P \times 38{,}880 = 2{,}332{,}800.$$

Hence,

$$P = \frac{2{,}332{,}800}{38{,}880} = 60 \text{ lb.},$$

= the amount of force necessary to apply to the belt. Ans.

GEAR-WHEELS.

42. A wheel that is provided with teeth that mesh with similar teeth on another wheel is called a **gear-wheel,** or **gear.** In Fig. 20 is shown a **spur gear.** On spur gears

FIG. 20.

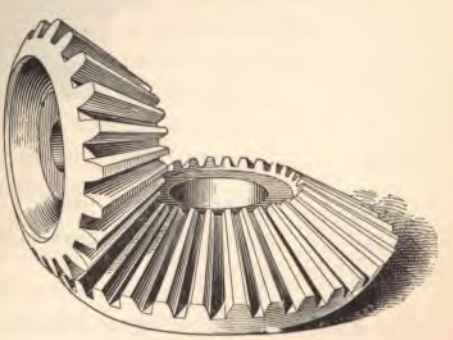

FIG. 21.

the teeth are usually parallel to the axis of the wheel or to its shaft.

Gears are said to be *in mesh* when the teeth of two wheels, respectively, engage each other or interlock.

43. In Fig. 21 is shown a pair of **bevel gears** in mesh. Of the two wheels shown, one is smaller than the other; when both wheels of a pair of bevel gears are of the same diameter they are called **miter gears.** In Fig. 22 is shown

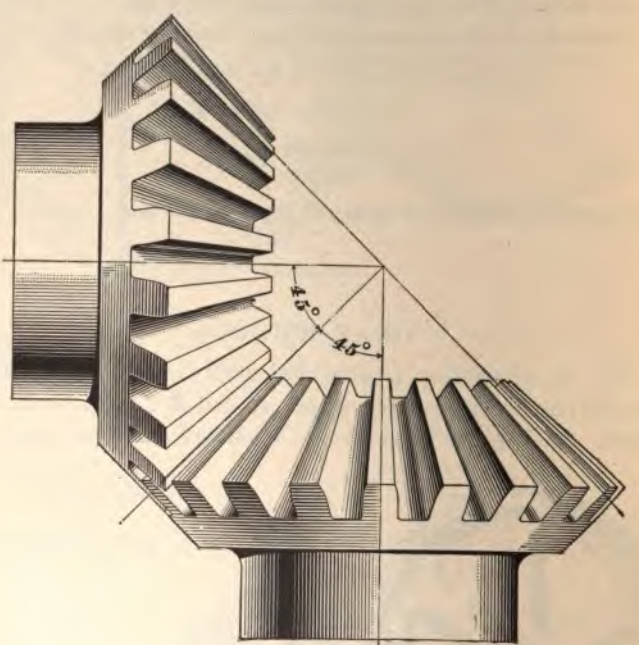

FIG. 22.

a pair of miter gears in mesh. It is obvious that the angle that the teeth of these gears make with the axis of the shaft must be 45°.

44. Of a pair of gear-wheels (either spur or bevel) having different diameters, the smaller is called a **pinion.**

In Fig. 23 is shown a revolving screw, or **worm,** as it is called, that meshes with a **worm-wheel.** It is used to transmit motion from one shaft to another at right angles to it.

As the worm is nothing else than a screw, each revolution given to it will rotate the wheel a distance equal to the pitch of the worm; consequently, if there are 40 teeth in the worm-wheel, a single-threaded worm must make 40 revolutions in order to turn the wheel once.

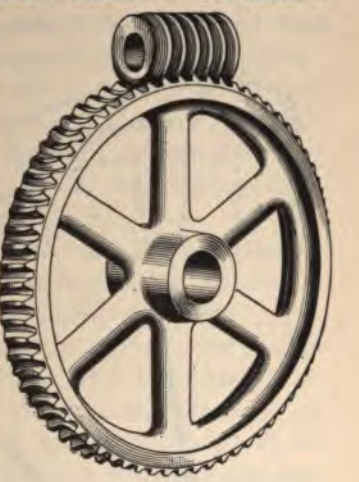

FIG. 23.

45. A **rack** is a straight bar that has gear-teeth cut on it. It may be considered as part of a gear-wheel in which the diameter is infinitely large. The teeth of racks are proportioned by the same rules as those of gear-wheels.

TEETH OF GEAR-WHEELS.

46. The object in designing the teeth of gear-wheels should be to so shape them that the motion transmitted will be exactly the same as with a corresponding pair of wheels, or cylinders, without teeth and running in contact without slipping. Such cylinders are called **pitch cylinders,** and are always represented on the drawing of a gear-wheel by a line called the **pitch circle** (see Fig. 24). The pitch circle is also called the **pitch line.**

The diameter of the pitch circle is called the **pitch diameter.** When the word "diameter" is applied to gears, it is always understood to mean the *pitch diameter*, unless especially stated as the "diameter over all" or "diameter at the root."

47. Circular and Diametral Pitch.—The distance from a point on one tooth to a corresponding point on the next tooth, *measured along the pitch circle*, is the **circular**

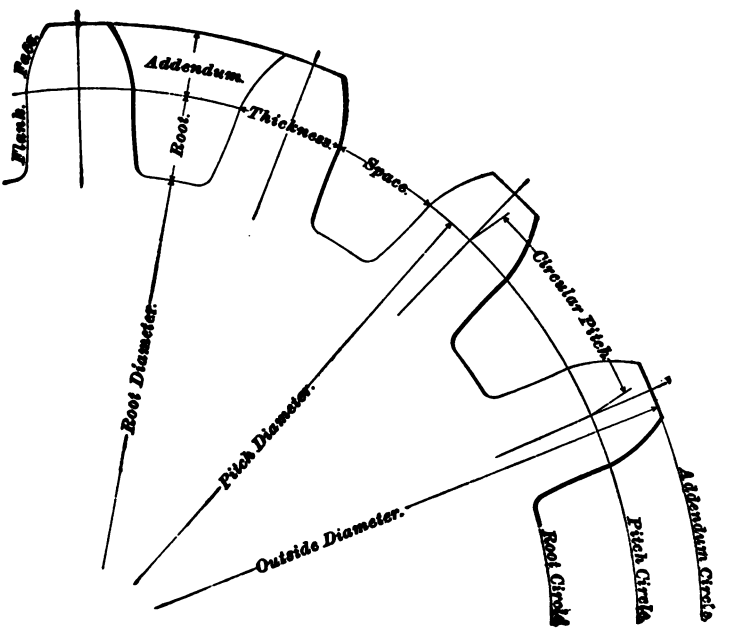

FIG. 24.

pitch. This is obtained by dividing the circumference (pitch circle) by the number of teeth, and is used in laying out the teeth of large gears and, also, when calculating their strength.

It would be very convenient to have the circular pitch expressed in manageable numbers like 1-inch, $\frac{3}{4}$ inch, etc.; but as the circumference of a gear is 3.1416 times its diameter, this requires awkward numbers for the diameters. Thus, a wheel of 40 teeth, 1-inch pitch, would have a circumference on the pitch circle of 40 inches and a diameter of 12.732 inches. Of the two it is more convenient, in the great majority of cases, to have the diameters expressed in numbers that can be easily handled. In order, however, to

have the pitch in a convenient form also, a pitch has been devised that is *expressed in terms of the diameter* and called the **diametral pitch.**

The diametral pitch is not a measurement like the circular pitch, but a *ratio. It is the ratio of the number of teeth in the gear to the number of inches in the diameter; or, it is the number of teeth on the circumference of the gear for 1 inch diameter of the pitch circle.* It is obtained by dividing the number of teeth by the diameter.

A gear, for example, has 60 teeth and is 10 inches in diameter. The diametral pitch is the ratio of 60 to $10 = \frac{60}{10} = 6$; this gear would be called a 6-pitch gear. From the definition it follows that teeth of any particular diametral pitch are of the same size and have the same width on the pitch line, whatever may be the diameter of the gear. Thus, if a 12-inch gear has 48 teeth, it will be 4 pitch. A 24-inch gear to have teeth of the same size will have twice 48, or 96 teeth, and as $96 \div 24 = 4$, has the same diametral pitch as before.

48. Other Definitions.—The other necessary definitions applying to the parts of a gear can be readily understood from Fig. 24. The thickness of the tooth and the width of the space are measured on the pitch circle. A tooth is composed of two parts, the **addendum,** or part outside of the pitch circle, and the **root,** which is inside.

A line through the outside end of the addendum is called the **addendum circle,** or **addendum line,** and one through the inside part of the root is called the **root circle,** or **root line.** The amount by which the width of the space is greater than the thickness of the tooth is called the **backlash,** or **side clearance.**

49. Proportions for Gear-Teeth.—With gears of large size, and often with cast gears of all sizes, the circular-pitch system is used. In these cases, it is usual to have the addendum, whole depth, and thickness of the tooth conform to arbitrary rules based on the circular pitch.

The usual proportions are for cast gears: Addendum $= .3 \times$ circular pitch. Root $= .4 \times$ circular pitch. Thickness of tooth $= .48 \times$ circular pitch.

The gears most often met with are the cut gears of small and medium size like those, for example, on machine tools, which are almost invariably diametral-pitch gears. The teeth are cut from the solid with standard milling cutters, proportioned with the diametral pitch as a basis. This system is also coming into use for cast gearing. In all diametral-pitch gears, the addendum, in inches, is made equal to 1 divided by the diametral pitch, and the working depth to twice the addendum. The end clearance is usually taken equal to $\frac{1}{8}$ the addendum for cut gears, though The Brown & Sharpe Manufacturing Company use $\frac{1}{10}$ the thickness of the tooth on the pitch line as the clearance. The side clearance, or "backlash," is barely enough to give a good working fit, and seldom exceeds $\frac{1}{60}$ the pitch.

Using the above proportions, a 4-pitch gear will have the addendum $= 1 \div 4 = \frac{1}{4}$ inch; the working depth will be $2 \times \frac{1}{4} = \frac{1}{2}$ inch; and the clearance, if made $\frac{1}{8}$ the addendum, $\frac{1}{8} \times \frac{1}{4} = \frac{1}{32}$ inch. The whole length of the tooth will be $\frac{1}{2} + \frac{1}{32} = \frac{17}{32}$ inch. The thickness of the tooth will be one-half the circular pitch, nearly. In a 10-pitch wheel, the addendum will be $\frac{1}{10}$ inch and the length of the tooth $\frac{17}{80}$ inch; in a $2\frac{1}{2}$-pitch, it will be $1 \div 2\frac{1}{2} = \frac{2}{5}$ inch and the length $\frac{17}{200}$ inch.

FORMS OF GEAR-TEETH.

50. The forms of teeth used in ordinary practice form part of certain curves known as the *epicycloid*, *hypocycloid*, and *involute*.

51. The Epicycloidal Tooth.—In the so-called epicycloidal tooth, which more properly is called a *cycloidal* tooth, the face of the tooth is part of an epicycloid, and the flank, part of a hypocycloid.

An **epicycloidal curve** is the path described by any point of a circle rolling, without slipping, on the outside of another circle. A **hypocycloid** is the path described by any point of a circle rolling, without slipping, on the inside of another circle.

Epicycloidal teeth can always be recognized by their appearance; they are formed by two curves that, commencing at the pitch circle, curve in opposite directions. Fig. 25 clearly exhibits the characteristic tooth form.

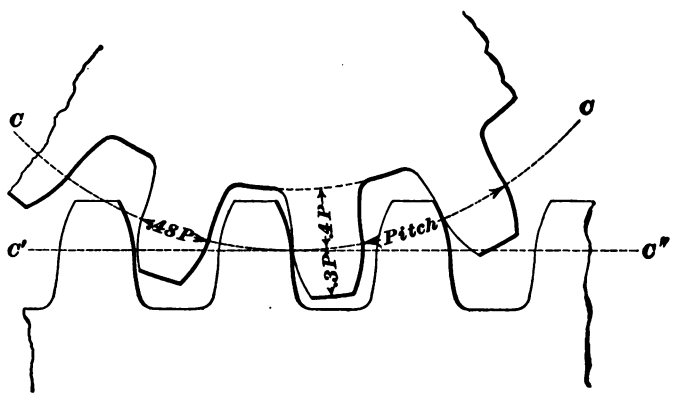

FIG. 25.

The use of epicycloidal teeth is being gradually abandoned, as they possess some practical defects, the chief defect being that the center-to-center distance of the two gears must be practically perfect in order to insure a uniform velocity of the driven gear.

52. Involute Teeth.—In Fig. 26 is shown the involute form of tooth, which is composed of but one curve.

The **involute** is the path described by any point of a string that is being wound on or off a cylinder, the cylinder being stationary. In the involute system, the sides of the teeth of the rack are straight lines, as shown in Fig. 26.

Involute teeth have two great advantages over epicycloidal teeth: (1) *They are stronger for the same pitch, as they are thicker at the root.* (2) *The gears may be spread*

slightly apart so that their pitch circles do not run tangent to

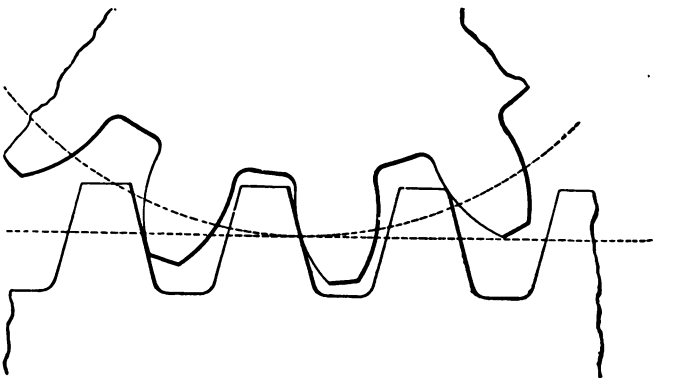

FIG. 26.

one another, without affecting the perfect action of the teeth to an appreciable extent.

GEAR CALCULATIONS.

53. The Circular-Pitch System.—For calculating the pitch diameter, number of teeth, etc. of gear-wheels, we have, for the circular-pitch system, the following rules,

where P = circular pitch in inches;
T = number of teeth;
D = pitch diameter of the gear in inches.

54. To find the pitch diameter of a gear-wheel in inches, when the pitch and number of teeth are given:

Rule 13.—*The pitch diameter equals the product of the pitch and the number of teeth divided by 3.1416.*

Or,
$$D = \frac{P\,T}{3.1416}.$$

EXAMPLE.—What is the diameter of the pitch circle of a gear-wheel that has 75 teeth and whose pitch is 1.675 inches?

SOLUTION.—Applying rule **13**, we have

$$D = \frac{1.675 \times 75}{3.1416} = 40 \text{ in.} \quad \text{Ans.}$$

55. To find the number of teeth in a gear-wheel when the diameter and pitch are given:

Rule 14.—*The number of teeth equals the product of 3.1416 and the diameter divided by the pitch.*

Or, $$T = \frac{3.1416\,D}{P}.$$

EXAMPLE.—The diameter of a gear-wheel is 40 inches and the pitch of the teeth is 1.675 inches; how many teeth are there in the wheel?

SOLUTION.—Applying the rule just given, we have

$$T = \frac{3.1416 \times 40}{1.675} = 75 \text{ teeth.} \quad \text{Ans.}$$

56. To find the pitch of a gear-wheel when the diameter and the number of teeth are given:

Rule 15.—*The pitch of the teeth equals the product of 3.1416 and the diameter divided by the number of teeth.*

Or, $$P = \frac{3.1416\,D}{T}.$$

EXAMPLE.—The diameter of a gear-wheel is 40 inches and it has 75 teeth; what is the pitch of the teeth?

SOLUTION.—By rule **15**, we have

$$P = \frac{3.1416 \times 40}{75} = 1.675 \text{ in. pitch.} \quad \text{Ans.}$$

57. The Diametral-Pitch System.—The diameter of the gear-wheel, the number of teeth, etc. are given by the following rules, where P_d = diametral pitch; D_o = outside diameter; N = number of teeth; and the other letters have the same meaning as in the three preceding rules.

58. To find the pitch diameter of the gear-wheel when the number of teeth and the pitch are given:

Rule 16.—*Divide the number of teeth by the diametral pitch.*

Or, $$D = \frac{N}{P_d}.$$

EXAMPLE.—A wheel is to have 40 teeth, 4 pitch; what is its pitch diameter?

SOLUTION.—By applying rule **16**, we have

$$D = \frac{40}{4} = 10 \text{ in.} \quad \text{Ans.}$$

59. To find the diameter over all, that is, the diameter of the blank from which the gear-wheel is cut, the number of teeth and the diametral pitch being given:

Rule 17.—*Add 2 to the number of teeth and divide by the diametral pitch.*

Or,
$$D_o = \frac{N+2}{P_d}.$$

EXAMPLE.—In the last example, what is the diameter over all the blank?

SOLUTION.—Applying the rule just given, we get

$$D_o = \frac{40+2}{4} = 10\tfrac{1}{2} \text{ in.} \quad \text{Ans.}$$

60. The number of teeth and the outside diameter of the gear-wheel being known, to find the diametral pitch:

Rule 18.—*Add 2 to the number of the teeth and divide by the outside diameter.*

Or,
$$P_d = \frac{N+2}{D_o}.$$

EXAMPLE.—A gear-wheel has 60 teeth and is $6\frac{1}{2}$ inches in diameter over all; what is the diametral pitch?

SOLUTION.—By applying rule **18**, we get

$$P_d = \frac{60+2}{6\frac{1}{2}} = 10. \quad \text{Ans.}$$

61. To find the diametral pitch, the number of teeth and the pitch diameter being known:

Rule 19.—*Divide the number of teeth by the pitch diameter.*

Or,
$$P_d = \frac{N}{D}.$$

EXAMPLE.—A wheel has 90 teeth and its pitch diameter is 30 inches; what is the diametral pitch?

SOLUTION.—By rule **19**, we get

$$P_d = \frac{90}{30} = 3. \quad \text{Ans.}$$

62. The pitch diameter and the diametral pitch being given, to obtain the number of teeth:

Rule 20.—*Multiply the pitch diameter by the diametral pitch.*

Or, $$N = D P_d.$$

EXAMPLE.—How many teeth are there in a gear-wheel having a pitch diameter of 36 inches and a diametral pitch of 5?

SOLUTION.—Applying the rule just given, we get

$$N = 36 \times 5 = 180 \text{ teeth.} \quad \text{Ans.}$$

63. The diameter over all and the diametral pitch being given, to find the number of teeth:

Rule 21.—*Subtract 2 from the product of the outside diameter and the diametral pitch.*

Or, $$N = D_o P_d - 2.$$

EXAMPLE.—How many 10-pitch teeth has a gear-wheel having an outside diameter of $8\frac{1}{8}$ inches?

SOLUTION.—By rule **21**, we get

$$N = 8\tfrac{1}{8} \times 10 - 2 = 80 \text{ teeth.} \quad \text{Ans.}$$

64. The standard diametral pitches used for cut gears are as follows: 2, $2\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{3}{4}$, 3, $3\frac{1}{2}$, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 36, 40, 48. Gears having a diametral pitch differing from those given here are, usually, either very large or very small.

65. Diameters and Distances Between Centers.—The distance between the centers of two gear-wheels being known and the ratio of their speeds, the diameter of the pitch circle of the smaller wheel is given by the following rule,

where A = center-to-center distance;
R = revolutions per minute of large gear;
r = revolutions per minute of small gear;
d = pitch diameter of small gear.

Rule 22.—*Multiply twice the center-to-center distance by the number of revolutions of the large gear, and divide the product by the sum of the revolutions of the large and small gear.*

Or, $$d = \frac{2AR}{R+r}.$$

EXAMPLE.—Given, the distance between centers = $5\frac{1}{2}$ inches; the small gear is to make 24 revolutions for every 8 revolutions of the large gear. What is the diameter of each gear?

SOLUTION.—By rule **22**,

$$d = \frac{2 \times 5\frac{1}{2} \times 8}{8 + 24} = 2\frac{3}{4} \text{ in., the diameter of the small wheel.}$$

Then, as the center distance is equal to the sum of the radii of the two wheels, the radius of the large wheel is $5\frac{1}{2} - \frac{2\frac{3}{4}}{2} = 4\frac{1}{8}$ in., and its diameter is $4\frac{1}{8} \times 2 = 8\frac{1}{4}$ in. Ans.

66. Speed and Number of Teeth.—To calculate the number of teeth or the speed of one of two gear-wheels that are to gear together:

Let N = number of revolutions per minute of the driving wheel;
n = number of revolutions per minute of the driven wheel;
T = number of teeth in the driving wheel;
t = number of teeth in the driven wheel.

Rule 23.—*The number of teeth in the driving wheel equals the product of the number of teeth and number of revolutions of the driven wheel divided by the number of revolutions of the driving wheel.*

Or, $$T = \frac{tn}{N}.$$

EXAMPLE.—The driven wheel has 27 teeth and will make 66 revolutions per minute; if the driving wheel makes 99 revolutions per minute, how many teeth are there in the driving wheel?

SOLUTION.—Applying rule **23**, we have

$$T = \frac{27 \times 66}{99} = 18 \text{ teeth.} \quad \text{Ans.}$$

67. The number of revolutions per minute of the driving wheel and the driven wheel and the number of teeth in the driving wheel being given, to find the number of teeth in the driven wheel:

Rule 24.—*The number of teeth in the driven wheel equals the product of the number of teeth and revolutions per minute of the driving wheel divided by the number of revolutions per minute of the driven wheel.*

Or, $$t = \frac{TN}{n}.$$

EXAMPLE.—The driving wheel has 18 teeth and makes 99 revolutions per minute, and the driven wheel must make 66 revolutions per minute; how many teeth must there be in the driven wheel?

SOLUTION.—Applying the rule just given, we have

$$t = \frac{18 \times 99}{66} = 27 \text{ teeth.} \quad \text{Ans.}$$

68. The number of teeth in the driving wheel and the driven wheel and the number of revolutions per minute of the driving wheel being given, to find the number of revolutions per minute of the driven wheel:

Rule 25.—*The number of revolutions per minute of the driven wheel equals the product of the number of teeth and number of revolutions of the driving wheel divided by the number of teeth of the driven wheel.*

Or, $$n = \frac{TN}{t}.$$

EXAMPLE.—There are 18 teeth in the driving wheel and it makes 99 revolutions per minute; how many revolutions per minute will the driven wheel make if it has 27 teeth?

SOLUTION.—Applying rule **25**, we have

$$n = \frac{18 \times 99}{27} = 66 \text{ R. P. M.} \quad \text{Ans.}$$

69. The number of teeth in the driving wheel and the driven wheel and the number of revolutions per minute of the driven wheel being given, to find the number of revolutions per minute of the driving wheel:

Rule 26.—*The number of revolutions of the driving wheel equals the product of the number of teeth and revolutions of the driven wheel divided by the number of teeth of the driving wheel.*

Or, $$N = \frac{t\,n}{T}.$$

EXAMPLE 1.—If there are 27 teeth in the driven wheel and if it makes 66 revolutions per minute, how many revolutions per minute will the driving wheel make if it has 18 teeth?

SOLUTION.—Applying the rule just given, we have

$$N = \frac{27 \times 66}{18} = 99 \text{ R. P. M.} \quad \text{Ans.}$$

EXAMPLE 2.—In Fig. 27, the crank-shaft makes 60 revolutions per minute; the governor pulley is 4 inches in diameter; the bevel gear on the governor pulley shaft has 19 teeth; the bevel gear that meshes with it and drives the governor has 30 teeth. The governor is to make 95 revolutions per minute; what should be the size of the pulley on the crank-shaft?

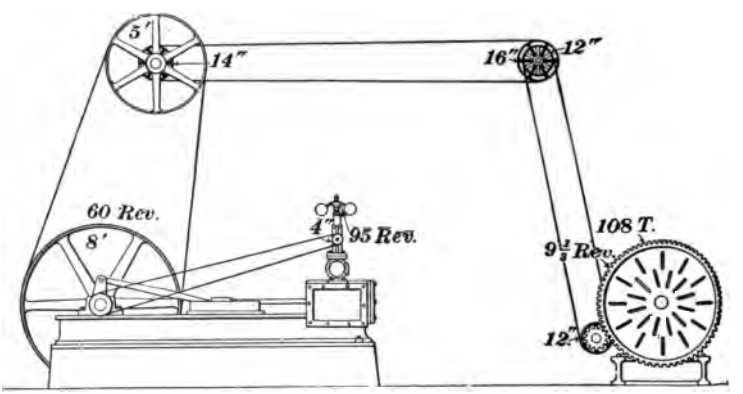

FIG. 27.

SOLUTION.—First determine the number of revolutions of the 4-inch pulley in order that the governor shall turn 95 times per minute. Applying rule 26, $N = \frac{t\,n}{T} = \frac{30 \times 95}{19} = 150$ revolutions of gear on pulley shaft = revolutions of governor pulley. Now, applying rule 3, the diameter of the pulley on the crank-shaft $= \frac{d\,n}{N} = \frac{4 \times 150}{60} = 10$ in.

Ans.

EXAMPLE 3.—In Fig. 27, the flywheel is 8 feet in diameter and drives a 5-foot pulley on the main shaft. A 14-inch pulley on the main shaft drives a 16-inch pulley on the countershaft. A 12-inch pulley on the countershaft drives a 12-inch pulley on a shaft on which is a pinion that meshes into a large gear attached to the face plate of a large lathe, and which has 108 teeth. How many teeth must the pinion have in order that the face plate may make $9\frac{1}{2}$ revolutions per minute?

SOLUTION.—Applying rule **5**, to find the revolutions per minute of the main shaft, $\frac{8 \times 60}{5} = 96$ R. P. M. Applying the same rule again to find the revolutions of the countershaft, $\frac{14 \times 96}{16} = 84$ R. P. M. Applying it once more to find the revolutions of the pulley that turns the small gear, $\frac{12 \times 84}{12} = 84$ R. P. M. Applying rule **23**, $\frac{108 \times 9\frac{1}{2}}{84}$ = 12 teeth in pinion or driver. Ans.

EXAMPLES FOR PRACTICE.

1. The driving pulley makes 110 R. P. M. and is 21 inches in diameter; what should be the size of the driven pulley in order to make 385 R. P. M.? Ans. 6 in.

2. The main shaft of a certain shop makes 120 R. P. M. It is desired to have the countershaft make 150 R. P. M. There are on hand pulleys 16 inches, 24 inches, 28 inches, 35 inches, and 38 inches in diameter. Can two of these be used, or must a new pulley be ordered? Ans. Use the 28-inch and 35-inch pulleys.

3. The pinion (driver) makes 174 R. P. M. and the follower 24 R. P. M.; how many teeth must the pinion have if the follower has 87 teeth? Ans. 12 teeth.

4. If an engine flywheel is 66 inches in diameter and makes 160 R. P. M., what must be the diameter of the pulley on the main shaft to make 128 R. P. M.? Ans. $82\frac{1}{2}$ in.

5. What is the pitch diameter of a gear whose circular pitch is $1\frac{1}{4}$ inches and has 28 teeth? Ans. 11.14 in.

6. How many teeth are there in a gear whose circular pitch is .7854 inch and which is 23 inches in diameter? Ans. 92 teeth.

7. What is the circular pitch of a gear whose diameter is 20.372 inches and which has 128 teeth? Ans. $\frac{1}{2}$ in.

8. What is the pitch diameter of a gear-wheel having 80 teeth, 7 diametral pitch? Ans. $11\frac{3}{7}$ in.

9. What is the over-all diameter of a gear-wheel of 9 diametral pitch and 47 teeth? Ans. $5\frac{4}{9}$ in.

10. What is the diametral pitch of a gear-wheel having 90 teeth and an outside diameter of $5\frac{1}{4}$ inches? Ans. 16 pitch.

11. How many teeth of 20 diametral pitch can be cut in a gear-wheel having an outside diameter of $2\frac{1}{10}$ inches? Ans. 40 teeth.

12. If the center-to-center distance of two gear-wheels is 6 inches and the small gear is to make 4 revolutions for 1 revolution of the large gear, what must be the diameter of the large gear?

Ans. 9.6 in.

13. In a pair of gear-wheels, the driver has 48 teeth and the driven wheel 56 teeth. If the driven wheel makes 98 revolutions per minute, how many revolutions must the driver make? Ans. $114\frac{1}{3}$ R. P. M.

14. In a train of gears, the drivers have 16, 30, 24, and 18 teeth, respectively; the followers have 12, 24, 36, 40 teeth, respectively. If the first driver makes 80 R. P. M., how many R. P. M. will the last follower make? Ans. 40 R. P. M.

FIXED AND MOVABLE PULLEYS.

70. Pulleys are also used for hoisting or raising loads, in which case the frame that supports the axle of the pulley is called the **block.**

71. A **fixed pulley** is one whose block is not movable, as in Fig. 28. In this case, if the weight W be lifted by

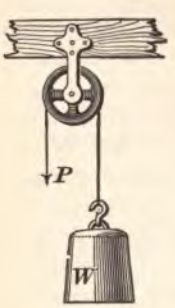

FIG. 28.

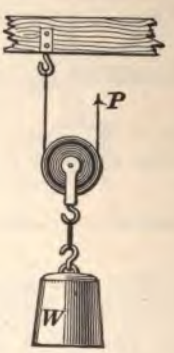

FIG. 29.

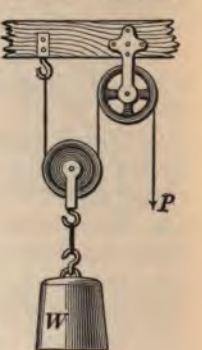

FIG. 30.

pulling down P, the other end of the cord W will evidently move the same distance upwards that P moves downwards; hence, P must equal W.

72. A **movable pulley** is one whose block is movable, as in Fig. 29. One end of the cord is fastened to the beam and the weight is suspended from the pulley, the other end of the cord being drawn up by the application of a force P. A little consideration will show that if P acts through a certain distance, say 1 foot, W will move through *half* that distance, or 6 inches; hence, a pull of 1 pound at P will lift 2 pounds at W.

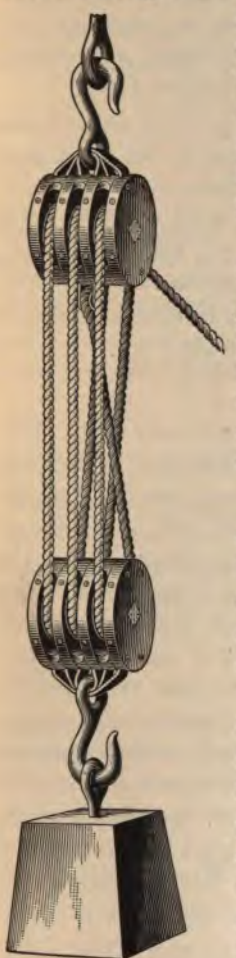

FIG. 31.

The same would also be true if the free end of the cord were passed over a *fixed pulley*, as in Fig. 30, in which case the fixed pulley merely changes the direction in which P acts, so that a weight of 1 pound hung on the free end of the cord will balance 2 pounds hung from the *movable pulley*.

73. A **combination of pulleys,** as shown in Fig. 31, is sometimes used. In this case there are three movable and three fixed pulleys, and the amount of movement of W, owing to a certain movement of P, is readily found.

It will be noticed that there are 6 *parts* of the rope, not counting the free end; hence, if the movable block be lifted 1 foot, P remaining in the same position, there will be 1 foot of slack in each of the 6 parts of the rope, or 6 feet in all. Therefore, P must move 6 feet in order to take up this slack, or P moves 6 times as far as W. Hence, 1 pound at P will support 6 pounds at W, since the *force multiplied by the distance through which it moves equals the weight multiplied by the distance through which it moves.* It will also be noticed that there are three movable pulleys, and that $3 \times 2 = 6$.

LAW OF COMBINATION OF PULLEYS.

74. Rule 27.—*In any combination of pulleys where one continuous rope is used, a load on the free end will balance a weight on the movable block as many times as great as itself as there are parts of the rope supporting the load, not counting the free end, assuming that there are no friction losses.*

The above law is good whether the pulleys are side by side, as in the ordinary block and tackle, or whether they are in line and beneath one another.

EXAMPLE.—In a block and tackle having 5 movable pulleys, how great a force must be applied to the free end of the rope to raise 1,250 pounds, assuming that there are no friction losses?

SOLUTION.—Since there are 5 movable pulleys, there must be 10 parts of the rope to support them. Hence, according to the above law, a force applied to the free end will support a load 10 times as great as itself, or the force $= \frac{1,250}{10} = 125$ lb. Ans.

75. Frictional Losses.—Owing to the friction of the sheaves and the friction and resistance to bending of the rope, it is not possible to move a weight with a block and tackle by applying the theoretical force calculated by the above rule. The actual pull required depends on such factors as the condition of the rope and the friction and number of the sheaves. By experiment it has been determined that under average conditions the load that can be raised with a block and tackle having 4 sheaves will average about 75 per cent. of the theoretical load; with 5 sheaves, about 70 per cent.; with 6 sheaves, about 66 per cent.; with 7 sheaves, about 63 per cent.; and with 8 sheaves, about 60 per cent. No records of tests with a larger number of sheaves have been made public, but it is fair to assume that the decrease in the load that can be lifted will vary in about the same ratio.

Let f_a = the force applied at the free end of the rope;
c = the ratio of the theoretical to the actual load corresponding to the number of sheaves;
m = the number of movable pulleys;
l = the load to be raised.

Then, the force that is actually required to raise a given load, may be found approximately by

Rule 28.—*Divide 100 times the load by the product of twice the number of movable pulleys and the number of per cent. corresponding to the number of sheaves used.*

Or, $$f_a = \frac{100\,l}{2\,m\,c}.$$

EXAMPLE.—In an ordinary block and tackle having three movable pulleys, as shown in Fig. 31, how great a force must actually be applied to raise 2,000 pounds?

SOLUTION.—According to Art. **75**, c may be taken as 66, there being 6 sheaves. Then applying rule **28**, we get

$$f_a = \frac{100 \times 2{,}000}{2 \times 3 \times 66} = 505 \text{ lb., nearly.} \quad \text{Ans.}$$

76. The load that may actually be raised with a block and tackle will be approximately given by the following rule:

Rule 29.—*Multiply the force to be applied to the free end of the rope by twice the number of movable pulleys and by the number of per cent. corresponding to the number of sheaves used. Divide the product by 100.*

Or, $$l = \frac{f_a\,2\,m\,c}{100}.$$

EXAMPLE.—If there are enough men pulling on the free end of the rope of a block and tackle having two movable pulleys to exert a force of 250 pounds, what weight may they expect to raise?

SOLUTION.—There being 4 sheaves, by Art. **75**, $c = 75$. Then by rule **29**,

$$l = \frac{250 \times 2 \times 2 \times 75}{100} = 750 \text{ lb.} \quad \text{Ans.}$$

77. If the free end of the hauling rope passes first around a stationary sheave, as in Fig. 31, it does not make any difference in what direction in the plane of the sheave the rope is pulled. If it passes first around a movable sheave, however, as in Fig. 29, the pull must be exerted in a line parallel to the line of action of the resistance, or a line joining the

centers of the movable and stationary sheaves, in order to obtain the maximum effect. If the rope pulls on the movable sheave at an angle, its effect will be a displacement sideways, instead of straight up.

78. The **Weston differential pulley block,** more commonly known as a **chain hoist,** is shown in Fig. 32. With the help of this device, one man can hoist a load far beyond what can be done with the ordinary block and tackle. There are two pulleys slightly different in size alongside of each other in the stationary block and rigidly connected so as to turn together. An endless chain passes over both pulleys and supports the movable block in one of its bights. The other loop, or bight, is free and forms the hauling part. This kind of a pulley block possesses the valuable property that the load can be stopped anywhere by simply ceasing to haul on the hoisting part of the free loop.

FIG. 32.

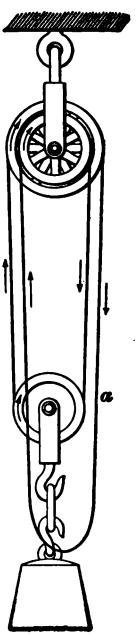

FIG. 33.

In Fig. 33 the differential pulley block is shown in diagrammatic form. Suppose that the part *a* of the free bight is pulled downwards, that is, in the direction of the arrow. Evidently, the upper pulleys will both turn in the direction of the hands of a watch, as indicated by the curved arrow. Then, the left-hand part of the bight supporting the movable pulley will move upwards, and the right-hand part downwards. But as the large and small pulley of the upper block move together, it follows that the left-hand side of the bight supporting the movable sheave will run faster

over the large pulley than the right-hand side of the bight will run downwards over the small pulley. The result is that the upward motion of the movable block will be equal to one-half the difference of the distances passed over in the same time by fixed points in both sides of the bight.

79. Differential chain hoists are invariably so constructed that one man can hoist any load up to the maximum capacity of the hoists. This maximum load will usually be found stamped on the upper block, and there is, hence, no need in practice of calculating the force that must be exerted to hoist a load with a chain hoist.

EXAMPLES FOR PRACTICE.

1. In a block and tackle with two movable pulleys, (*a*) what force will be required to lift 300 pounds, assuming that there are no friction losses? (*b*) What force may be expected to be actually required, taking friction into account?

Ans. { (*a*) 75 lb. (*b*) 100 lb.

2. A man weighing 120 pounds throws his whole weight on the free end of the rope of a block and tackle with 3 movable pulleys. What weight can he raise, (*a*) assuming that there are no friction losses? (*b*) taking friction into account?

Ans. { (*a*) 720 lb. (*b*) 475 lb., about.

THE INCLINED PLANE.

80. An **inclined plane** is a slope, or flat surface, making an angle with another surface or base plane.

81. Three cases may arise in practice with an inclined plane having a horizontal base plane: (1) Where the force acts parallel to the plane, as

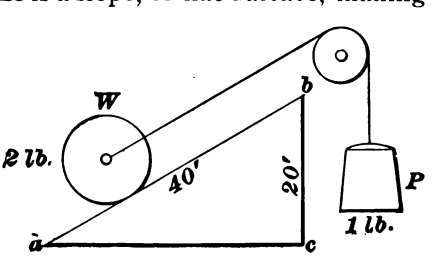

FIG. 34.

in Fig. 34. (2) Where the force acts parallel to the base, as in Fig. 35. (3) Where the force acts at an angle to the plane or to the base, as in Fig. 36.

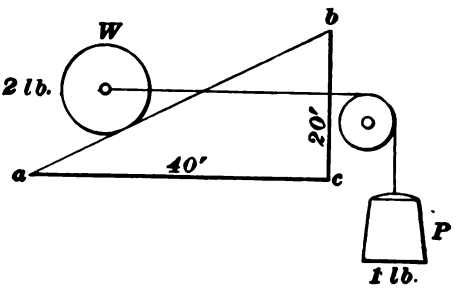

FIG. 35.

82. Relation Between Force and Weight.—In Fig. 34, the relation existing between the force and the weight is easily found. The weight ascends a distance equal to *c b*, or the height of the inclined plane, while the force acts through a distance equal to *a b*, or the length of the inclined plane. Therefore,

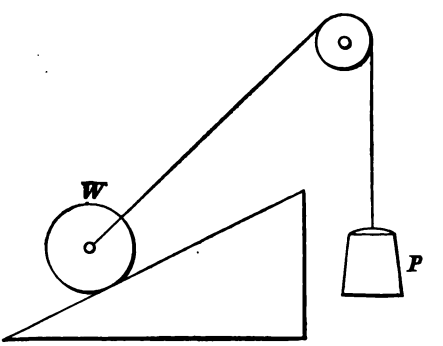

FIG. 36.

Rule 30.—*To find the force, multiply the weight by the height of the plane and divide the product by the length of the plane. To find the weight that can be raised, multiply the force by the length of the plane and divide this product by the height of the plane.*

Or, let
P = force;
l = length of the plane;
b = base of the plane;
h = height of plane;
W = weight.

Then, $$P = \frac{Wh}{l} \text{ and } W = \frac{Pl}{h}.$$

EXAMPLE 1.—The length of an inclined plane is 40 feet and its height is 5 feet; what force is required to sustain a weight of 100 pounds?

SOLUTION.—Applying rule **30**, we have

$$P = \frac{100 \times 5}{40} = 12\tfrac{1}{2} \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 2.—The length of an inclined plane is 8 feet and its height is 15 inches; what weight will a force of 120 pounds support?

SOLUTION.—Applying the rule just given,

$$W = \frac{120 \times 96}{15} = 768 \text{ lb.} \quad \text{Ans.}$$

83. In Fig. 35, the force is supposed to act parallel to the base for any position of W; therefore, while W is moving from the level $a\,c$ to b, or through the height $c\,b$ of the inclined plane, P will move through a distance equal to the length of the base $a\,c$. When the force acts parallel to the base, we have

Rule 31.—*To find the force required, multiply the weight by the height of the plane and divide the product by the length of the base of the plane. To find the weight that can be raised, multiply the force by the length of the base of the plane and divide the product by the height of the plane.*

Or, $$P = \frac{W h}{b} \text{ and } W = \frac{P b}{h}.$$

EXAMPLE 1.—With a base 30 feet long and a height of 6 feet, what force will sustain a weight of 75 pounds?

SOLUTION.—By rule **31**,

$$P = \frac{75 \times 6}{30} = 15 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 2.—A force of 12 pounds sustains a weight on a plane whose base is 6 feet long and whose height is 18 inches; find the weight.

SOLUTION.—Applying the rule just given, we have

$$W = \frac{12 \times 6}{1\frac{1}{2}} = 48 \text{ lb.} \quad \text{Ans.}$$

For Fig. 36, no rule can be given. The ratio of the power varies as the position of W varies; it may be determined by the principle of the resolution of forces. As W

shifts its position, the angle that its cord makes with the face of the plane varies, and the magnitude of the force P depends on this angle; the smaller the angle the less is P required to be in order to support a given weight on a given plane.

84. Applications of the Inclined Plane.—The wedge is a movable inclined plane that in various modifications serves for many different purposes. In the form of a stake wedge, as shown in Fig. 37, it serves to move heavy weights or to separate closely joined pieces of material. In the form of a cold chisel or knife, it serves to sever substances. In the form of a key, it serves to move the brasses in the different pin-joint connections of a steam engine. Familiar examples of inclined planes about machinery are the keys used for fastening a crank to the crank-shaft or a pulley to the shaft; in some designs of cross-heads, the cotters securing the crosshead to the piston rod are an example of an inclined plane; as are also the adjustable shoes in many designs of crossheads for steam engines.

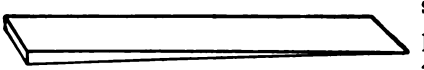

FIG. 37.

85. Taper of Keys.—In keys, cotters, and stake wedges, it is customary to express the inclinations as a certain amount per foot, or taper per foot. Thus, in Fig. 38, the inclination would be $3 - 2 = 1$ inch in 12 inches,

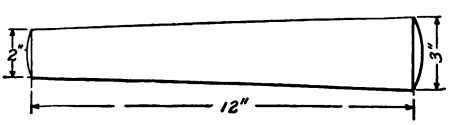

FIG. 38.

and the taper would be, as there are 12 inches in a foot, 1 inch per foot. The taper per foot can be ascertained by the following rule,

where a = the large diameter in inches;
b = the small diameter in inches;
l = the length in inches;
T = taper in inches per foot.

Rule 32.—*Subtract the small diameter from the large diameter, multiply the remainder by 12, and divide the product by the length of the key.*

Or, $$T = \frac{(a - b)\ 12}{l}.$$

EXAMPLE.—A key measures 4 inches at the large end and 3 inches at the small end; if its length is 16 inches, what is the taper per foot?

SOLUTION.—Applying the rule just given, we get

$$T = \frac{(4 - 3) \times 12}{16} = \tfrac{3}{4} \text{ in. per ft.} \quad \text{Ans.}$$

86. Where tapered keys are used for adjusting brasses, it is often desirable to know how much the brasses will be closed in on the pin by a given movement of the key, or how much to back out a key that has been driven home until the brasses nip the pin, the clearance between the brasses and the pin necessary for satisfactory running having been decided on.

Let T = taper in inches per foot;
d = distance the key is driven in inches;
x = amount the brass is moved in inches.

Rule 33.—*To find the amount the brass has been moved, multiply the taper by the distance the key is driven and divide the product by 12. To find how far to back out a key, multiply the amount the brass is to be moved back by 12 and divide by the taper.*

Or, $$x = \frac{T d}{12} \text{ and } d = \frac{12\, x}{T}.$$

EXAMPLE.—In Fig. 39, the key has been driven home until the brasses nip the pin; it has been determined to run the brasses with a clearance of $\frac{1}{64}$ inch; how much must the key be backed out, its taper being $\frac{3}{4}$ inch per foot?

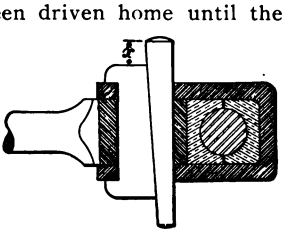

FIG. 39.

SOLUTION.—By rule **33**,

$$d = \frac{12 \times \frac{1}{64}}{\frac{3}{4}} = \tfrac{1}{4} \text{ in.} \quad \text{Ans.}$$

A ready way of measuring the distance the key is driven back is to measure from the top surface of the cotter to the upper edge of the key while the key is home. Add the amount the key is to be driven back, and move it until a rule shows it to have been driven the right amount. This method is preferable to making scriber marks on the flat sides of the key, as there is no liability of getting mixed up by the multiplicity of marks sure to be found after years of service.

87. Keys for fastening pulleys to shafts in order to cause them to rotate together belong usually to one of the three classes shown in Fig. 40. The one shown at *a* is a concave key that is hollowed out to fit the shaft. As it holds the pulley by friction alone, it is only suitable for light work. In case such a key slips, its holding power can be increased a little by hollowing out its concave side to a radius smaller than

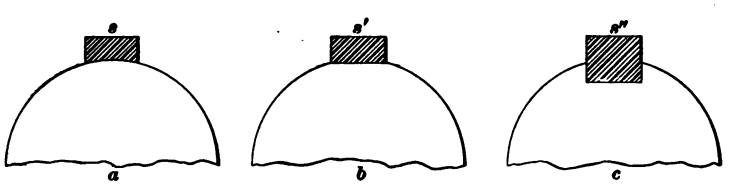

FIG. 40.

the radius of the shaft. The flat key is shown in Fig. 40 at *b*; a flat surface is cut on the shaft, and, consequently, the key is more effective than the concave key, as far as slipping under a rotative strain is concerned. The sunk key shown in Fig. 40 at *c* is the most effective, since it is impossible for the pulley to turn on the shaft without shearing off the key. Keys for fastening pulleys, etc. are usually given a taper of from $\frac{1}{8}$ to $\frac{1}{4}$ inch per foot.

EXAMPLES FOR PRACTICE.

1. An inclined plane is 30 feet long and 7 feet high; what force is required to roll a barrel of flour weighing 196 pounds up the plane, the friction being neglected? Ans. 45.7 lb.

2. What is the taper per foot of a stake wedge measuring $\frac{1}{32}$ inch at the small end and $\frac{1}{4}$ inch at the large end, its length being 4 inches?

Ans. $\frac{7}{32}$ in. per ft.

3. Given, a key having a taper of 1 inch per foot. How much must it be backed out to give the brasses a working clearance of $\frac{1}{32}$ inch?

Ans. $\frac{1}{8}$ in.

THE SCREW.

88. A **screw** is a cylinder having a helical projection winding around its circumference. This helix is called the **thread** of the screw. The distance that a point on the helix is drawn back or advanced in the direction of the length of the screw during one turn is called the **pitch** of the screw. The screw in Fig. 41 is turned in a nut *a* by means of a force applied at the end of the handle *P*. For one complete revolution of the handle, the screw will be advanced lengthwise an amount equal to the pitch. If the nut is fixed and a weight placed upon the end of the screw, as shown, it will be raised vertically, by one revolution of the screw, a distance equal to the pitch. During this revolution, the force at *P* will move through a distance equal to the circumference of a circle whose radius is *PF*. Therefore, $W \times$ *pitch of thread* $= P \times$ *circumference of circle through which P moves.*

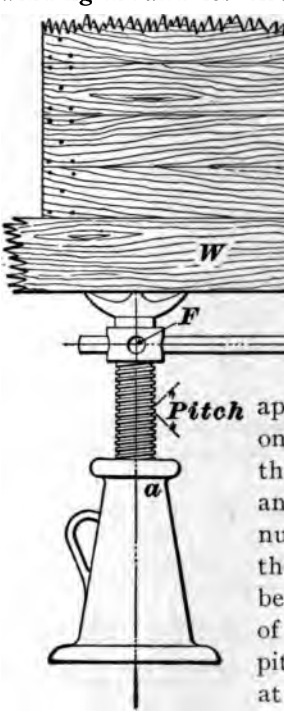

FIG. 41.

89. Owing to the friction between the nut and the screw and between the pivot of the screw and its cap, the weight that can be raised with a screw, or the force that can be

exerted by it in the direction of its axis, is far less than that calculated on the assumption that friction does not exist. The friction depends on the pitch, the condition of the thread on the screw and in the nut, and the nature of the lubricant.

From a long series of experiments, Mr. Wilfred Lewis has deduced a formula for determining a factor by which to multiply the weight that could be raised if there were no friction, so as to determine the probable actual weight that can be raised.

Let d = diameter of screw in inches;
p = pitch in inches;
E = the number by which to multiply the theoretical weight in order to obtain the approximate actual weight.

Rule 34.—*Add the diameter of the screw to the pitch and divide the pitch by this sum; the quotient so obtained will be the factor by which to multiply the theoretical weight in order to determine the approximate actual weight.*

Or,
$$E = \frac{p}{p + d}.$$

EXAMPLE.—With a screw having 4 threads to the inch and 2 inches in diameter, what is the factor by which to determine how much of the theoretical load can probably be raised?

SOLUTION.—Since there are 4 threads per inch, the pitch is $1 \div 4$, or $\frac{1}{4}$ inch. Then applying rule **34**, we get

$$E = \frac{\frac{1}{4}}{\frac{1}{4} + 2} = .11. \quad \text{Ans.}$$

90. The weight that can be lifted when friction is neglected, or the force that can be exerted by a screw when a given force is exerted on the lever, is given by the following rule,

where W = weight or force exerted by the screw;
P = force applied to the end of the lever;
p = pitch;
r = distance from the axis of the screw to the point of application of the force P.

Rule 35.—*Multiply the force applied to the lever, in pounds, by the distance of its point of application from the axis of the screw, in inches, and multiply this product by 6.2832; divide the result by the pitch in inches and the quotient will be the theoretical force that will be exerted by the screw.*

Or, $$W = \frac{6.2832\ P r}{p}.$$

91. To find the theoretical force that must be applied to the lever to raise a given weight or to exert a given force:

Rule 36.—*Multiply the weight or given force by the pitch and divide this product by the product of 6.2832 and the distance from the axis of the screw to the point on the lever where the force is applied.*

Or, $$P = \frac{W p}{6.2832\ r}.$$

92. To obtain the probable actual weight that may be raised, multiply the result obtained by rule **35** by the value obtained from rule **34.** To obtain the probable force that must be applied to the lever in order to lift a given weight, divide the result obtained from rule **36** by the value obtained from rule **34.**

EXAMPLE 1.—A screw jack has a screw $1\frac{1}{4}$ inches diameter with 5 threads to the inch. If a man pulls at the end of the lever 14 inches from the axis with a force of 40 pounds, what weight may he expect to raise?

SOLUTION.— 5 threads per inch $= \frac{1}{5}$-inch pitch.

By rule **35**, $W = \frac{6.2832 \times 40 \times 14}{\frac{1}{5}} = 17{,}593$ pounds, nearly.

By rule **34**, $E = \frac{\frac{1}{5}}{\frac{1}{5} + 1\frac{1}{4}} = .1$, nearly.

Then, as just stated, the probable weight that may be raised is

$$17{,}593 \times .1 = 1{,}759.3 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 2.—The pull that an ordinary man can exert at the end of a lever is usually taken at 40 pounds; would it be possible for one man

to raise a weight of 3 tons with a jack-screw having 4 threads to the inch and $2\frac{1}{2}$ inches in diameter when the longest lever available is 24 inches?

SOLUTION.—According to rule **36**, the theoretical force that must be exerted is

$$\frac{3 \times 2000 \times \frac{1}{4}}{6.2832 \times 24} = 9.9 \text{ pounds, nearly.}$$

By rule **34**, $$E = \frac{\frac{1}{4}}{\frac{1}{4} + 2\frac{1}{2}} = .091, \text{ nearly.}$$

Then, as just stated, the probable actual force required is 9.9. $9.9 \div .091 = 109$ pounds, about, or more than a single man may be expected to exert steadily. Ans.

SCREWS USED AS FASTENINGS.

93. In machine construction, screws are used more frequently as fastening devices than as a means of transmitting motion or producing pressure. Screws used as fastening devices may be divided into two general classes, which are distinguishable from each other by the form of thread used: (1) *Metal screws*, or screws intended to be screwed into metals. (2) *Wood screws*, or screws intended for wood.

94. **Metal screws** have either a sharp **V** thread or the United States standard thread shown in Fig. 42. In the

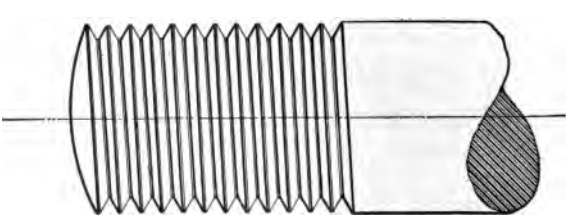

FIG. 42.

latter, as also in the sharp **V** thread, the sides of the thread form a 60° angle. In the United States standard thread, the thread is flattened at the top and bottom, as shown in

the figure. The advantage of this is that it makes a stronger screw for equal pitches and diameter.

95. Metal screws are divided by the trade into *machine bolts*, *capscrews*, and *machine screws*.

96. In **machine bolts,** or **bolts,** for short, one of which is shown in Fig. 43 (*a*), the shank and head are left rough, or just as they come from the bolt-heading machine. Machine bolts are regularly made in lengths varying by

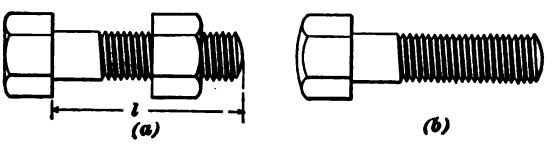

FIG. 43.

half inches from $1\frac{1}{2}$ to 8 inches. Above 8 inches and up to 20 inches the length varies by whole inches. The length of a machine bolt is always measured under the head, as the distance l in Fig. 43 (*a*).

97. The standard diameters in which machine bolts are regularly made are $\frac{1}{4}$, $\frac{5}{16}$, $\frac{3}{8}$, $\frac{7}{16}$, $\frac{1}{2}$, $\frac{9}{16}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$, and 1 inch. Sizes differing from these are special and can only be obtained on special order. By the *diameter of a bolt* is always meant its diameter over the top of the thread. Machine bolts can be obtained with either hexagonal or square heads and with hexagonal or square nuts.

98. **Capscrews,** one of which is shown in Fig. 43 (*b*), are bolts with either hexagonal or square heads; they have the shanks turned and the heads finished all over by milling and turning. Unlike machine bolts, they are usually furnished without nuts, unless especially ordered.

99. Machine bolts and capscrews are made with a pitch of thread corresponding to the United States standard thread, as shown in Table II.

TABLE II.

STANDARD THREADS PER INCH.

Diameter. Inches.	Threads per Inch.	Diameter. Inches.	Threads per Inch.
$\frac{1}{4}$	20	$1\frac{1}{4}$	7
$\frac{5}{16}$	18	$1\frac{3}{8}$	6
$\frac{3}{8}$	16	$1\frac{1}{2}$	6
$\frac{7}{16}$	14	$1\frac{5}{8}$	$5\frac{1}{2}$
$\frac{1}{2}$	13	$1\frac{3}{4}$	5
$\frac{9}{16}$	12	$1\frac{7}{8}$	5
$\frac{5}{8}$	11	2	$4\frac{1}{2}$
$\frac{3}{4}$	10	$2\frac{1}{4}$	$4\frac{1}{2}$
$\frac{7}{8}$	9	$2\frac{1}{2}$	4
1	8	$2\frac{3}{4}$	4
$1\frac{1}{8}$	7	3	$3\frac{1}{2}$

The only deviation from this table of threads is in $\frac{1}{2}$-inch square and hexagonal capscrews, where the usual number of threads for a sharp V thread is 12 per inch instead of 13, as called for. Capscrews are usually furnished with a sharp V thread, but can now be obtained with the United States standard thread. Machine bolts usually have the United States standard thread and the number of threads per inch called for in the table.

100. Machine screws are small screws with a slotted head. They are known to the trade as *flat heads*, shown in Fig. 44 (*a*); *round heads*, shown in Fig. 44 (*b*); *button heads*, shown in Fig. 44 (*c*), and *fillister heads*, shown in Fig. 44 (*d*).

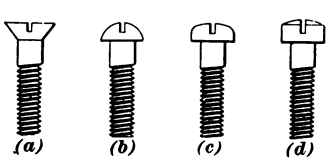

FIG. 44.

101. Screws for uniting metals are made in a variety of forms that differ from those here shown to suit special

conditions. Some of these are setscrews, carriage bolts, tire bolts, boiler patch bolts, elevator bolts, etc. Most of these can only be obtained on special order, although some of the more common sizes may be found in stock.

102. **Wood screws** are either flat headed or round headed, with a slotted head. When wood screws have a square head for a wrench, instead of a slot for a screwdriver, they are known as *lagscrews*, or *coach screws*.

VELOCITY RATIO AND EFFICIENCY.

103. By the term **velocity ratio** is meant the ratio of the distance through which the force acts to the corresponding distance the weight moves. Thus, if the force acts through a distance of 12 inches while the weight moves 1 inch, the velocity ratio is 12 to 1, or 12, that is, P moves 12 times as fast as W.

If the velocity ratio be known, the weight that any machine will raise, under the assumption that there are no frictional resistances, can be found by multiplying the force by the velocity ratio. If the velocity ratio is 8.7 to 1, or 8.7, $W = 8.7 \times P$, since $W \times 1 = 8.7$.

104. In any machine, the force actually required to raise a weight or overcome a resistance *is always greater than the quotient obtained by dividing this weight, or resistance, by the velocity ratio of the machine.*

Thus, if there were no friction, a machine whose velocity ratio was 5 would, by an application of 100 pounds of force, raise a weight of 500 pounds. Now, suppose that the friction in the machine is equivalent to 10 pounds of force; then, it would take 110 pounds of force to raise 500 pounds.

If in the above illustration friction were neglected, we would have 110 lb. $\times$ 5 = 550 lb. as the weight that 110 pounds would raise; but owing to the frictional

resistance, it only raised 500 pounds. Then, we have for the ratio between the two, $\frac{500}{550} = .91$. That is, 500 : 550 :: .91 : 1.

105. Efficiency.—The ratio between the weight actually raised and the force multiplied by the velocity ratio is called the **efficiency of the machine.** For example, if the weight actually raised by a machine, say a screw, is 1,600 pounds, and the force multiplied by the velocity ratio is 2,400 pounds, the efficiency of this machine is $\frac{1600}{2400} = .66\frac{2}{3}$, or $66\frac{2}{3}$ per cent.

EXAMPLE.—In a machine having a combination of pulleys and gears, the velocity ratio of the whole is 9.75; a force of 250 pounds just lifts a weight of 1,626 pounds; what is the efficiency of the machine?

SOLUTION.—Efficiency $= \frac{1,626}{250 \times 9.75} = .6671$ or 66.71 per cent. Ans.

Since the total amount of friction varies with the load, it follows that the efficiency will also vary for different loads.

106. If the efficiency of a machine is known, the force actually required to raise a given load may be found by dividing the load by the product of the velocity ratio of the machine and the efficiency. Thus, if a certain machine has a velocity ratio of 10.6 and its efficiency is 60 per cent., the force that must actually be applied to raise a load of 840 pounds is $\frac{840}{10.6 \times .60} = 132.1$ pounds, nearly. If there have been no losses through friction, etc., the force required will be $840 \div 10.6 = 79.25$ pounds, nearly.

107. If the efficiency is known, the weight that a certain force will raise may be found by multiplying together the force, velocity ratio, and the efficiency. Thus, if a certain machine has a velocity ratio of $6\frac{1}{2}$ and an efficiency of 78 per cent., a force of 140 pounds will raise a weight of $140 \times 6\frac{1}{2} \times .78 = 709.9$ pounds.

EXAMPLES FOR PRACTICE.

1. The distance from the axis of a screw to the point on the handle where the force is applied is 12 inches. The screw has 8 threads per inch and is 1 inch in diameter. (*a*) What force is necessary to raise a weight of 1,248 pounds, assuming that there is no friction? (*b*) What probable force will be required?

Ans. (*a*) 2.07 lb., nearly.
(*b*) 19 lb., about.

2. What weight can actually be raised with a screw jack having a screw 1¼ inches in diameter and 6 threads per inch, when a man is pulling with a force of 40 pounds at the end of a bar 16 inches long?

Ans. 2,838 lb., about.

3. In example 2, what is the velocity ratio? Ans. 603, nearly.

4. In example 2, what is the efficiency? Ans. 11¼ per cent.

MECHANICS OF FLUIDS.

HYDROSTATICS.

1. **Hydrostatics** treats of liquids at rest under the action of forces.

2. Liquids are very nearly *incompressible*. A pressure of 15 pounds per square inch compresses water less than $\frac{1}{20000}$ of its volume.

3. **Hydrostatic Pressure.**—Fig. 1 represents two cylindrical vessels of exactly the same size. The vessel *a* is fitted with a wooden block of the same size as, and free to move in, the cylinder; the vessel *b* is filled with water, whose depth is the same as the length of the wooden block in *a*. Both vessels are fitted with air-tight pistons *P*, each of whose areas are 10 square inches.

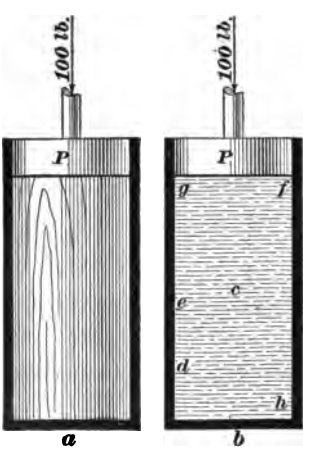

FIG. 1.

Suppose, for convenience, that the weights of the pistons, block, and water be neglected, and that a force of 100 pounds be applied to both pistons. The pressure per square inch will be $\frac{100}{10} = 10$ pounds. In the vessel *a* this pressure will be transmitted to the bottom of the vessel, and will be 10 pounds per square inch; it is easy to see that there will

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in $a = 6 \times 6 \times 24 = 864$ cubic inches; and the weight of the water is $864 \times .03617 = 31.25$ pounds. Hence, the total pressure on the bottom of the vessel a is 31.25 pounds, or $\frac{31.25}{6 \times 6} = .868$ pound per square inch.

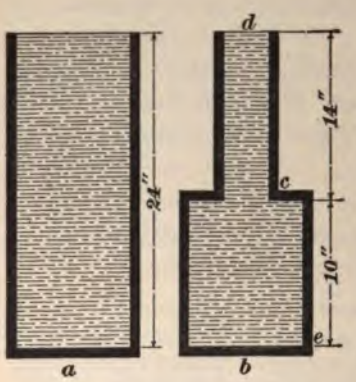

FIG. 3.

The pressure in b due to the weight contained in the part $b\,c$ is $6 \times 6 \times 10 \times .03617 = 13.02$ pounds.

The weight of the part contained in $c\,d$ is $2 \times 2 \times 14 \times .03617 = 2.0255$ pounds, and the weight per square inch of area in $c\,d$ is $\frac{2.0255}{4} = .5064$ pound.

8. According to Pascal's law, this weight (pressure) is transmitted equally in all directions, therefore, an extra weight of .5064 pound is imposed on every square inch of the bottom of $b\,c$; the area of this is $6 \times 6 = 36$ square inches, and the pressure on it due to the water contained in $c\,d$ is, therefore, $36 \times .5064 = 18.23$ pounds; thus, we have a total pressure on the bottom of vessel b of $13.02 + 18.23 = 31.25$ pounds, the same as in vessel a. As a result of the above law, there is also an upward pressure of .5064 pound acting on every square inch of the *top* of the enlarged part $b\,c$.

If an additional pressure of 10 pounds per square inch were applied to the upper surface of both vessels, the total pressure on each bottom would be $31.25 + (6 \times 6 \times 10) = 31.25 + 360 = 391.25$ pounds.

If this pressure were to be obtained by means of a weight placed on each piston (as shown in Fig. 1 at a and b), we would have to put a weight of $6 \times 6 \times 10 = 360$ pounds on the piston in vessel a, Fig. 3, and one of $2 \times 2 \times 10 = 40$ pounds on the piston in vessel b.

9. The general law for the downward pressure on the bottom of any vessel:

Rule 1.—*The pressure on the bottom of a vessel containing a fluid is independent of the shape of the vessel, and is equal to the weight of a column of the fluid, the area of whose base is equal to that of the bottom of the vessel and whose altitude is the distance between the bottom and the upper surface of the fluid, increased by the pressure per unit of area on the upper surface of the fluid multiplied by the area of the bottom of the vessel, in case there is any pressure on the surface.*

10. Suppose that the vessel *b*, in Fig. 3, were inverted, as shown in Fig. 4, the pressure on the bottom would still be .868 pound per square inch, but it would require a weight of 3,490 pounds on a piston at the upper surface to make the pressure on the bottom 391.25 pounds, instead of a weight of 40 pounds, as in the other case.

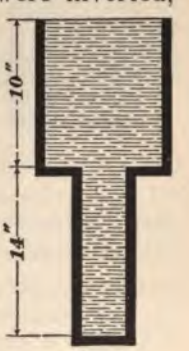

FIG. 4.

EXAMPLE.—A vessel filled with salt water, weighing .037254 pound per cubic inch, has a circular bottom 13 inches in diameter. The top of the vessel is fitted with a piston 3 inches in diameter, on which is laid a weight of 75 pounds; what is the total pressure on the bottom, if the depth of the water is 18 inches?

SOLUTION.—Applying the rule, we have

$13 \times 13 \times .7854 \times 18 \times .037254 = 89.01$ pounds, the pressure due to the weight of the water.

$\frac{75}{3 \times 3 \times .7854} = 10.61$ pounds per square inch, due to the weight on the piston.

$$13 \times 13 \times .7854 \times 10.61 = 1{,}408.29 \text{ pounds.}$$

$$1{,}408.29 + 89.01 = 1{,}497.3 \text{ lb.} = \text{total pressure.} \quad \text{Ans.}$$

11. Upward Pressure.—In Fig. 5 is represented a vessel of exactly the same size as that shown in Fig. 4. There is no upward pressure on the surface *c* due to the weight of the water in the large part *c d*, but there is an upward pressure on *c* due to the weight of the water in the

small part *b c*. The pressure per square inch due to the weight of the water in *b c* was found to be .5064 pound; the area of the upper surface *c* of the large part *c d* is $(6 \times 6) - (2 \times 2) = 36 - 4 = 32$ square inches, and the total upward pressure due to the weight of the water is $.5064 \times 32 = 16.2$ pounds.

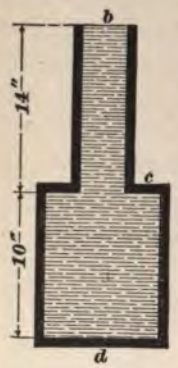

FIG. 5.

If an additional pressure of 10 pounds per square inch were applied to a piston fitting in the top of the vessel, the total upward pressure on the surface *c* would be $16.2 + (32 \times 10) = 336.2$ pounds.

12. General law for upward pressure:

Rule 2.—*The upward pressure on any submerged horizontal surface equals the weight of a column of the liquid whose base has an area equal to the area of the submerged surface and whose altitude is the distance between the submerged surface and the upper surface of the liquid, increased by the pressure per unit of area on the upper surface of the fluid multiplied by the area of the submerged surface, in case of any pressure on the upper surface.*

EXAMPLE.—A horizontal surface 6 inches by 4 inches is submerged in a vessel of water 26 inches below the upper surface; if the pressure on the water is 16 pounds per square inch, what is the total upward pressure on the horizontal surface?

SOLUTION.—Applying the rule, we get $6 \times 4 \times 26 \times .03617 = 22.57$ pounds for the upward pressure due to the weight of the water, and $6 \times 4 \times 16 = 384$ pounds for the upward pressure due to the outside pressure of 16 pounds per square inch.

Therefore, $384 + 22.57 = 406.57$ lb. = the total upward pressure.

Ans.

13. Lateral (Sidewise) Pressure.—Suppose the top of the vessel shown in Fig. 6 is 10 inches square and that the projections at *a* and *b* are 1 inch square.

The pressure per square inch on the bottom of the vessel due to the weight of the liquid is $1 \times 1 \times 18 \times$ the weight of a cubic inch of the liquid.

The pressure at a depth equal to the distance of the upper surface of *b* is $1 \times 1 \times 17 \times$ the weight of a cubic inch of the liquid.

Since both of these pressures are transmitted in every direction, they are also transmitted laterally (sidewise), and the *pressure per unit of area on the projection b is a mean between the two*, and equals $1 \times 1 \times \frac{17+18}{2}$ $\times$ the weight of a cubic inch of the liquid.

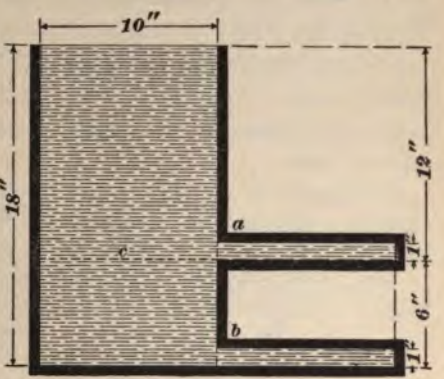

FIG. 6.

To find the lateral pressure on the projection *a*, imagine that the dotted line *c* is the bottom of the vessel; then, the conditions would be the same as in the preceding case, except that the depth is not so great.

The lateral pressure per square inch on *a* is thus seen to be $1 \times 1 \times \frac{11+12}{2} \times$ the weight of a cubic inch of the liquid.

14. General law for lateral pressure:

Rule 3.—*The pressure on any vertical surface due to the weight of the liquid is equal to the weight of a column of the liquid whose base has the same area as the vertical surface and whose altitude is the depth of the center of gravity of the vertical surface below the upper surface of the liquid.*

Any additional pressure is to be added, as in the previous cases.

EXAMPLE.—A well 3 feet in diameter and 20 feet deep is filled with water; what is the pressure on a strip of wall 1 inch wide, the top of which is 1 foot from the bottom of the well? What is the pressure on the bottom? What is the upward pressure per square inch 2 feet 6 inches from the bottom?

SOLUTION.—Applying the rule, the area of the strip is equal to its length (= circumference of well) multiplied by its height. The length $= 36 \times 3.1416 = 113.1$ inches; height = 1 inch; hence, area of

strip $= 113.1 \times 1 = 113.1$ square inches. Depth of center of gravity of strip $= (20 - 1)$ feet $+ \frac{1}{2}$ inch, the half width of strip $= 228\frac{1}{2}$ inches. Consequently, $113.1 \times 228.5 \times .03617 = 934.75$ lb. = the total pressure on the strip. Ans.

The pressure on each square inch of the strip is $\frac{934.75}{113.1} = 8.265$ lb., nearly. Ans.

$36 \times 36 \times .7854 \times 20 \times 12 \times .03617 = 8{,}836$ lb., the pressure on the bottom. Ans.

$20 - 2.5 = 17.5$. $1 \times 17.5 \times 12 \times .03617 = 7.596$ lb., the upward pressure per square inch 2 ft. 6 in. from the bottom. Ans.

15. The effects of lateral pressure are illustrated in Fig. 7. In the figure, e is a tall vessel having a stop-cock near its base and arranged to float upon the water, as shown. When this vessel is filled with water, the lateral pressures at any two points of the surface of the vessel opposite each other are equal. Being equal and acting in opposite directions, they destroy each other, and no motion can result; but, if the stop-cock is opened, there will be no resistance to the pressure acting on that part, and the water will flow out; at the same time, the reaction of the jet issuing from the stop-cock causes the vessel to move backwards through the water in a direction opposite to that of the issuing jet.

FIG. 7.

16. Liquids Influenced by Gravity.—Since the pressure on the bottom of a vessel due to the weight of the liquid is dependent only on the height of the liquid and not on the shape of the vessel, it follows that if a vessel has a number of radiating tubes (see Fig. 8) the water in each

tube will be on the same level, no matter what may be the shape of the tubes. For, if the water were higher in one tube than in the others, the downward pressure on the bottom due to the height of the water in this tube would be greater than that due to the height of the water in the other tubes. Consequently, the upward pressure would also be greater; the equilibrium would be destroyed and the water

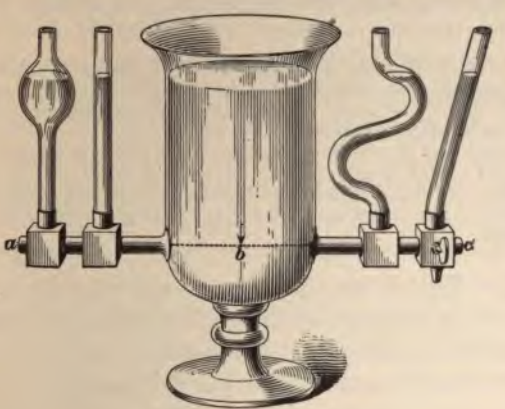

FIG. 8.

would flow from this tube into the vessel and rise in the other tubes until it was at the same level in all, when it would be in equilibrium. This principle is expressed in the familiar saying, *water seeks its level.*

An application of this principle is the glass water gauge used for showing the level of the water in a steam boiler or tank.

17. The above principle explains why city water reservoirs are located on high elevations and why water on leaving the hose nozzle spouts so high.

If there were no resistance by friction and air, the water would spout to a height equal to the level of the water in the reservoirs. If a long, vertical pipe, whose length was equal to the vertical distance between the nozzle and the level of the water in the reservoir, were attached to the nozzle, the

water would just reach the end of the pipe. If the pipe were lowered slightly, the water would trickle out.

Fountains, canal locks, and artesian wells are examples of the application of this principle.

EXAMPLE.—The water level in a city reservoir is 150 feet above the level of the street; what is the pressure of the water per square inch on the hydrant?

SOLUTION.—Applying rule **1**, $1 \times 150 \times 12 \times .03617 = 65.106$ lb. per sq. in. Ans.

NOTE.—In measuring the height of the water to find the pressure that it produces, the **vertical** height, or distance, between the level of the water and the point considered is always taken. This vertical height is called the **head.**

The weight of a column of water 1 inch square and 1 foot high is $62.5 \div 144 = .434$ pound, nearly. Hence, if the depth (head) be given, the pressure per square inch may be found by multiplying the depth in feet by .434. The constant .434 is the one ordinarily used in practical calculations.

18. In Fig. 9, let the area of the piston *a* be 1 square inch; of *b*, 40 square inches. According to Pascal's law, 1 pound placed on *a* will balance 40 pounds placed on *b*.

Suppose that *a* moves downwards 10 inches, then 10 cubic inches of water will be forced into the tube *b*. This will be distributed over the entire area of the tube *b*, in the form of a cylinder whose cubical contents must be 10 cubic inches, whose base has an area of 40 square inches, and whose altitude must be $\frac{10}{40} = \frac{1}{4}$ inch; that is, a movement of 10 inches of the piston *a* will cause a movement of $\frac{1}{4}$ inch in the piston *b*. This is another illustration of the well-known principle of machines: *The force multiplied by the distance through which it acts equals the weight multiplied by the distance through which it moves*, since, if 1 pound on the piston *a* represents the force *P*, the equivalent weight *W* on *b* may be obtained from the equation $W \times \frac{1}{4} = P \times 10$, whence $W = 40\,P = 40$ pounds.

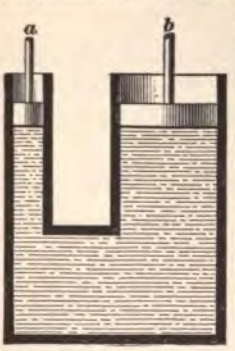

FIG. 9.

Another familiar fact is also recognized, for the velocity

ratio of P to W is $10 : \frac{1}{4}$, or 40; and since in any machine the weight equals the force multiplied by the velocity ratio, $W = P \times 40$, and when $P = 1$, $W = 40$. An interesting application of this principle is the hydraulic jack, by the aid of which one man can lift a very large load.

19. A **Watson-Stillman hydraulic jack** is shown in section in Fig. 10 (*a*). In this illustration, *a* is a lever that is depressed by the operator when he desires to raise the load. This lever freely fits a rectangular hole or socket in a shaft *b* that extends clear through the ram head. The shaft *b* carries the crank *c* to which the piston *d* is hinged. A small valve is placed in the center of the piston and serves to open or close communication between the water space above and below the piston. The lower end of the ram is fitted with a small check valve, which normally is held closed by the small spring shown beneath the valve. The inside of the jack is filled with a mixture of alcohol and water, about 2 parts of alcohol to 3 parts of water, which prevents the freezing of the liquid when the jack is used in cold weather. The operation is as follows:

The jack being clear down, as shown in Fig. 10 (*a*), it is placed under the load and the operator alternately depresses and raises the lever through its full range, the motion being limited by a projection on the under side of the lever. The slightest depression of the lever closes the valve in the piston. The downward movement of the piston continuing, the water between the bottom of the piston and the end of the ram is subjected to a pressure that opens the valve in the end of the ram. This allows the water under pressure to pass into the space beneath the ram. The pressure so transmitted to the lower end of the ram forces it upwards. The lever having been depressed to its limit is again raised. This raises the piston, causes the piston valve to open, and allows the water above the piston to flow beneath it. The pressure of the water below the ram promptly closes the ram valve as soon as the piston commences to move upwards. The operation may now be repeated as often as required.

As will readily be seen, the ram cannot descend under the weight of the load, as the only water passage leading from

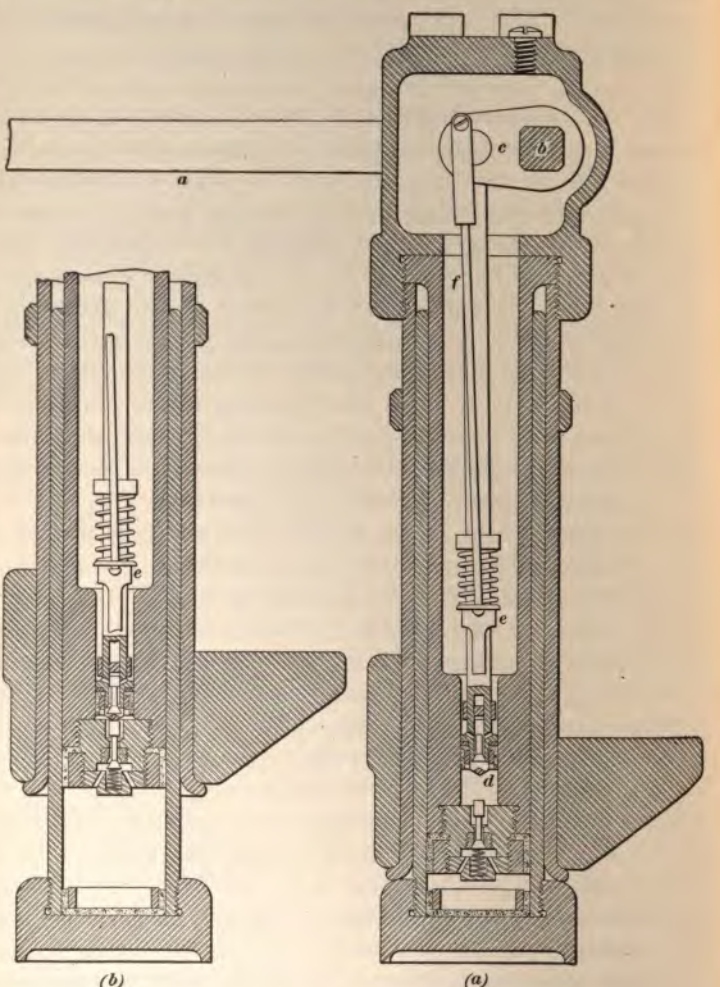

FIG. 10.

the space below the ram is closed by the ram valve, which is kept tightly closed by the pressure of the water beneath

it and also by the spring. Hence, in order to lower the ram, this ram valve must be opened by some means. As previously mentioned, the lever has a projection on its under side to limit its travel and the travel of the piston. This projection is made of such a length that when the lever is down as far as it will go, the piston is still a short distance from the upper end of the ram valve stem.

In order to lower the ram, the lever is withdrawn from its socket and inserted again with the projection upwards. It can now be depressed far enough to cause the end of the piston to strike and open the ram-head valve, as shown in Fig. 10 (*b*), where the parts of the jack are shown in the relative positions occupied while lowering. But before the water in the bottom of the ram can escape to the top, the valve in the piston must also be opened. This is accomplished by a heavy spring that forces the sleeve *e* downwards. The lower end of the sleeve has a cotter working in a slot in the piston rod; the cotter bearing against the top of the valve forces the latter downwards. The sleeve is connected to the crank *c* by the so-called lowering wire *f*, in such a manner that the cotter will not strike the piston valve while the jack is raising the load. The ram will descend as long as the lever is kept hard down; it can be stopped instantly by raising the lever slightly.

20. If the area of the piston is 1 square inch and the area of the ram is 10 square inches, the velocity ratio will be 10. If the length of the lever between the hand of the operator and the fulcrum is 10 times the length between the fulcrum and the piston, the velocity ratio of the lever will be 10, and the total velocity ratio of the hand to the piston will be $10 \times 10 = 100$. Taking the weight of the operator at 150 pounds, his whole weight to be thrown on the lever, the weight that can be raised is $150 \times 100 = 15{,}000$ pounds, or $7\frac{1}{2}$ tons. But if the average movement of the hand is 4 inches per stroke, it will require $\frac{100}{4} = 25$ strokes to raise the load on the jack a distance of 1 inch, and it is again seen that what is gained in force is lost in speed.

21. Other applications of this principle are seen in various hydraulic machines used in boiler shops. A familiar example is the hand test pump used in testing boilers under hydraulic pressure.

EXAMPLE 1.—A vertical cylinder is tested for the tightness of its heads by filling it with water. A pipe whose inside diameter is $\frac{1}{4}$ inch and whose length is 20 feet is screwed into a hole in the upper head and is then filled with water. What is the pressure per square inch on each head if the cylinder is 40 inches in diameter and 60 inches long?

SOLUTION.— $40^2 \times .7854 = 1{,}256.64$ square inches = area of head.

$1 \times 60 \times .03617 = 2.17$ pounds pressure per square inch on the bottom head due to the weight of the water in the cylinder.

$(\frac{1}{4})^2 \times .7854 = .04909$ square inch, the area of the pipe.

$.04909 \times 20 \times 12 \times .03617 = .426$ pound = the weight of water in the pipe = the pressure on a surface area of .04909 square inch.

The pressure per square inch due to the water in the pipe is $\frac{1}{.04909} \times .426 = 8.68$ lb. per sq. in. upon the upper head. Ans.

The total pressure per square inch on the lower head is $8.68 + 2.17 = 10.85$ lb. Ans.

EXAMPLE 2.—In the other example, if the pipe be fitted with a piston weighing $\frac{1}{4}$ pound and a 5-pound weight be laid on it what is the pressure per square inch on the upper head?

SOLUTION.—In addition to the pressure of .426 pound on the area of .04909 square inch, there is now an additional pressure on this area of $5 + \frac{1}{4} = 5.25$ pounds, and the total pressure on this area is $.426 + 5.25 = 5.676$ pounds. $\frac{1}{.04909} \times 5.676 = 115.6$ lb. = the pressure per square inch. Ans.

22. When calculating the weight that can be raised with a hydraulic jack, no allowance was made for the power lost in overcoming the friction between the cup leathers of the piston and the ram, and the cup leather of the ram and the cylinder; this varies according to the condition of the leathers, and, of course, the smoothness of the ram and cylinder; when the leathers are in good condition, the loss is about 5 per cent. of the total pressure on the ram; when the leathers are old, stiff, and dirty, the loss may amount to 15 per cent. or more.

SPECIFIC GRAVITY.

23. The **specific gravity of a body** is the ratio between its weight and the weight of a like volume of water.

24. Since gases are so much lighter than water, it is usual to take the specific gravity of a gas as the ratio between the weight of a certain volume of the gas and the weight of the same volume of air.

EXAMPLE.—A cubic foot of cast iron weighs 450 pounds; what is its specific gravity, a cubic foot of water weighing 62.42 pounds?

SOLUTION.—According to the definition,

$$\frac{450}{62.42} = 7.21. \quad \text{Ans.}$$

TABLE I.

SPECIFIC GRAVITY AND WEIGHT PER CUBIC FOOT OF VARIOUS METALS.

Substance.	Specific Gravity.	Weight per Cubic Foot. Pounds.
Platinum	21.50	1,343.8
Gold	19.50	1,218.8
Mercury	13.60	850.0
Lead (cast)	11.35	709.4
Silver	10.50	656.3
Copper (cast)	8.79	549.4
Brass	8.38	523.8
Wrought Iron	7.68	480.0
Cast Iron	7.21	450.0
Steel	7.84	490.0
Tin (cast)	7.29	455.6
Zinc (cast)	6.86	428 8
Antimony	6.71	419.4
Aluminum	2.50	156.3

TABLE II.

SPECIFIC GRAVITY AND WEIGHT PER CUBIC FOOT OF VARIOUS WOODS.

Substance.	Specific Gravity.	Weight per Cubic Foot. Pounds.
Ash	.845	52.80
Beech	.852	53.25
Cedar	.561	35.06
Cork	.240	15.00
Ebony (American)	1.331	83.19
Lignum-vitæ	1.333	83.30
Maple	.750	46.88
Oak (old)	1.170	73.10
Spruce	.500	31.25
Pine (yellow)	.660	41.20
Pine (white)	.554	34.60
Walnut	.671	41.90

TABLE III.

SPECIFIC GRAVITY AND WEIGHT PER CUBIC FOOT OF VARIOUS LIQUIDS.

Substance.	Specific Gravity.	Weight per Cubic Foot. Pounds.
Acetic acid	1.062	66.4
Nitric acid	1.217	76.1
Sulphuric acid	1.841	115.1
Muriatic acid	1.200	75.0
Alcohol	.800	50.0
Turpentine	.870	54.4
Sea water (ordinary)	1.026	64.1
Milk	1.032	64.5

25. The specific gravities of different bodies are given in printed tables; hence, if it is desired to know the weight of a body that cannot be conveniently weighed, *calculate its cubical contents and multiply the specific gravity of the body by the weight of a like volume of water, remembering that a cubic foot of water weighs 62.42 pounds.*

EXAMPLE 1.—How much will 3,214 cubic inches of cast iron weigh? Take its specific gravity as 7.21.

SOLUTION.—Since 1 cubic foot of water weighs 62.42 pounds, 3,214 cubic inches weigh

$$\frac{3,214}{1,728} \times 62.42 = 116.098 \text{ pounds.}$$

Then, $116.098 \times 7.21 = 837.067$ lb. Ans.

TABLE IV.

SPECIFIC GRAVITY AND WEIGHT PER CUBIC FOOT OF VARIOUS GASES AT 32° F. AND UNDER A PRESSURE OF 1 ATMOSPHERE.

Substance.	Specific Gravity.	Weight per Cubic Foot. Pounds.
Atmospheric air	1.0000	.08073
Carbonic acid	1.5290	.12344
Carbonic oxide	.9674	.07810
Chlorine	2.4400	.19700
Oxygen	1.1056	.08925
Nitrogen	.9736	.07860
Smoke (bituminous coal)	.1020	.00815
Smoke (wood)	.0900	.00727
*Steam at 212° F	.4700	.03790
Hydrogen	.0692	.00559

*The specific gravity of steam at any temperature and pressure compared with air at the same temperature and pressure is .622.

EXAMPLE 2.—What is the weight of a cubic inch of cast iron?

SOLUTION.— $\frac{62.42}{1,728} \times 7.21 = .26044$ lb. Ans.

One cubic foot of pure distilled water at a temperature of 39.2° Fahrenheit weighs 62.425 pounds, but *the value usually taken in making calculations is 62.5 pounds.*

EXAMPLE 3.—What is the weight in pounds of 7 cubic feet of oxygen?

SOLUTION.—One cubic foot of air weighs .08073 pound, and the specific gravity of oxygen is 1.1056, compared with air; hence,

$$.08073 \times 1.1056 \times 7 = .62479 \text{ lb., nearly. Ans.}$$

TABLE V.

SPECIFIC GRAVITY AND WEIGHT PER CUBIC FOOT OF VARIOUS SUBSTANCES.

Substance.	Specific Gravity.	Weight per Cubic Foot. Pounds.
Emery	4.00	250
Glass (average)	2.80	175
Chalk	2.78	174
Granite	2.65	166
Marble	2.70	169
Stone (common)	2.52	158
Salt (common)	2.13	133
Soil (common)	1.98	124
Clay	1.93	121
Brick	1.90	118
Plaster of Paris (average)	2.00	125
Sand	1.80	113

BUOYANT EFFECTS OF WATER.

26. In Fig. 11 is shown a 6-inch cube entirely submerged in water. The lateral pressures are equal and in opposite directions. The upward pressure acting on the lower surface of the cube is $6 \times 6 \times 21 \times .03617$; the downward pressure acting on the top of the cube is $6 \times 6 \times 15 \times .03617$; and the difference is $6 \times 6 \times 6 \times .03617$, which equals the volume of the cube in cubic inches times the weight of 1 cubic inch of water. That is, the upward pressure exceeds the downward pressure by the weight of a volume of water equal to the volume of the body.

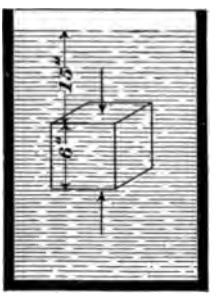

FIG. 11.

This excess of upward pressure over the downward pressure acts against gravity; that is, the water presses the body *upwards* with a greater force than it presses it *downwards;* consequently, *if a body is immersed in a fluid, it will lose in weight an amount equal to the weight of the fluid it displaces.* This is called the **principle of Archimedes,** because it was first stated by him.

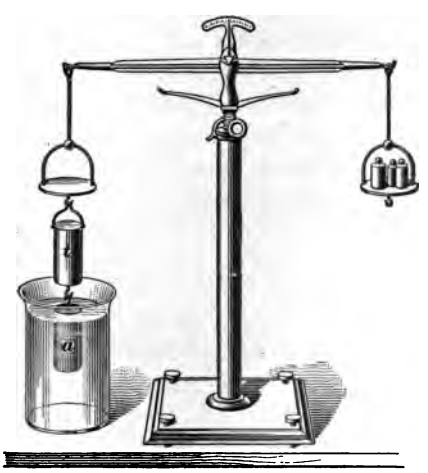

FIG. 12.

27. This principle may be experimentally demonstrated with the beam scales, as shown in Fig. 12. From one scale pan suspend a hollow cylinder of metal *t*, and below that a solid cylinder *a* of the same size as the hollow part of the upper cylinder. Put weights in the other scale pan until they exactly balance the two cylinders. If *a* be immersed in water, the scale pan

containing the weights will descend, showing that *a* has lost some of its weight. Now fill *t* with water, and the volume of water that can be poured into *t* will equal that displaced by *a*. The scale pan that contains the weights will gradually rise until *t* is filled, when the scales again balance.

If a body be lighter than the liquid in which it is immersed, the upward pressure will cause it to rise and project partly out of the liquid, until the weight of the body and the weight of the liquid displaced are equal. If the immersed body be heavier than the liquid, the downward pressure plus the weight of the body will be greater than the upward pressure, and the body will fall downwards until it touches bottom or meets an obstruction. If the weights of equal volumes of the liquid and the body are equal, the body will remain stationary and will be in equilibrium in any position or depth beneath the surface of the liquid.

28. An interesting experiment in confirmation of the above facts may be performed as follows: Drop an egg into a glass jar filled with fresh water. The mean density of the egg being a little greater than that of the water, it will fall to the bottom of the jar. Now, dissolve salt in the water, stirring it so as to mix the fresh and salt water. The salt water will presently become denser than the egg and the egg will rise. Now, if fresh water be poured in until the egg and water have the same density, the egg will remain stationary in any position that it may be placed below the surface of the water.

29. The principle of Archimedes gives a very easy and accurate method of finding the volume of an irregularly shaped body. Thus, subtract its weight in water from its weight in air and divide by .03617; the quotient will be the volume in cubic inches, or divide by 62.5 and the quotient will be the volume in cubic feet.

If the specific gravity of the body is known, its cubical contents can be found by dividing its weight by its specific gravity, and then dividing again by either .03617 or 62.5.

EXAMPLE.—A certain body has a specific gravity of 4.38 and weighs 76 pounds; how many cubic inches are there in the body?

SOLUTION.— $\frac{76}{4.38 \times .03617} = 479.72$ cu. in. Ans.

THE HYDROMETER.

30. Instruments called **hydrometers** are in general use for determining quickly and accurately the specific gravities of liquids and some forms of solids. They are of two kinds, viz.: (1) *Hydrometers of constant weight, as* **Beaume's;** (2) *hydrometers of constant volume, as* **Nicholson's.**

A **hydrometer of constant weight** is shown in Fig. 13. It consists of a glass tube, near the bottom of which are two bulbs. The lower and smaller bulb is loaded with mercury or shot, so as to cause the instrument to remain in a vertical position when placed in the liquid. The upper bulb is filled with air, and its volume is such that the whole instrument is lighter than an equal volume of water.

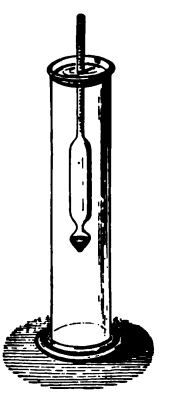

FIG. 13.

The point to which the hydrometer sinks when placed in water is usually marked, the tube being graduated above and below in such a manner that the specific gravity of the liquid can be read directly. It is customary to have two instruments, one with the zero point near the top of the stem for use in liquids heavier than water, and the other with the zero point near the bulb for use in liquids lighter than water.

These instruments are more commonly used for determining the degree of concentration or dilution of certain liquids, as acids, alcohol, milk, solutions of sugar, etc., rather than their actual specific gravities. They are then known as *acidometers, alcoholometers, lactometers, saccharometers, salinometers*, etc., according to the use to which they are put.

31. **Hydrometers of constant volume** are not in common use and are rarely found outside of laboratories.

HYDROKINETICS.

FLOW OF WATER IN PIPES.

32. Experience has demonstrated that for satisfactory work, the flow of water in the suction pipes of boiler feed-pumps and other comparatively small pumps should not exceed 200 feet per minute, and it should not be more than 500 feet in the delivery pipe for a duplex pump, or 400 feet for a single-cylinder pump.

Knowing the volume of water that is to flow through or to be discharged from a pipe in 1 minute, the area of the suction and delivery pipes can readily be determined.

33. The volume of water in cubic feet discharged from a pipe in 1 minute is equal to the velocity in feet per minute times the area of the pipe in square feet. Then, the area of the pipe equals $\frac{\text{volume in cubic feet per minute}}{\text{velocity in feet per minute}}$. As there are 144 square inches in a square foot, the area of the pipe in square inches is $\frac{144 \times \text{volume in cubic feet per minute}}{\text{velocity in feet per minute}}$.

34. If the volume is expressed in gallons per minute, then, as there are 7.48 gallons in a cubic foot, the area of the pipe in square inches will be $\frac{144 \times \text{volume in gallons*}}{7.48 \times \text{velocity in feet per minute}}$. Hence, the following rule,

where n = gallons per minute;
v = velocity in feet per minute;
A = area of pipe in square inches.

Rule 4.—*To find the area of a pipe in square inches, divide 19.25 times the number of gallons per minute by the velocity in feet per minute.*

Or, $$A = \frac{19.25\, n}{v}.$$

* The gallon here referred to is the Winchester or wine gallon of 231 cubic inches capacity and in common use in the United States of America.

EXAMPLE.—If a duplex pump is used, what area of feed-pipe is required for a boiler into which 25 gallons of water per minute is to be pumped?

SOLUTION.—The allowable velocity being 500 feet, by applying rule 4, we get

$$A = \frac{19.25 \times 25}{500} = .9625 \text{ sq. in.} \quad \text{Ans.}$$

35. The quantity of water, expressed in gallons, that will flow through a given pipe in 1 minute, its velocity being known, is given by the following rule:

Rule 5.—*Multiply the area of the pipe in square inches by the velocity in feet per minute. Divide the product by 19.25.*

Or,
$$n = \frac{A\,v}{19.25}.$$

EXAMPLE.—How many gallons of water per minute will flow through a pipe having an area of 2 square inches, the velocity of flow being 450 feet per minute?

SOLUTION.—Applying the rule just given, we get

$$n = \frac{2 \times 450}{19.25} = 46.75 \text{ gal.} \quad \text{Ans.}$$

36. The velocity with which water will flow through the delivery pipe of a pump when the area of the water cylinder, the area of the delivery pipe, and the piston speed of the pump are known, is given by the following rule,

where v = velocity in feet per minute;
A = area of delivery pipe in square inches;
a = area of water piston in square inches;
S = piston speed in feet per minute.

Rule 6.—*Multiply the area of the water piston by the piston speed, and divide this product by the area of the delivery pipe.*

Or,
$$v = \frac{a\,S}{A}.$$

EXAMPLE.—If the water piston of a pump has an area of 12 square inches and moves at a speed of 100 feet per minute, what will be the

velocity of the water in the delivery pipe if the latter has an area of 2 square inches?

SOLUTION.—Applying rule **6**, we get

$$v = \frac{12 \times 100}{2} = 600 \text{ ft. per min.} \quad \text{Ans.}$$

STANDARD PIPE DIMENSIONS.

37. The great majority of the pipes used about steam plants are made of wrought iron and are almost invariably made in accordance with the Briggs standard. It will be noticed that the diameter of the pipe by which it is known to the trade is not the *actual* diameter; hence, in calculating the amount of water that will flow through a pipe, the actual diameter or actual internal area must be taken from Table VI.

38. As wrought-iron pipes are not made in sizes differing from those given in the table, it will be apparent that only in rare instances can a pipe be selected that will have the area calculated by rule **4**. In practice the nearest commercial size of pipe would be selected.

EXAMPLE.—What commercial size of delivery pipe should be used for a single-cylinder pump to deliver 90 gallons of water per minute?

SOLUTION.—For a single-cylinder pump, the velocity of flow should not be more than 400 feet per minute. Then, applying rule **4**, we get

$$A = \frac{19.25 \times 90}{400} = 4.33 \text{ sq. in.}$$

According to Table VI, the commercial size of pipe having an area nearest this is 2½ inches; therefore, a 2½-inch pipe should be used. Ans.

Rule **4** will be found to agree quite closely with the practice of the leading pump manufacturers. In case they should, however, recommend a larger size of pipe, it is advisable to follow their advice.

Pipes made in accordance with the Briggs standard, when below 1 inch nominal size, are butt-welded and proved to 300 pounds per square inch by hydraulic pressure. Pipes above 1 inch are lap-welded and proved to 500 pounds.

TABLE VI.

TABLE OF STANDARD DIMENSIONS OF WROUGHT-IRON WELDED PIPES.

Nominal Diameter. Inches.	Actual Internal Diameter. Inches.	Actual Internal Area. Square Inches.	Actual External Diameter. Inches.	Number of Threads Per Inch.
⅛	.27	.057	.40	27
¼	.36	.104	.54	18
⅜	.49	.192	.67	18
½	.62	.305	.84	14
¾	.82	.533	1.05	14
1	1.05	.863	1.31	11½
1¼	1.38	1.496	1.66	11½
1½	1.61	2.038	1.90	11½
2	2.07	3.355	2.37	11½
2½	2.47	4.783	2.87	8
3	3.07	7.388	3.50	8
3½	3.55	9.887	4.00	8
4	4.07	12.730	4.50	8
4½	4.51	15.939	5.00	8
5	5.04	19 990	5.56	8
6	6.06	28.889	6.62	8
7	7.02	38.737	7.62	8
8	7.98	50.039	8.62	8
9	9.00	63.633	9.62	8
10	10.02	78.838	10.75	8

PIPE FITTINGS.

39. In piping a steam plant, it is rarely possible to run the piping in a straight line, it being usually necessary to introduce one or more elbows or similar fittings to reach the point desired. The effect of T and L fittings is to increase the resistance to the flow of the water through the pipes,

thus requiring the pump to do more work for the same quantity of water delivered.

As pumps for boiler feeding are always built with a large steam cylinder, there is enough excess of power to allow the pump under ordinary conditions to force the required quantity of water through the pipe. On the suction side of the pump, however, the force impelling the water to flow into the pump is quite small, and a very slight resistance will be sufficient to interfere with the flow of the water into the pump. Hence, it is important that the suction pipe be as straight as possible; if it is impossible to make it straight, larger sizes of pipe should be used, or easy bends with as large a radius as possible should be substituted for the elbows. This applies especially to pumps that must lift the water more than 10 feet. It is impossible to lay down any hard-and-fast rules as to what numbers of elbows should not be exceeded in a suction and delivery pipe; judgment will have to be used. Generally speaking, they should be as few as possible.

EXAMPLES FOR PRACTICE.

1. Suppose a cylinder to be filled with water and placed in an upright position. If the diameter of the cylinder is 19 inches and its total length inside 26 inches, what will be the total pressure on the bottom when a pipe $\frac{1}{2}$ inch in diameter and 12 feet long is screwed into the cylinder head and filled with water? The pipe is vertical.

Ans. 1,743.2 lb.

2. In the last example, what is the total pressure against the upper head? Ans. 1,476.6 lb.

3. In example 1, a piston is fitted to the upper end of the pipe and an additional force of 10 pounds is applied to the water in the pipe. What is the total pressure (*a*) on the bottom of the cylinder? (*b*) on the upper head?

Ans. { (*a*) 16,184 lb
(*b*) 15,906 lb

4. In example 3, what is the pressure per square inch in the pi 2 inches from the upper cylinder head? Ans. 56.0656 lb. per sq.

5. A water tower 80 feet high is filled with water. A pipe 4 inc in diameter is so connected to the side of the tower that its cente 3 feet from the bottom. If the pipe is closed by a flat cover, wha the total pressure against the cover? Ans. 42

6. In the last example, what is the upward pressure per square inch 10 feet from the bottom of the tower? Ans. 30.3828 lb. per sq. in.

7. A cube of wood, one edge of which measures 3 feet, is sunk until the upper surface is 40 feet below the level of the water; what is the total force that tends to move the cube upwards? Ans. 1,687.5 lb.

8. A body weighs 72 pounds when immersed in water and 321 pounds in air; what is its volume in cubic feet? Ans. 3.984 cu. ft.

9. The specific gravity of a body being 2.5 and its weight 81 pounds, what is its volume in cubic inches? Ans. 235.73 cu. in.

10. What commercial size of pipe should be used for the suction and delivery pipes of a single-cylinder pump to deliver 50 gallons of water per minute? Ans. { Suction pipe, 2½ in. / Delivery pipe, 2 in.

11. How many gallons per hour, at a velocity of 400 feet per minute, will flow through a 1-inch pipe? Ans. 1,076 gal., nearly.

12. The water piston of a pump is 3½ inches; the piston speed is 54 feet per minute; and a 1¼-inch delivery pipe is used; what is the velocity in the delivery pipe? Ans. 255 ft. per min., nearly.

PNEUMATICS.

PRESSURE OF GASES.

40. Pneumatics is that branch of mechanics that treats of the properties and pressures of gases.

41. The most striking feature of all gases is their great expansibility. *If we inject a quantity of gas, however small, into a vessel, it will expand and fill that vessel.* If a bladder or football be partly filled with air and placed under a glass jar (called a receiver), from which the air has been exhausted, the bladder or football will immediately expand, as shown in Fig. 14. The force that a gas always exerts when confined in a limited

FIG. 14.

space is called *tension*. The word **tension** in this case means pressure, and is only used in this sense in reference to gases.

42. As *water* is the most common type of fluids, so *air* is the most common type of gases. It was supposed by the ancients that air had no weight, and it was not until about the year 1650 that the contrary was proved. A cubic inch of air, under ordinary conditions, weighs .31 grain, nearly. At a temperature of 32° F. and a pressure of 14.7 pounds per square inch, the ratio of the weight of air to water is about 1 : 774; that is, air is only $\frac{1}{774}$ as heavy as water. It has been shown that if a body were immersed in water and weighed less than the volume of water displaced, the body would rise and project partly out of the water. The same is true, to a certain extent, of air. If a vessel made of light material is filled with a gas lighter than air, so that the total weight of the vessel and gas is less than the air they displace, the vessel will rise. It is on this principle that balloons are made.

PRESSURE OF THE ATMOSPHERE.

43. Since air has weight, it is evident that the enormous quantity of air that constitutes the atmosphere must exert a considerable pressure on the earth. This is easily proved by taking a long glass tube closed at one end and filling it with mercury. If the finger be placed over the open end so as to keep the mercury from running out and the tube inverted and placed in a cup of mercury, as shown in Fig. 15, the mercury will fall, then rise, and after a few oscillations will come to rest at a height above the top of the mercury in the cup equal to about 30 inches at sea level.

This height will always be the same under the same atmospheric conditions. Now, if the atmosphere has weight, it must press upon the upper surface of the mercury in the cup with equal intensity upon every square unit, except upon that part of the surface occupied by the tube. In order that there may be equilibrium, the weight of the mercury in the tube must be equal to the pressure of the air

upon a portion of the surface of the mercury in the cup equal in area to the inside of the tube. Suppose that the area of the inside of the tube is 1 square inch, then, since mercury is 13.6 times as heavy as water, the weight of the mercurial column is $.03617 \times 13.6 \times 30 = 14.7574$ pounds. The actual height of the mercury is a little less than 30 inches, and the actual weight of a cubic inch of distilled water is a little less than .03617 pound. When these considerations are taken into account, the average weight of the mercurial column at the level of the sea, when the temperature is 60° F., is 14.69 pounds, or practically 14.7 pounds. Since this weight, when exerted upon 1 square inch of the liquid in the glass, just produces equilibrium, it is plain that the pressure of the outside air is 14.7 pounds upon every square inch of surface.

FIG. 15.

44. Vacuum.—The space between the upper end of the tube and the upper surface of the mercury is called a **vacuum,** meaning that it is an entirely empty space and does not contain any substance—solid, liquid, or gaseous. If there were a gas of some kind there, no matter how small the quantity might be, it would expand and fill the space, and its tension would cause the column of mercury to fall and become shorter, according to the amount of gas or air present. The condition then existing in the space would be called a **partial vacuum.**

45. The Measurement of Vacuum.—The degree to which the air has been exhausted from a closed vessel in

which there is a partial vacuum is measured by the height to which a mercurial column in a vertical tube, whose top is connected to the vessel, will rise under the pressure of the atmosphere. Thus, when we say that there is a vacuum of 29 inches in a vessel, we mean that enough air has been exhausted from the vessel to enable the surrounding air to support a mercurial column 29 inches high, as shown in Fig. 16, where *A* is the vessel in which there is a partial vacuum. In other words, the pressure in the vessel is less than the pressure of the atmosphere by a column of mercury 29 inches in height.

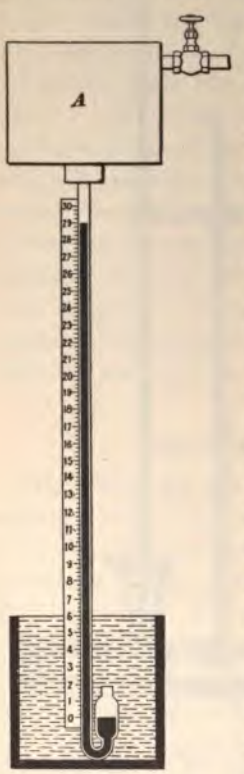

FIG. 16.

When there is a partial vacuum inside a closed vessel, the air remaining in the vessel is under a tension, or pressure, less than its original tension of 14.7 pounds per square inch. This tension, in inches of mercury, is equal to the difference between the height at which the mercurial column connected to the vessel stands and that at which it would stand if the vacuum were perfect. Consider that the mercury column will be in equilibrium when the hydrostatic pressure on its base equals the atmospheric pressure. The hydrostatic pressure on the base is the sum of two pressures: (1) The pressure due to the weight of the mercury column; (2) the pressure in the space above the mercury.

From this it follows that if the atmospheric pressure and the pressure due to the weight of the mercurial column are given, the pressure above the column must be that due to their difference. As the atmosphere will force a mercurial column to a height of 30 inches when there is a perfect vacuum above it, it follows that to find the pressure in a vessel

in which there is a partial vacuum, the number of inches of height of the mercurial column is to be subtracted from 30. Thus, if the vacuum in a vessel is 26 inches, the pressure in the vessel is 30 — 26 = 4 inches of mercury.

46. In practice it is convenient to always use the same unit of pressure, which is 1 pound per square inch. We know that 14.7 pounds per square inch will support a mercurial column 30 inches high. Hence, 1 inch of height of a mercurial column represents a pressure of $\frac{14.7}{30} = .49$ pound per square inch. Then, to reduce inches of mercury to pounds per square inch, multiply their number by .49.

EXAMPLE.—A gauge shows a vacuum of 22 inches in a condenser. What is the absolute pressure in the condenser?

SOLUTION.—The pressure, in inches of mercury, is 30 — 22 = 8 inches, or 8 × .49 = 3.92 lb. per sq. in. Ans.

47. The Vacuum Gauge.—For engineering work, the glass tube of Fig. 16 would be a rather inconvenient form of gauge for measuring the vacuum. Hence, special metallic gauges, known as **vacuum gauges,** have been designed. Their dial is graduated to show the degree of vacuum, in inches of mercury. In steam-engineering work, they are used chiefly in connection with condensers for steam engines. It should be borne in mind that a vacuum gauge does not indicate the pressure in the vessel to which it is attached, but instead shows, in inches of mercury, how much the pressure has been *lowered below the atmospheric pressure.* In this respect a vacuum gauge differs essentially from a **pressure gauge,** which shows how much the pressure has been *increased* either above the pressure of a perfect vacuum, which is zero, as in case of a gauge registering the pressure of the atmosphere, or above the atmospheric pressure, as in case of the ordinary pressure gauge.

48. If the tube of Fig. 15 had been filled with a liquid lighter than mercury, the height of the column required to balance the atmospheric pressure would be greater. The

height of the column depends on the specific gravity of the liquid. Thus, if the liquid had a specific gravity of 1, as water, its height would be $\frac{30 \times 13.6}{1} = 408$ inches, or 34 feet. This means that if a tube be filled with water, inverted, and placed in a dish of water, in a manner similar to the experiment made with the mercury, the height of the column of water will be 34 feet.

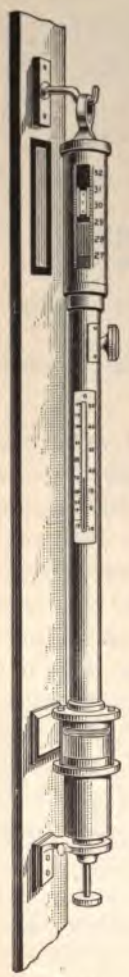

FIG. 17.

49. The Barometer.—As is well known, the atmosphere does not exert exactly the same pressure at all times; the pressure varies with conditions. As the height of the mercury in the glass tube of Fig. 15 depends chiefly on the atmospheric pressure, it is evident that an instrument constructed on this principle will indicate the varying pressure by the height of the column. Such an instrument is called a **mercurial barometer.**

50. A standard form of barometer is shown in Fig. 17. The barometer is simply a pressure gauge that registers the pressure of the air. In this case the cup and tube at the bottom are protected by a brass or iron casing. At the top of the tube is a graduated scale. Attached to the casing is an accurate thermometer for determining the temperature of the outside air at the time the barometric observation is taken. This is necessary, since mercury expands when the temperature is increased and contracts when the temperature falls; for this reason a standard temperature is assumed, and all barometer readings are reduced to this temperature. This standard temperature is usually taken at 32° F., at which temperature the height of the mercurial column is 30 inches under normal conditions at sea level. Another

correction is made for the altitude of the place above sea level, and a third correction for the effects of capillary attraction.

51. A mercurial barometer is not only a very bulky instrument, but is also quite heavy and, hence, is not very well adapted for transportation. To overcome these drawbacks, a form of portable barometer known as an **aneroid barometer,** has been designed, which operates on a somewhat different principle. Such a barometer is shown in

FIG. 18.

Fig. 18. The principal part of the aneroid barometer is a cylindrical, air-tight box of metal closed by a corrugated top of thin, elastic metal. The air is exhausted from the box, which is then sealed. Evidently, the pressure of the air on the outside of the cover will cause the cover to curve inwards, as the space inside of the cover is void of pressure, until the

resistance due to the elasticity of the cover, aided by the resistance of a spring beneath it, is equal to the force exerted by the air. Now, if the air pressure increases, the cover will be curved further inwards; if the air pressure decreases, the elasticity of the cover, aided by the spring beneath, will cause a reduction of its inward curvature. These movements of the cover are transmitted and multiplied by a combination of delicate levers that act on the index hand shown in the figure and cause it to move to the right or left over a graduated scale.

These barometers are compensated (self-correcting) for variations in temperature. The better grade of these instruments have two graduations; the inner graduation corresponds to the graduations of the mercurial barometer; that is, it reads to inches of mercury. The outer graduation represents elevations above and below sea level, the distance between each graduation line representing a difference in elevation of 10 feet.

These instruments are made in various sizes, from the size of a watch up to 8 or 10 inches in diameter. They are very portable, occupying but a small space, are very light, and are quite delicate.

52. Both the mercurial and the aneroid barometers operate on the same general principle; viz., that if two opposite forces act on a body, it will be in equilibrium when the forces are equal and will be set in motion when they are unequal, provided the difference in the magnitude of the two forces is sufficient to overcome the resistance.

The two styles of barometer differ from each other only in the method by which equilibrium is established. In the mercurial barometer, the weight of the mercurial column inside the tube equalizes the outside air pressure; in the aneroid barometer, the outside pressure is equalized by the resistance of the flexible cover and spring beneath it.

53. Variations of Pressure at Different Elevations. With air, as with water, the lower we get the greater is the pressure, and the higher we get the less is the pressure. At

the level of the sea, the height of the mercurial column is about 30 inches; at 5,000 feet above the sea, it is 24.7 inches; at 10,000 feet above the sea, it is 20.5 inches; at 15,000 feet, it is 16.9 inches; at 3 miles, it is 16.4 inches; and at 6 miles above the sea level, it is 8.9 inches.

Air being an elastic fluid, the density or weight of the atmosphere also varies with the altitude; that is, a cubic foot of air at an elevation of 5,000 feet above the sea level will not weigh as much as a cubic foot at sea level. This is proved conclusively by the fact that at a height of 3½ miles the mercurial column measures but 15 inches, indicating that half the weight of the entire atmosphere is below that height. It is known that the height of the earth's atmosphere is at least 50 miles; hence, the air just before reaching the limit must be in an exceedingly rarefied state. It is by means of barometers that great heights are measured. The aneroid barometer has the heights marked on the dial, so that they can be read directly. With the mercurial barometer, the heights must be calculated from the reading.

54. Pressure in Different Directions.—The atmospheric pressure is everywhere present and presses all objects in all directions with equal force. If a book is laid upon the table, the air presses upon it in every direction with an equal average force of 14.7 pounds per square inch. It would seem as though it would take considerable force to raise a book from the table, since, if the size of the book were 8 inches by 5 inches, the pressure upon it would be $8 \times 5 \times 14.7 = 588$ pounds; but there is an equal pressure *beneath* the book that counteracts the pressure on the top. It would now seem as though it would require a great force to open the book, since there are two pressures of 588 pounds each acting in opposite directions and tending to crush the book; so it would, but for the fact that there is a layer of air beneath each leaf, which acts upwards and downwards with a pressure of 14.7 pounds per square inch.

If a piece of flat glass be laid upon a flat surface that has been previously moistened with water, it will require

considerable force to separate them; this is because the water helps to fill the pores in the flat surface and glass and thus creates a partial vacuum between the glass and the surface, thereby reducing the counter pressure beneath the glass.

55. Tension of Gases.—In Fig. 15, the space above the column of mercury was said to be a vacuum, and it was also stated that if any gas or air were present, it would expand and its tension would force the column of mercury downwards. If sufficient gas be admitted to cause the mercury to stand at 15 inches, the tension of the gas is evidently $\frac{14.7}{2} = 7.35$ pounds per square inch, since the pressure of the outside air, 14.7 pounds per square inch, now balances only 15 instead of 30 inches of mercury; that is, it balances only one-half as much as it would if there were no gas in the tube; therefore, the pressure (tension) of the gas in the tube is 7.35 pounds. If more gas be forced into the tube until the top of the mercurial column is just level with the mercury in the cup, the gas in the tube will then have a tension equal to the outside pressure of the atmosphere. Suppose that the bottom of the tube is fitted with a piston, and that the total length of the inside of the tube is 36 inches. If the piston be shoved upwards so that the space occupied by the gas is 18 inches long instead of 36 inches, the temperature remaining the same as before, it will be found that the tension of the gas within the tube is 29.4 pounds. It will be noticed that the volume occupied by the gas is only half that in the tube before the piston was moved, while the pressure is twice as great, since $14.7 \times 2 = 29.4$ pounds. If the piston be shoved up, so that the space occupied by the gas is only 9 inches instead of 18 inches, the temperature still remaining the same, the pressure will be found to be 58.8 pounds per square inch. The volume has again been reduced one-half and the pressure increased twofold, since $29.4 \times 2 = 58.8$ pounds. The space now occupied by the gas is 9 inches long, whereas,

before the piston was moved, it was 36 inches long; as the tube is assumed to be of uniform diameter throughout its length, the volume is now $\frac{9}{36} = \frac{1}{4}$ of its original volume, and its pressure is $\frac{58.8}{14.7} = 4$ times its original pressure. Moreover, if the temperature of the confined gas remains the same, the pressure and volume will always vary in a similar way.

56. Absolute and Gauge Pressures.—From the above explanation, it will be apparent that the pressure in the tube is the pressure above that of a perfect vacuum. Pressures reckoned above vacuum are called **absolute pressures.** The only pressure gauges that indicate absolute pressures are the mercurial and aneroid barometers; ordinary pressure gauges, such as the common steam gauge, indicate pressures above the pressure of the atmosphere. Pressures above that of the atmosphere are commonly called **gauge pressures.** Gauge pressures are changed to absolute pressures by adding 14.7 pounds per square inch to their readings. Truly speaking, the pressure indicated by a barometer, reduced to pounds pressure, should be added to the gauge pressure, since the value 14.7 pounds only represents the mean atmospheric pressure under normal conditions at *sea level.*

57. Mariotte's Law.—The law that states the effect of compressing and expanding gases is called **Mariotte's law** and is as follows:

The temperature remaining the same, the volume of a given quantity of gas varies inversely as the absolute pressure.

The meaning of the law is this: If the volume of a gas be diminished to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, etc. of its former value, the tension will be increased 2, 3, 5, etc. times, or if the outside pressure be increased 2, 3, 5, etc. times, the volume of the gas will be diminished to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, etc. of its original volume, the temperature remaining constant. It also means that if a gas is under a certain pressure, and this pressure is diminished to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{10}$, etc. of its original intensity, the volume of

the confined gas will be increased 2, 4, 10, etc. times—its tension decreasing at the same rate.

Suppose 3 cubic feet of air to be under a pressure of 60 pounds per square inch in a cylinder fitted with a movable piston; then the product of the volume and pressure is $3 \times 60 = 180$. Let the volume be increased to 6 cubic feet; then the pressure will be 30 pounds per square inch, and $30 \times 6 = 180$, as before. Let the volume be increased to 24 cubic feet; it is then $\frac{24}{3} = 8$ times its original volume, and the pressure is $\frac{1}{8}$ of its original pressure, or $60 \times \frac{1}{8} = 7\frac{1}{2}$ pounds, and $24 \times 7\frac{1}{2} = 180$, as in the two preceding cases. It will now be noticed that if a gas be enclosed within a confined space and allowed to expand without losing any heat, *the product of the pressure and the corresponding volume for any one position of the piston is the same as for any other position.* If the piston were forced inwards so as to compress the air, the same results would be obtained.

58. Application of Mariotte's Law.—If the volume of the vessel and the pressure of the gas are known, and it is desired to know the pressure after the first volume has been changed:

Rule 7.—*Divide the product of the first, or original, volume and pressure by the new volume; the result will be the new pressure.*

Or, let p = original absolute pressure;
p_1 = final absolute pressure;
v = volume corresponding to the pressure p;
v_1 = volume corresponding to the pressure p_1.

Then, $$p_1 = \frac{p\,v}{v_1}.$$

EXAMPLE.—At the point of cut-off in a steam engine, the amount of steam in the cylinder is 862 cubic inches. The pressure at this point is 120 pounds per square inch. What will be the pressure of the steam when the piston has reached the end of its stroke and the volume is 1,800 cubic inches?

SOLUTION.—Applying the rule, we get

$$p_1 = \frac{862 \times 120}{1{,}800} = 57.47 \text{ lb. per sq. in. absolute.} \quad \text{Ans.}$$

59. If it is required to determine the volume after a change in the pressure:

Rule 8.—*Divide the product of the original volume and pressure by the new pressure; the result will be the new volume.*

Or, using the same letters as before,

$$v_1 = \frac{p\,v}{p_1}.$$

EXAMPLE 1.—At the commencement of compression, the volume of the steam is 380 cubic inches and the pressure is 18 pounds per square inch. At the end of compression, the pressure is 112 pounds per square inch. What is the final volume?

SOLUTION.—Applying the rule, we get

$$v_1 = \frac{380 \times 18}{112} = 61.07 \text{ cu. in.} \quad \text{Ans.}$$

EXAMPLE 2.—A vessel contains 10 cubic feet of air at a pressure of 15 pounds per square inch and has 25 cubic feet of air of the same pressure forced into it; what is the resulting pressure?

SOLUTION.—The original volume $= 10 + 25 = 35$ cubic feet; the original pressure is 15 pounds per square inch; the final volume is 10 cubic feet. Hence, applying rule **7**, we get

$$p_1 = \frac{35 \times 15}{10} = 52.5 \text{ lb. per sq. in. absolute.} \quad \text{Ans.}$$

It must be remembered that in the preceding examples the temperature is supposed to remain constant. When the temperature changes during expansion and compression, the problem becomes a rather difficult one and cannot be solved by ordinary arithmetic.

EXAMPLES FOR PRACTICE.

1. A vessel contains 25 cubic feet of gas at a pressure of 18 pounds per square inch; if 125 cubic feet of gas having the same pressure are forced into the vessel, what will be the resulting pressure?

Ans. 108 lb. per sq. in.

2. The volume of steam in the cylinder of a steam engine at cut-off is 1.35 cubic feet and the pressure is 85 pounds per square inch; the pressure at the end of the stroke is 25 pounds per square inch. What is the new volume? Ans. 4.59 cu. ft.

3. A receiver contains 180 cubic feet of gas at a pressure of 20 pounds per square inch; if a vessel holding 12 cubic feet be filled from the larger vessel until its pressure is 20 pounds per square inch, what will be the pressure in the larger vessel?

Ans. $18\frac{3}{8}$ lb. per sq. in.

4. A spherical shell has a part of the air within it removed, forming a partial vacuum; if the outside diameter of the shell is 18 inches and the pressure of the air within is 5 pounds per square inch, what is the total pressure tending to crush the shell? Ans. 9,873.42 lb.

PNEUMATIC MACHINES.

THE AIR PUMP.

60. *The* **air pump** *is an instrument for removing air from a given space.* A section of the principal parts is

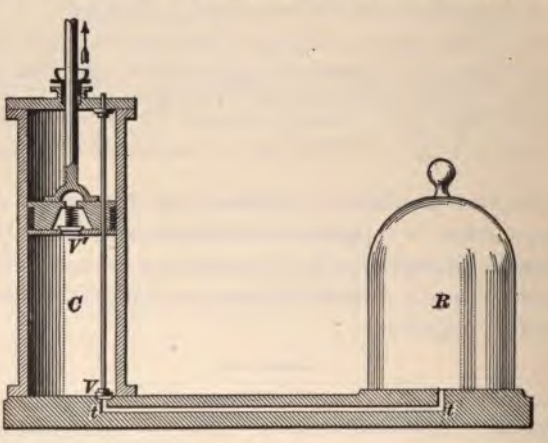

FIG. 19.

shown in Fig. 19 and the complete instrument in Fig. 20. The closed vessel R is called the **receiver,** and the space

that it encloses is that from which it is desired to remove the air. It is usually made of glass, and the edges are ground so as to be perfectly air-tight. When made in the form shown, it is called a **bell-jar receiver.** The receiver rests upon a horizontal plate, in the center of which is an opening communicating with the pump cylinder *C* by means of the passage *t t*. The pump piston accurately fits the cylinder, and has a valve *V'* opening upwards. Where the passage *t t* joins the cylinder, another valve *V* is placed, which also opens upwards. When the piston is raised, the valve *V'* closes, and, since no air can get into the cylinder from above, the piston leaves a vacuum behind it. The pressure upon *V* being now removed, the tension of the air in the receiver *R* causes *V* to rise; the air in the receiver and passage *t t* then expands so as to occupy the additional space provided by the upward movement of the piston.

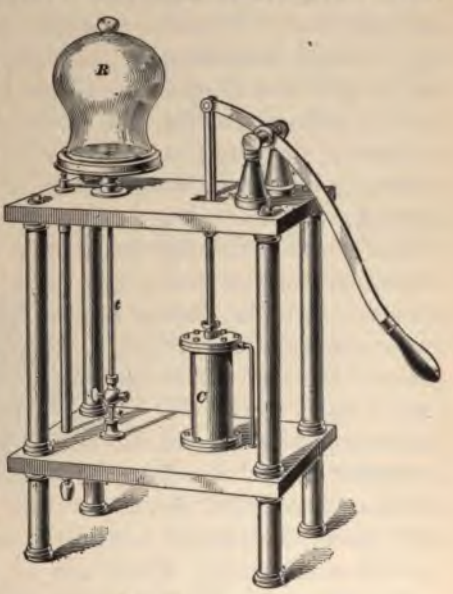

FIG. 20.

The piston is now pushed down, the valve *V* closes, the valve *V'* opens, and the air in *C* escapes. The lower valve *V* is sometimes supported, as shown in Fig. 19, by a metal rod passing through the piston and fitting it somewhat tightly. When the piston is raised or lowered, this rod moves with it. A button near the upper end of the rod confines its motion within very narrow limits, the piston sliding upon the rod during the greater part of the journey. In the complete form of the instrument shown

in Fig. 20, communication between receiver and pump is made by means of the tube t.

61. Degrees and Limits of Exhaustion.—Suppose that the volume of R and t together is four times that of C, Fig. 19, and that there are, say, 200 grains of air in R and t, and 50 grains in C when the piston is at the top of the cylinder. At the end of the first stroke, when the piston is again at the top, 50 grains of air in the cylinder C will have been removed, and the 200 grains in R and t will occupy the space R, t, and C. The ratio between the sum of the spaces R and t and the total space $R + t + C$ is $\frac{4}{5}$; hence, $200 \times \frac{4}{5} = 160$ grains = the weight of air in R and t after the first stroke. After the second stroke, the weight of the air in R and t would be $(200 \times \frac{4}{5}) \times \frac{4}{5} = 200 \times (\frac{4}{5})^2 = 200 \times \frac{16}{25} = 128$ grains. At the end of the third stroke, the weight would be $[200 \times (\frac{4}{5})^2] \times \frac{4}{5} = 200 \times (\frac{4}{5})^3 = 200 \times \frac{64}{125} = 102.4$ grains. At the end of n strokes the weight would be $200 \times (\frac{4}{5})^n$. It is evident that *it is impossible to remove all the air that is contained in R and t by this method.* It requires an exceedingly good air pump to reduce the tension of the air in R to $\frac{1}{50}$ inch of mercury. When the air has become so rarefied as this, the valve V' will not lift, and, consequently, no more air can be exhausted.

62. The Dashpot.—The pressure of the atmosphere is utilized in Corliss engines to close the steam valves rapidly. For this purpose a so-called **dashpot** is used. The dashpot of a Bullock-Corliss engine is shown in two positions in Fig. 21. It consists of the base A, which is fastened to the floor, and a plunger B pivoted to the end of a crank-arm keyed to the stem of the steam valve. The base is accurately turned and bored to fit the plunger; packing rings are fitted to both the plunger and the base in order to make an air-tight joint.

The annular space around the central stationary piston is in communication with the outside air by a small passage, the opening of which can be increased or decreased, at will,

by means of the valve *C*. In operation, the plunger is first in the position shown in Fig. 21 (*b*). It is then picked up by the valve gear of the engine and drawn up until it is in the position shown in Fig. 21 (*a*). The large piston on the lower end of the plunger is in communication with the atmosphere through the valve *C*, and hence the pressure of the atmosphere on both sides of the piston is equal. The upper part of the plunger, however, has an equal pressure on both its sides only when it is down, as in Fig. 21 (*b*). When it is drawn up, the air in the space *D* expands and a partial vacuum is formed. The valve gear next releases the plunger, and as the atmospheric pressure is much greater than the pressure within *D*, the plunger rapidly descends. During its descent, the large piston at the end of the plunger gradually compresses the air in the annular space beneath it, and is

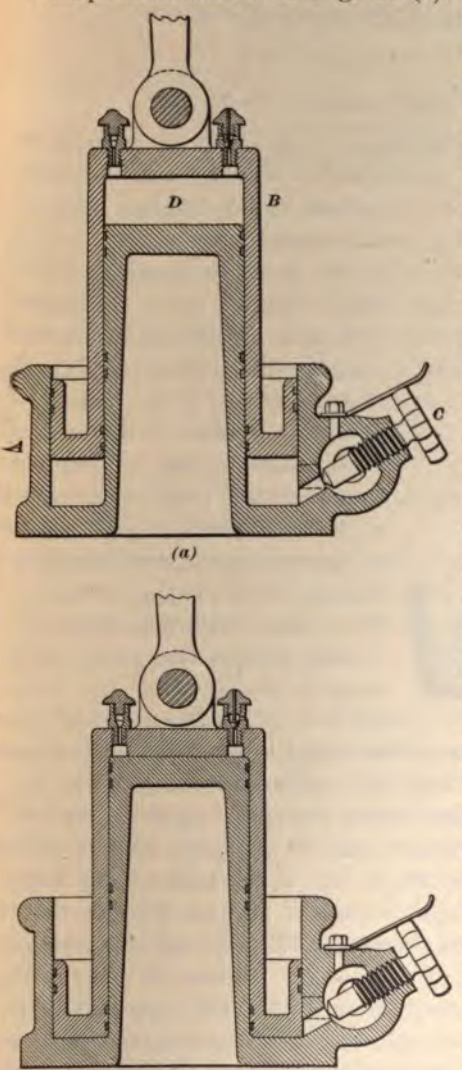

FIG. 21.

thus gradually brought to rest. The valve *C* serves to regulate the amount of compression and at the same time admits air during the up stroke of the plunger.

THE SIPHON.

63. Theory of the Siphon.—The action of the **siphon** illustrates the effect of atmospheric pressure. It is simply a bent tube having unequal branches, open at both ends, and is used to convey a liquid from a higher point to a lower over an intermediate point that is higher than either. In Fig. 22, *A* and *B* are two vessels, *B* being lower than *A*, and *A C B* is the bent tube, or siphon. Suppose this tube to be filled with water and placed in the vessels as shown, with the short branch *A C* in the vessel *A*. The water will flow from the vessel *A* into the vessel *B* as long as the level of the water in *B* is below the level of the water in *A* and the level of the water in *A* is above the lower end of the tube *A C*. The atmospheric pressure on the surfaces of *A* and *B* tends to force the water up the tubes *A C* and *B C*. When the siphon is filled with water, each of these pressures is counteracted in part by the pressure of the water in that branch of the siphon that is immersed in the water on which the pressure is exerted. The atmospheric pressure opposed to the weight of the longer column of water will, therefore, be resisted more strongly than that opposed to the weight of the shorter column; consequently, the pressure exerted on the shorter column will be greater than that on the longer column, and this excess of pressure will produce motion.

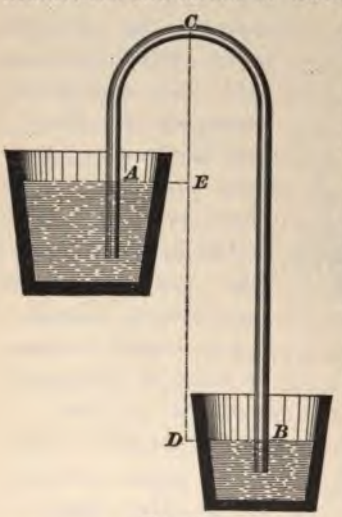

FIG. 22.

64. Let A = the area of the tube;

$h = D\,C$ = the vertical distance between the surface of the water in B and the highest point of the center line of the tube;

$h_1 = E\,C$ = the distance between the surface of the water in A and the highest point of the center line of the tube.

The weight of the water in the short column is $.03617 \times A\ h_1$, and the resultant atmospheric pressure tending to force the water up the short column is $14.7 \times A - .03617 \times A\ h_1$. The weight of the water in the long column is $.03617\ A\ h$, and the resultant atmospheric pressure tending to force the water up the long column is $14.7\ A - .03617\ A\ h$. The difference between these two is $(14.7\ A - .03617\ A\ h_1) - (14.7\ A - .03617\ A\ h) = .03617\ A\ (h - h_1)$. But $h - h_1 = E\,D$ = the difference between the levels of the water in the two vessels. In the above, h and h_1 were taken in inches and A in square inches.

It will be noticed that the short column must not be higher than 34 feet for water, or the siphon will not work, since the pressure of the atmosphere will not support a column of water that is higher than 34 feet.

65. Fig. 23 shows the actual construction of a siphon. It is desired to convey the water from D to E. The point of discharge must always be lower than the point from which the water is taken, otherwise a siphon cannot work. The siphon consists of ordinary iron pipe jointed in any convenient manner so as to be air-tight. It has three valves A, B, and C. The suction end is provided with a perforated strainer c to keep out large particles of foreign matter.

In order to start the siphon, it is necessary to remove the air in the pipe. This is done by closing the valves A and B and opening the valve C, which should always be located at the highest point of the siphon. Water is then poured into the funnel above the valve C until the pipe is filled. When

no more water can be poured in, that is, when all the air has been driven out, the valve C is closed and the valves A and B are opened; the siphon will now start.

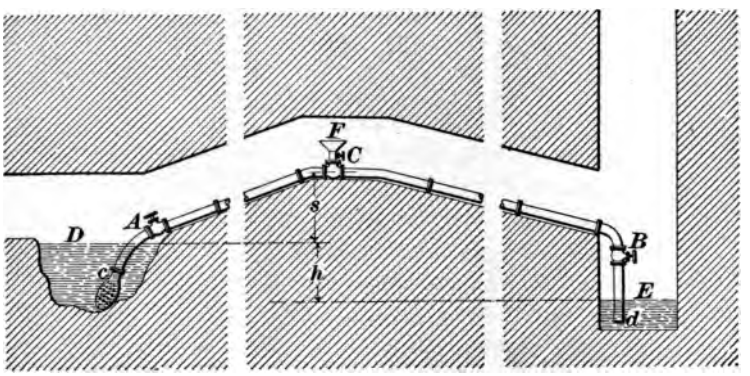

FIG. 23.

In practice, it has been found that the distance s must not exceed 28 feet at sea level, and that the siphon will work more satisfactory if it is less. The greater the distance h between the two water levels, the better the siphon will work.

66. In order that a siphon will work properly, it is necessary that air should be kept out of the pipe, or, if it does get in, means should be provided for its escape. The joints of the pipe must be perfectly air-tight. But some air will always be carried in with the water, and this air will collect at the highest point. The bad effects of this can be minimized by having a straight horizontal pipe at the highest point instead of a sharp bend.

67. A device that will remove the air is shown in Fig. 24. Here A is an air-tight vessel connected with the siphon by two small pipes B and C. The pipe B extends nearly to the top of A, while the pipe C barely enters the bottom. Each pipe has a valve D and E. A valve F and funnel G are placed on top of the vessel. When the siphon ceases to flow, which is an indication that air has collected (it is here

assumed that the suction end has not become uncovered), the valves D and E are closed and the valve F is opened. The vessel is now filled with water, the valve F is closed, and D and E are opened. The water will now flow through C into the siphon and force out the air collected there, which passes through B to the top of A. When all air is out of the siphon, D and E are shut and F is opened. The vessel A is now filled with water, F is shut, D and E are

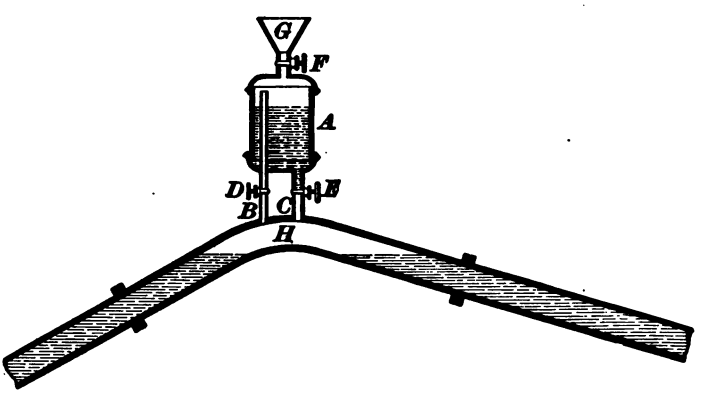

FIG. 24.

opened and left open. Any air that enters the siphon will, instead of collecting at H, ascend the pipe B and force out a small amount of water through C. This will continue until A is filled with air, when the valves D and E should be closed and A refilled. This arrangement may also be used to fill the siphon for the purpose of setting it to work.

The highest point of the water in A should not be more than 28 feet at sea level above the water level at the suction end.

PUMPS.

68. A **pump** is a machine for conveying liquids from one level to a higher level or for performing work equivalent to such an operation.

69. Classification of Pumps.—Pumps may be divided into three general divisions, according to the service they perform, viz., *suction pumps*, *lifting pumps*, and *force pumps*. They may also be divided into two general classes, *single-acting* and *double-acting pumps*, according as they take water on one side or on both sides of the water piston. According to the arrangement of the pump cylinders, they are classified as *simple*, *duplex*, or *triplex pumps*. As pumps displace the liquids in various ways, they may also be divided according to the method of displacement, into *reciprocating*, *centrifugal*, and *rotary pumps*. Reciprocating pumps only will be considered here.

70. The Suction Pump.—A section of an ordinary suction pump is shown in diagrammatic form in Fig. 25.

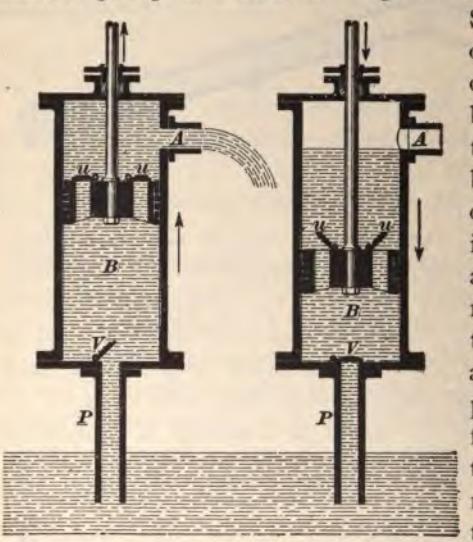

FIG. 25.

Suppose the piston, or **bucket** as it is commonly termed, to be at the bottom of the cylinder and to be just on the point of moving upwards in the direction of the arrow. As the piston rises it leaves a partial vacuum behind it, and the atmospheric pressure on the surface of the water in the well causes it to rise in the pipe P for the same reason that the mercury rises in the barometer tube. The water rushes up the pipe and lifts the suction valve V, filling the empty space in the cylinder B caused by the displacement of the piston. When the piston has reached the end of its stroke, the water entirely fills the space between the bottom of the piston and the

bottom of the cylinder, and also the pipe P. The instant the piston begins its down stroke, the water in the chamber B begins to flow back into the well, and its downward flow forces the valve V to its seat, thus preventing any further escape of the water. As the piston descends, the water must give way to it, and since the suction valve V is closed, the bucket valves u, u must open, and thus allow the water to pass through the piston, as shown in the right-hand figure. When the piston has reached the end of its downward stroke and commenced its upward movement again, the water flowing through the piston quickly closes the valves u, u. All the water resting on the top of the piston is then lifted by the piston on its upward stroke and discharged through the spout A; the valve V again opens and the water fills the space below the piston, as before.

71. It is evident that the distance between the piston when at the top of its stroke and the surface of the water in the well must not exceed 34 feet, the highest column of water that the pressure of the atmosphere will sustain, since, otherwise, the water in the pipe would not rise and fill the cylinder as the piston ascended. In practice, this distance should not exceed 28 feet. This is due to the fact that there is a little air left between the bottom of the piston and the bottom of the cylinder, a little air leaks through the valves, which are not perfectly air-tight, and a pressure is needed to raise the valve against its own weight, which, of course, acts downwards.

There are many varieties of the suction pump, differing principally in the construction of the valves and piston, but the principle is the same in all.

72. The Lifting Pump.—In some cases it is desired to raise water higher than it can be forced by the pressure of the atmosphere into the chamber of a simple suction pump, such as is shown in Fig. 25. To accomplish this the pump chamber with its bucket and valves are set at a distance above the supply not exceeding that to which the air will successfully force the water. A closed pipe P', Fig. 26,

called the *delivery*, or *discharge*, *pipe*, is then led from the upper part of the chamber to the point where it is desired to deliver the water. Such a pump is often called a **lifting pump.**

In order to prevent the leakage of water around the piston rod, a stuffingbox S is provided. The lower end of the discharge pipe P' is sometimes fitted with a valve c to prevent the water flowing back into the pump chamber; this

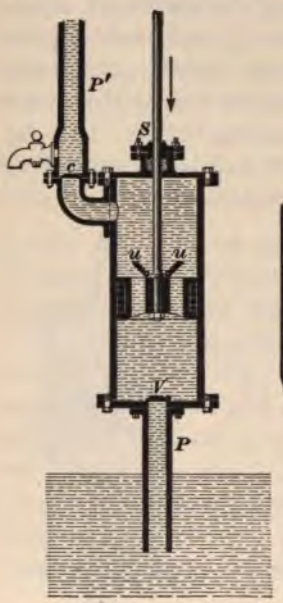

FIG. 25.

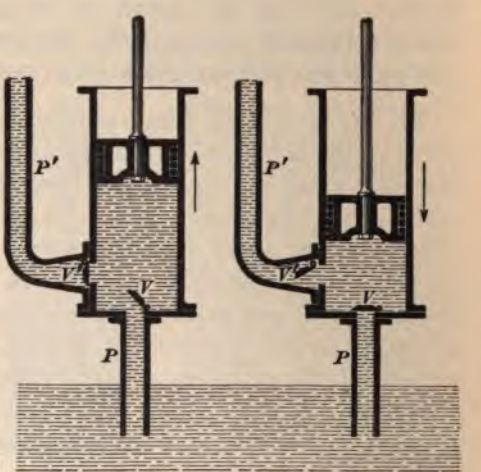

FIG. 27.

valve is not essential to the operation of the pump, however, since the valve V prevents the water in the chamber and discharge pipe discharging during the downward motion of the piston.

While it is customary to consider lifting pumps and suction pumps as two different types of pumps, there is in reality no difference in their operation, as a careful study of Figs. 25 and 26 will show. The only difference is that the water is discharged by a suction pump *at* the level of the

pump, while a lifting pump discharges the water *above* the level of the pump.

73. Force Pumps.—The force pump differs from the lifting pump in one important particular, that is, in the fact that its piston is solid. A section of a force pump is shown in Fig. 27. As the piston ascends, as shown in the left-hand figure, the pressure of the atmosphere forces the water up the suction pipe P; the water opens the suction valve V and flows into the pump cylinder. When the piston moves down, as shown in the right-hand figure, the suction valve is closed and the delivery valve V' opened. The water in the pump cylinder is now forced up the delivery pipe P'. When the piston again begins to move upwards, the delivery valve is closed by the water above it and the suction valve opened by the pressure of the atmosphere on the water below it.

74. Plunger Pumps.—When force pumps are used to convey water to great heights or to force water against heavy pressures, the great pressure of the water makes it extremely difficult to keep the water from leaking past the piston, and the constant repairing and renewal of the piston packing becomes a nuisance and involves serious expense. To obviate this drawback, plunger pumps have been designed, one of which is shown diagrammatically in Fig. 28. The action

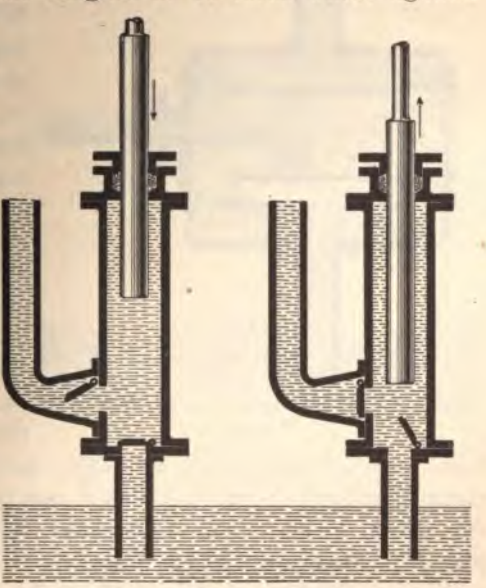

FIG. 28.

does not differ in any way from that of the piston force pump. During the up stroke of the plunger, the suction valve is open and the delivery valve is closed; during the down stroke, the suction valve is closed and the delivery valve is open.

75. The force pumps shown so far are **single-acting**, that is, the water is forced into the delivery pipe only during the downstroke or forward stroke of the piston or plunger. Force pumps, either of the piston or plunger pattern, may be constructed so as to force water into the delivery pipe both during the forward and return stroke. They are then called **double-acting.**

76. A **double-acting force pump** of the piston pattern is shown in Fig. 29. Such a pump has two sets of suction valves and delivery valves, one set for each side of the piston. With the piston moving in the direction of the arrow, the pressure of the atmosphere forces the water up the suction pipe P into the left-hand end of the pump cylinder, the left-hand suction valve opens and the left-hand delivery valve is closed. The piston in moving to the right, displaces the water in the right-hand end of the pump cylinder; as a consequence the right-hand suction valve is closed and the right-hand delivery valve opens. The water now flows up the delivery pipe P'. Imagine that the piston is at the end of its stroke and commences to move to the left. Its first movement

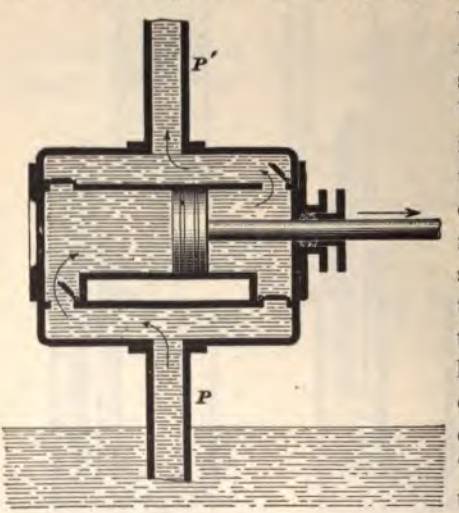

FIG. 29.

promptly closes the left-hand suction valve and opens the left-hand delivery valve. It also closes the right-hand delivery valve and opens the right-hand suction valve.

It is thus seen that with the arrangement given, which shows the principle of operation of all double-acting pumps, the piston will discharge water both during the forward and the return stroke. While the pump shown is a horizontal pump, it may be vertical as well.

STRENGTH OF MATERIALS.

GENERAL PRINCIPLES.

1. When a force is applied to a body, it changes either its form or its volume. A force, when considered with reference to the internal changes it tends to produce in any solid, is called a **stress.**

Thus, if we suspend a weight of 2 tons by a rod, the stress in the rod is 2 tons. This stress is accompanied by a lengthening of the rod, which increases until the internal stress or resistance is in equilibrium with the external weight.

2. Classification of Stresses.—Stresses may be classified as follows: *Tensile*, or *pulling stress; transverse*, or *bending stress; compressive*, or *pushing stress; shearing*, or *cutting stress; torsional*, or *twisting stress.*

3. A **unit stress** is the amount of stress on a unit of area, and may be expressed either in pounds or tons per square inch or per square foot; or, it is the load per square inch or per square foot on any body.

Thus, if 10 tons are suspended by a wrought-iron bar that has an area of 5 square inches, the unit stress is 2 tons per square inch, because $\frac{10}{5} = 2$ tons.

4. Strain is the deformation or change of shape of a body resulting from stress.

For example, if a rod 100 feet long is pulled in the direction of its length, and if it is lengthened 1 foot, it is strained $\frac{1}{100}$ its length, or 1 per cent.

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5. Elasticity is the property by virtue of which a body regains its original form after the external force on it is withdrawn, provided the stress has not exceeded the elastic limit. It is a property possessed by all bodies.

Consequently, we see from this that all material is lengthened or shortened when subjected to either tensile or compressive stress, and the change of the length within the elastic limit is directly proportional to the stress.

For stresses within the elastic limits, materials are perfectly elastic, and return to their original length on removal of the stresses; but when their elastic limits are exceeded, the changes of their lengths are no longer regular, and a permanent *set* takes place. The destruction of the material has then begun.

6. The **measure of elasticity** of any material is the change of length under stress within the elastic limit.

7. The **elastic limit** is that unit stress under which the permanent set becomes visible.

8. The elasticity of wrought iron and of all grades of steel is practically the same; that is, *within the elastic limit* each material will change an equal amount of length under the same stress. The elastic limit, however, is not the same for steel as for iron; it is higher for soft steel than for wrought iron, and, in general, the harder and stronger the steel the higher will be its elastic limit. As a consequence, steel will lengthen or shorten more than wrought iron, and hard steel more than soft, before its elasticity or ability to return to its original dimensions is injured.

TENSILE STRENGTH OF MATERIALS.

9. The **tensile strength** of any material is the *resistance* offered by its fibers to being pulled apart.

10. The *tensile strength* of any material *is proportional to the area of its cross-section.*

Consequently, when it is required to find the safe tensile strength of any material, we have only to find the area at the minimum cross-section of the body, and multiply it by the load per square inch that it can safely carry, as given in the following table under the heading "Safe Loads."

NOTE.—The minimum cross-section referred to in the above paragraph is that section of the material which is pierced with holes, such as bolt or rivet holes in iron, or knots in wood, if there are any.

TABLE I.

TENSILE STRENGTH OF MATERIALS.

Material.	Ultimate Tensile Strength. Pounds per Square Inch.	Safe Loads. Pounds per Square Inch.		
		Sudden.	Gradual.	Steady.
Timber.....	10,000	600	900	1,200
Cast iron.....	16,000– 20,000	1,600	2,400	3,200
Gun-metal cast iron	30,000– 35,000	2,000	3,000	4,000
Wrought iron.....	45,000– 55,000	4,500	9,000	13,000
Extra soft steel...	45,000– 55,000	6,000	10,000	14,000
Flange steel.....	52,000– 62,000	6,000	10,000	14,000
Firebox steel.....	52,000– 62,000	6,000	10,000	14,000
Machinery steel...	60,000– 75,000	6,500	11,000	15,000
Axle steel.....	75,000– 90,000	7,000	14,000	20,000
Hard steel (rail steel).....	90,000–115,000	7,000	14,000	20,000
Tire steel.....	100,000–125,000	8,000	15,000	20,000
Crucible (tool) steel	125,000–180,000	15,000	23,000	30,000
Brass, cast.....	14,000– 20,000	1,400	2,200	3,000
Bronze, cast.....	30,000– 36,000	3,000	5,000	7,000
Tobin bronze.....	70,000– 80,000	7,000	11,000	14,000
Hard-drawn brass wire.....	30,000– 40,000	2,500	3,700	5,000
Copper, rolled.....	30,000– 36,000	3,500	6,000	8,000

11. In metals, a high tensile strength in itself is no indication of the ability of the metal to stand repeated applications of sudden stresses, as a high tensile strength usually involves a lesser degree of ductility than is obtained in metals of lower tensile strength. Steel of a tensile strength higher than 75,000 pounds is seldom used merely on account of its superior strength, but rather on account of its hardness, which enables it to better withstand abrasion.

12. The loads given in Table I are conservative safe loads deducted from experience and observation. They are given for material free from welds for such material as can be welded. While it is possible to make a weld as strong as the solid bar, the chances of the weld being imperfect are so great that it is unsafe to rely on such a degree of strength in welded bars. Furthermore, the value of the weld is an uncertain quantity that cannot be determined by an ocular inspection or any other inspection short of actually pulling the weld apart in a testing machine. Hence, it is advisable to reduce the safe loads given in the above table by 25 per cent. when a welded bar is subjected to a tensile stress. When judging the safe load of timber, due allowance must be made for knot holes and sappy spots.

13. It is often rather hard to determine whether a stress is steady, gradually varying, or gradually applied, or sudden. For example, considering the shell of a boiler, it would appear on first thought as though the tensile stress in the shell plates was a steady stress. But looking further into the problem, it will become apparent that the stress varies with the steam pressure, which gradually fluctuates within narrow or wider limits. Hence, most designers would consider the stress in a boiler shell as a gradually applied stress. In a piston rod or connecting-rod the load is applied almost instantly as soon as the crank passes over the center, and, hence, the stress would be considered to be a suddenly applied stress. When in doubt, it is good policy to err on the side of safety, that is, to choose a smaller safe load per square inch of section.

For special work, experience has indicated safe loads for different materials that fall below, or are above, those given in the table. Examples of this will be given later on.

14. The safe load per square inch of section is often called the **working stress per square inch,** or the **working load per square inch.** Care should be taken not to confound these terms with *working load*, *working stress*, *safe load*, or *safe tensile strength*, which, when applied without the limitation as to the unit of area, refer to the safe load the whole bar can carry.

RULES AND FORMULAS FOR TENSILE STRENGTH.

15. Let W = safe load in pounds;
A = area of minimum cross-section;
S = working stress in pounds per square inch, as given in Table I.

Rule 1.—*The working load in pounds for any bar subjected to a tensile stress is equal to the minimum sectional area of the bar, multiplied by the working stress in pounds per square inch, as given in the table.*

Or, $$W = A\,S.$$

EXAMPLE.—A bar of good wrought iron that is 3 inches square is to be subjected to a steady tensile stress; what is the maximum load that it should carry?

SOLUTION.—According to the table, a working stress of 13,000 pounds may be allowed. Applying the rule, we have

$$W = 3 \times 3 \times 13{,}000 = 117{,}000 \text{ lb.} \quad \text{Ans.}$$

Rule 2.—*The minimum sectional area of any bar subjected to a tensile stress should be equal to the load in pounds, divided by the working stress in pounds per square inch, as given in the table.*

Or, $$A = \frac{W}{S}.$$

EXAMPLE.—What should be the area of a machinery-steel bar to carry a steady load of 108,000 pounds?

SOLUTION.—According to the table, a safe load of 15,000 pounds per square inch of section may be allowed. Then, applying the rule,

$$A = \frac{108{,}000}{15{,}000} = 7.2 \text{ sq. in.} \quad \text{Ans.}$$

Rule 3.—*The working stress in pounds per square inch is equal to the load in pounds divided by the minimum sectional area of the bar.*

Or,
$$S = \frac{W}{A}.$$

EXAMPLE.—A piston rod 3 inches in diameter carries a piston 20 inches in diameter. With a steam pressure of 100 pounds to the square inch, what is the stress per square inch of section of the rod?

SOLUTION.—The load on the piston is $20^2 \times .7854 \times 100 = 31{,}416$ pounds. The area of the rod is $3^2 \times .7854 = 7.0686$ square inches. Then, applying the rule just given, we have

$$S = \frac{31{,}416}{7.0686} = 4{,}444.4 \text{ lb.} \quad \text{Ans.}$$

CHAINS.

16. **Chains** made of the same size iron vary in strength, owing to the different kinds of links from which they are made. It is a good practice to anneal old chains that have become brittle by overstraining. This renders them less liable to snap from sudden jerks. It reduces their tensile strength, but increases their toughness and ductility, which are sometimes more important qualities.

When annealing, care should be taken that a sufficient heat be applied, otherwise no benefit will be gained; the chains ought to be heated to a cherry red, say 1,300° F., at least.

17. The loads that can safely be lifted with chains are given in the following table. The safe load given is one-quarter of the steady load at which they may be expected to break. The table has been deduced from one published by the Lukens Iron and Steel Company.

TABLE II.

STRENGTH OF CHAINS.

Diameter of Rod of Which Link Is Made.	Safe Load. Pounds.	Diameter of Rod of Which Link Is Made.	Safe Load. Pounds.
$\frac{3}{16}$	430	1	12,500
$\frac{1}{4}$	770	$1\frac{1}{8}$	15,000
$\frac{5}{16}$	1,200	$1\frac{1}{4}$	18,000
$\frac{3}{8}$	1,750	$1\frac{3}{8}$	22,000
$\frac{7}{16}$	2,350	$1\frac{1}{2}$	26,000
$\frac{1}{2}$	3,100	$1\frac{5}{8}$	31,000
$\frac{9}{16}$	4,000	$1\frac{3}{4}$	36,000
$\frac{5}{8}$	4,800	$1\frac{7}{8}$	41,000
$\frac{11}{16}$	5,800	2	47,000
$\frac{3}{4}$	6,900	$2\frac{1}{4}$	56,000
$\frac{13}{16}$	8,000	$2\frac{1}{2}$	69,000
$\frac{7}{8}$	9,400	$2\frac{3}{4}$	84,000
$\frac{15}{16}$	11,000	3	100,000

STRENGTH OF ROPES.

HEMP ROPES.

18. The strength of hemp ropes does not depend entirely on the quality of the material and the cross-section of the rope, but to a large extent on the method of manufacture and the amount of twisting. A hemp rope is made up of fibers twisted together to form a **yarn,** several of which are laid up left-handed into **strands,** which in turn are twisted up right-handed into what is known as **plain-laid rope.**

Cable-laid or **hawser-laid rope** is left-handed rope of nine (9) strands.

Shroud-laid rope is formed by adding another strand to a plain-laid rope. Since the four strands on application of

strain would sink in and detract from the rope's strength by an unequal distribution of strain, they are laid up around a **heart,** which is a small rope, made soft and elastic, and about one-third the size of the strands.

19. Ordinary hemp rope is designated by its circumference; rope for transmission purposes, which is either iron, steel, or manila rope, is designated by its diameter. It is well to keep this in mind. The loads that can safely be hoisted with hemp ropes of different sizes is given in the following table prepared by Ensign F. R. Brainard, United States Navy.

TABLE III.

STRENGTH OF HEMP ROPES.

Circumference. Inches.	Safe Load. Pounds.	Circumference. Inches.	Safe Load. Pounds.
1.00	65	5.75	2,680
1.25	125	6.00	2,975
1.50	185	6.50	3,470
1.75	255	7.00	3,965
2.00	320	7.50	4,570
2.25	410	8.00	5,175
2.50	500	8.50	5,910
2.75	610	9.00	6,640
3.00	715	9.50	7,370
3.25	850	10.00	8,105
3.50	980	10.50	8,970
3.75	1,135	11.00	9,840
4.00	1,285	11.50	11,015
4.25	1,450	12.00	12,190
4.50	1,610	12.50	13,365
4.75	1,750	13.00	14,540
5.00	1,990	13.50	15,845
5.25	2,190	14.00	17,150
5.50	2,385	14.50	18,450

20. The above table applies to new ropes or ropes in first-class condition. If the rope is used for a block and tackle, the bending of the rope around the pulleys will cause a rapid deterioration of the inside, owing to the chafing and sliding of the strands and yarns upon one another. The outside appearance of a rope used for block and tackle is in itself no indication of its quality; ropes are frequently found that appear perfectly sound when judged by their outside appearance and yet are entirely unsafe. To inspect the rope, take it in both hands and untwist it sufficiently to expose the inner surfaces that have been chafing against one another. If a considerable number of broken fibers are found, the safe load should be reduced by 50 per cent. If a rather large quantity of the fibers have been reduced to powder, the rope should be condemned and preferably destroyed immediately, in order to remove any temptation to use it. A rope in this condition is liable to give way suddenly under a very light load.

Ropes used for slings and lashings, as a general rule, are ruined by the external chafing they receive and, hence, their safety can be determined by their outward appearance.

21. **Slings** are chiefly used for attaching tackles to machine parts during the erection and repair of machinery. They are made by taking a piece of rope a little more than twice the required length of the rope and joining the ends by splicing, the splice known as a **short splice** being usually employed. It should be distinctly understood that this splice is not intended nor adapted for ropes that are to pass over a pulley.

22. **Splicing a Sling.**—To make a short splice in a sling, untwist the strands for some distance from each end of the rope. Take an end in each hand and lay the strands within one another, as shown in Fig. 1 (*a*). The strands *1'*, *2'*, and *3'* may be tied to the left-hand end for convenience in handling. Now take the strand *1*, pass it over strand *2'* and under strand *1'*, as shown in Fig. 1 (*b*); then, draw it

tight. Next, take strand 2, pass it over 3′ and under 2′, drawing it up tightly. Now take strand 3, pass it over 1′ and under 3′; then draw it up. The splice will now appear as shown in Fig. 1 (c). Weave the strands 1′, 2′, and 3′ into the left-hand rope in the same manner. Continue weaving the right-hand and left-hand strands alternately until the length of the splice is about 6 times the diameter of the rope. Draw the strands tight and subject the splice to a strain. Then cut off the projecting ends of the strands.

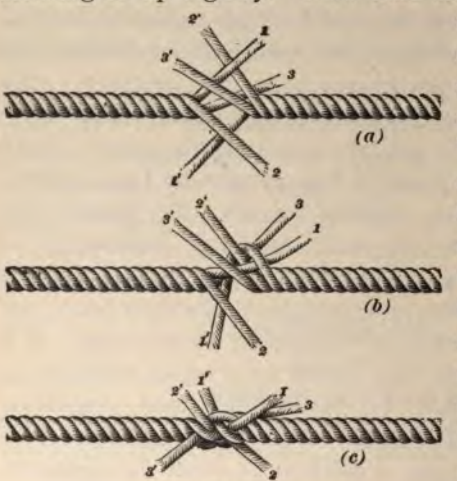


FIG. 1.

23. Use of Slings.—The weight that can safely be lifted by a sling depends not only on the size of the rope of which it is made, but also on the manner in which it is attached to the hook of the tackle block. Thus, in Fig. 2, the sling is simply hooked over the hook.

Owing to the sharp bend over the hook, the strength of the sling is greatly reduced, and the direct load for this manner of attachment should not exceed the safe load given in the table for one rope of equal circumference, although the load is borne by two parts of the rope.

When the sling is attached to the hook by doubling over, as shown in Fig. 3, its greatest strength is realized, and for the two parts of the rope parallel to each other, the stress in the rope will be equal to half the load, as the load is supported by two ropes. Then, with a sling doubled over as shown, a load twice as great can safely be hoisted than when applied as shown in Fig. 2.

In the foregoing statement, it is assumed that the sling is fastened to the work without a sharp bend, the equivalent of a single loop over a hook. The stress in the two parts of a sling will be least when they are parallel to each other; hence, a sling will then lift the greatest load with safety. The stress becomes greater when the angle between the two parts of the sling is increased; its magnitude for any angle can be determined by the method of the triangle of forces.

24. A quick way of attaching a rope to a hook is that shown in Fig. 4, and known as a **Blackwall hitch.** When

FIG. 2.

FIG. 3.

FIG. 4.

making this hitch, always make it as shown in the figure; that is, let the detached end lie between the hook and the end sustaining the load.

25. The proper size of a rope for a block and tackle is fixed by the size of the groove turned in the pulleys. Then, from the table, take the safe working load of the rope, and using this as the force applied to the free end of the rope, the maximum load a given block and tackle can lift safely can be calculated by the rule given in connection with the subject of pulleys.

MANILA ROPE.

26. Manila rope is used chiefly for hoisting purposes and for power transmission. The so-called "Stevedore" manila rope has the fibers lubricated with a mixture of tallow and plumbago during the process of manufacture. This lubrication prevents internal chafing and wear to a large extent.

Manila rope for hoisting and power-transmission purposes is usually made with four strands, that is, shroud laid. Hoisting rope is ordered by *circumference;* transmission rope by *diameter.*

27. The working stress on manila hoisting rope should not exceed that given in the table below, which was published by C. W. Hunt & Company.

TABLE IV.

STRENGTH OF MANILA HOISTING ROPE.

Circumference of Rope. Inches.	Working Stress.
3	350
$3\frac{1}{2}$	500
4	650
$4\frac{1}{2}$	800
5	1,000

Under ordinary conditions, with a working stress not exceeding that given in the table, a manila hoisting rope will last until a total load of about 6,000 tons has been hoisted with it. Under exceptionally favorable circumstances, where the rope runs over large sheaves, a manila rope has hoisted 20,000 tons. Manila ropes for hoisting purposes should not be spliced, as it is difficult to make a splice that will not pull out while running over the sheaves,

and the increased wear to be obtained from splicing a broken hoisting rope is usually very small.

28. Manila rope for power transmission is made from ½ inch up to 2 inches in diameter. The greatest horsepower can be transmitted by a rope when the rope speed is about 5,000 feet per minute. When run at greater speed, the effect of the centrifugal force in the rope rapidly decreases the horsepower that can be transmitted. The size of the pulley has an important bearing on the wear of the rope; in general, it should not be smaller than 40 times the diameter of the rope, and as much larger as it can conveniently be made.

WIRE ROPES.

29. Wire rope is made of iron and steel wire. It is stronger than hemp rope, and, to carry the same load, is of smaller diameter.

In substituting steel for iron rope, the object in view should be to gain an increase of wear from the rope, rather than to reduce the size. A steel rope to be serviceable should be of the best obtainable quality, because ropes made from low grades of steel are inferior to good iron ropes.

30. Rules for the strength of wire ropes:

Let W = maximum of working load in pounds;
C = circumference of rope in inches.

Rule 4.—*The maximum working load in pounds that should be allowed on any iron-wire rope is equal to the square of the circumference of the rope in inches, multiplied by 600.*

Or, $$W = 600\,C^2.$$

EXAMPLE.—What is the maximum load in pounds that should be carried by an iron rope whose circumference is 4½ inches?

SOLUTION.—Applying the rule just given, we have

$$W = 600 \times 4.5^2 = 12{,}150 \text{ lb.} \quad \text{Ans.}$$

Rule 5.—*The circumference of any iron-wire rope in inches is equal to the square root of the maximum working load in pounds, multiplied by .0408.*

Or, $$C = .0408\sqrt{W}.$$

EXAMPLE.—A maximum working load of 12,150 pounds is to be carried by an iron-wire rope; what should be the minimum circumference of the rope?

SOLUTION.—Applying rule **5**, we have

$$C = .0408\sqrt{12{,}150} = 4\tfrac{1}{2} \text{ in.} \quad \text{Ans.}$$

Rule 6.—*The above rules and formulas are also made applicable when computing the safe strength of steel-wire rope by substituting the constant 1,000 for the constant 600, and .0316 for .0408.*

EXAMPLE.—What is the maximum load in pounds that should be carried by a steel-wire rope, the circumference of which is $4\frac{1}{2}$ inches?

SOLUTION.—Applying rule **4**, we have

$$W = 1{,}000 \times 4.5^2 = 20{,}250 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE.—A maximum working load of 10,485 pounds is to be carried by a steel-wire rope; what should be the minimum circumference of the rope?

SOLUTION.—Applying rule **5**, we have

$$C = .0316\sqrt{10{,}485} = 3.24 \text{ in.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

1. What should be the diameter of a machinery-steel piston rod of a steam engine to resist tension, if the piston is 19 inches in diameter and the pressure is 85 pounds per square inch? Ans. $2\frac{1}{4}$ in., nearly.

2. What safe load will a cast-iron bar of rectangular cross-section $7\frac{1}{2}$ in. by $3\frac{1}{2}$ in. support if subjected to shocks? The bar is in tension. Ans. 42,000 lb.

3. What is the stress per square inch on a piece of timber 8 inches square that is subjected to a steady pull of 60,000 pounds? Ans. 937.5 lb. per sq. in.

4. What should be the allowable working load for a steel-wire rope whose circumference is $3\frac{3}{4}$ inches? Ans. 14,062.5 lb.

5. What should be the circumference of an iron-wire rope to support a load of 20,000 pounds? Ans. $5\frac{1}{4}$ in., nearly.

CRUSHING STRENGTH OF MATERIALS.

31. The **crushing strength** of any material is the resistance offered by its fibers to being pushed together.

32. To obtain only compression, the length of a rod should not be more than five times greater than its least diameter or its least thickness when it is a rectangular rod. Such a rod is called a **short column.**

If a bar is long compared with its cross dimensions, the load, if sufficiently great, will cause it to bend sidewise under the compressive force, and we have, then, not only compression, but compression compounded with bending.

33. When a bar or rod is longer than five times its least diameter or least thickness, it is called a **long column,** or **long pillar.**

34. The shape of the *ends* of a column has great influence on its strength. In Fig. 5 are shown three columns with differently shaped ends.

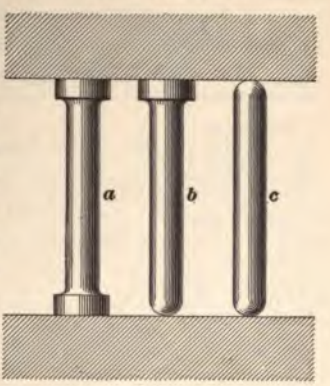


FIG. 5.

It has been proved by the aid of higher mathematics that, theoretically, a pillar, as *a*, having flat or fixed ends, is 4 times as strong as one that has round or movable ends, as the pillar *c*, and $1\frac{7}{9}$ times as strong as one having one flat and one round end, as the pillar *b*; *b* is thus $2\frac{1}{4}$ times as strong as *c*. It has also been found that if three pillars *a*, *b*, *c*, which have the same cross-section, are to carry the same load and be of equal strength, their lengths must be as the numbers 2, $1\frac{1}{2}$, and 1, respectively.

In practice, however, the ends of the pillars *b* and *c* are not generally made as shown by the figure, but have holes at their ends into which pins are fitted that are fastened to some other piece. In such cases, it has been found that *a* is

2 times as strong as c and that b is $1\frac{1}{2}$ times as strong as c. That is, in actual practice, a column fixed as at c is really $\frac{1}{2}$ as strong as one fixed as at a, instead of being only $\frac{1}{4}$ as strong, as given above.

An example of a column fixed at both ends is the piston rod of a steam engine. The valve stem of a slide-valve engine may be considered as an example of a column fixed at one end and movable at the other. The connecting-rod of a steam engine is a good example of a column having two movable ends.

35. Average values of the crushing strength of various materials used in engineering are given in the table below.

TABLE V.

CRUSHING STRENGTH OF MATERIALS.

Material.	Crushing Strength. Pounds per Sq. In.	Safe Loads. Pounds.		
		Sudden.	Gradual.	Steady.
Cast iron	80,000	7,000	11,000	14,000
Wrought iron	45,000	3,500	4,500	6,000
Machinery steel	60,000	4,000	6,000	8,000
Cast brass	9,000	800	1,200	1,500
Timber, dry	7,000	600	850	1,100
Timber, wet	3,000	250	380	500
Stone	6,000	....	....	200
Brick	2,000	....	....	100

STRENGTH OF COLUMNS.

36. In practice, columns subjected to a compressive stress are made of cast iron, wrought iron, steel, or timber. For short columns, the area, etc. can be calculated by the same rules that have been given for tensile stresses, substituting the working loads per square inch given in the above table.

37. The **safe working load on long columns** is given by the following rule, which is applicable to columns the length of which does not exceed 40 times their least diameter or their least thickness when rectangular. The rule given applies to columns uniform throughout their length. The rules for tapering columns involve considerable mathematical knowledge and are so rarely used in practice that they will not be given here.

Let C = safe crushing load, as given in the table;
S = sectional area in square inches;
L = length of column in inches;
d = least thickness of rectangular column, or diameter of round column, or dimension indicated by the arrowheads in the following table, in inches;
W = safe working load in pounds;
A = area of the two flanges in square inches;
B = area of the web in square inches;
a = constant corresponding to the cross-section of the column, as given in Tables VI, VII, and VIII.

Rule 7.—*The safe working load of a long column, in pounds, is equal to the safe working load corresponding to the kind of stress given in Table V multiplied by the sectional area of the column, in square inches, and the product divided by 1 plus the quotient obtained by dividing the square of the length of the pillar in inches by the square of the diameter (or least thickness, if rectangular) multiplied by the value of a.*

Or,
$$W = \frac{C S}{1 + \frac{L^2}{a d^2}}.$$

38. When using this formula, first obtain the value of C from Table V. Next, calculate the area of the cross-section of the pillar. Then find the value of a from one of the last three tables. Finally, be sure that the length of the column has been reduced to inches before substituting in the formula.

TABLE VI.

CONSTANTS FOR WROUGHT-IRON AND STRUCTURAL-STEEL PILLARS.

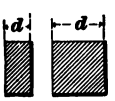
Cross-Section of Pillar.	When Both Ends of the Pillar Are Flat or Fixed.	When One End of the Pillar Is Flat or Fixed and the Other Round or Movable.	When Both Ends of the Pillar Are Round or Movable.
Round.	2,250	1,500	1,125
 Square or Rectangle.	3,000	2,000	1,500
Thin Square Tube.	6,000	4,000	3,000
Thin Round Tube.	4,500	3,000	2,250
Angle With Equal Sides.	1,500	1,000	750
Cross With Equal Arms.	1,500	1,000	750
I Beam.	$3{,}000 \times \frac{A}{A+B}$	$2{,}000 \times \frac{A}{A+B}$	$1{,}500 \times \frac{A}{A+B}$

TABLE VII.

CONSTANTS FOR CAST-IRON PILLARS.

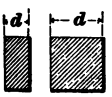
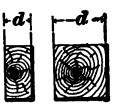
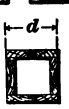
Cross-Section of Pillar.	When Both Ends of the Pillar Are Flat or Fixed.	When One End of the Pillar Is Flat or Fixed and the Other Round or Movable.	When Both Ends of the Pillar Are Round or Movable.
Round.	281.25	187.5	140.625
 Square or Rectangle.	375.00	250.0	187.500
Thin Square Tube.	750.00	500.0	375.000
Thin Round Tube.	562.50	375.0	281.250
Angle With Equal Sides.	187.50	125.0	93.750
Cross With Equal Arms.	187.50	125.0	93.750
I Beam.	$375 \times \frac{A}{A+B}$	$250 \times \frac{A}{A+B}$	$125 \times \frac{A}{A+B}$

TABLE VIII.

CONSTANTS FOR WOODEN PILLARS.

Cross-Section of Pillar.	When Both Ends of the Pillar Are Flat or Fixed.	When One End of the Pillar Is Flat or Fixed and the Other Round or Movable.	When Both Ends of the Pillar Are Round or Movable.
Round.	187.5	125.00	93.75
 Square or Rectangle.	250.0	166.66	125.00
 Hollow Square Made of Boards.	500.0	333.33	250.00

To find the proper value of a in any example, first turn to the table dealing with the material in question, and find the figure corresponding to the given cross-section; in the horizontal line containing this are three numbers corresponding to the different conditions of the ends of the column. From these numbers select the one corresponding to the given conditions of the column to be calculated, and this will be the required value of a.

NOTE.—If the length of the pillar is given in feet, be sure to reduce it to inches before substituting in the formula.

39. Rule **7** cannot be transformed to determine directly the diameter or area of a column of a given cross-section required to sustain a stated load. An area and a corresponding value of d must be assumed and the safe load

corresponding to these values calculated. If the safe load is smaller than the load to be sustained, a larger area and value of d must be chosen. If the calculated safe load is larger, a somewhat smaller area and value of d may be tried.

When applying the rule given to a column subject to wear, as, for instance, the piston rod of an engine, the diameter of the rod should be increased somewhat over that given by calculation in order to allow for subsequent truing up. This allowance may be from $\frac{1}{8}$ to $\frac{1}{4}$ of an inch, according to the judgment of the designer.

40. -A rough-and-ready rule for the size of a piston rod is to make it one-sixth the diameter of the cylinder of a simple engine. This rule of thumb allows us to choose a value of S and d that will answer very well indeed for the first trial.

ILLUSTRATION.—What size machinery-steel piston rod is required for a simple engine having a piston 36 inches in diameter to carry a steam pressure (gauge) of 100 pounds per square inch; the rod is 70 inches long?

According to the rule given, an approximate size of the piston rod will be $36 \div 6 = 6$ inches. The area corresponding to this is $6^2 \times .7854 = 28.27$ square inches. From Table VI, $a = 2{,}250$. The piston rod being subjected to suddenly applied stresses, by Table V, the allowable safe working load per square inch is 4,000 pounds. The working load on the piston rod is $36^2 \times .7854 \times 100 = 101{,}787.8$ pounds. Applying rule **7** and substituting values, we get

$$W = \frac{4{,}000 \times 28.27}{1 + \frac{70^2}{2{,}250 \times 6^2}} = 106{,}629 \text{ pounds},$$

which is a safe load on a 6-inch piston rod under the given conditions. This tends to show that the rod is rather large.

Now try a $5\frac{7}{8}$-inch rod. The area of the rod is $5\frac{7}{8}^2 \times .7854 = 27.1$ square inches, nearly.

Applying rule **7** again and substituting the new values of S and d, we get

$$W = \frac{4{,}000 \times 27.1}{1 + \frac{70^2}{2{,}250 \times 5\frac{7}{8}^2}} = 102{,}264 \text{ pounds, nearly.}$$

It is thus seen that a 5¾-inch rod is just about right. Most designers would allow ¼ inch for truing up, thus making the rod 6 inches in diameter.

In this particular case the size of rod found by calculation agrees with that given by the rough-and-ready rule. This cannot be in every case, however, as will readily be seen if a steam pressure of 150 pounds be substituted for 100 pounds and the rod be calculated for this pressure. It will then be found that a rod larger than 6 inches will be required.

41. In calculating the size of piston rods in actual work, it is good practice to assume at least a steam pressure of 100 pounds for the calculation, even though it is intended to carry a lower pressure. An extra margin of strength is thus provided which may be needed in the future, as cases are very frequent where engines designed for 75 pounds have ultimately been run at 100 pounds pressure.

42. In engineers' examinations, candidates for license are often asked to calculate the size of the piston rod for a given engine. A rule that is easily remembered is the following, which assumes the piston rod to be a short column and makes allowance for failure by bending by reducing the allowable working stress given in the table by 10 per cent., thus making the safe working stress for steel piston rods 3,600 pounds per square inch and, say, 3,100 pounds for iron.

This rule will give fairly good results within the limits of ordinary practice and will usually satisfy an examiner. For an actual design, however, rule **7**, which is more rational, is preferable.

Let S = area of piston rod;

l = load in pounds on the rod = area of piston $\times$ steam pressure;

c = safe working load in pounds.

Rule 8.—*Multiply the area of the piston by the steam pressure and divide the product by the safe working load corresponding to the material. The quotient will be the required area.*

Or, $$S = \frac{l}{c}.$$

EXAMPLE.—What size steel piston rod is required for a 36-inch cylinder with a steam pressure of 100 pounds?

SOLUTION.—The safe working load to be used in applying rule 8 is 3,600 pounds.

Then, $$S = \frac{36^2 \times .7854 \times 100}{3{,}600} = 28.27 \text{ square inches.}$$

The corresponding diameter is $\sqrt{\frac{28.27}{.7854}} = 6$ in. Ans.

EXAMPLES FOR PRACTICE.

1. A round wrought-iron column 4 inches in diameter and 60 inches long is subjected to a steady load. The column is fixed at one end and movable at the other. What load will it sustain?

Ans. 65,564 lb., nearly.

2. A solid machinery-steel column with both ends hinged is 4½ inches in diameter and 10 feet long. It is subjected to a suddenly applied stress. What is the safe working load? Ans. 38,979 lb.

3. A rectangular wooden column is 14 feet long. One end is fixed and one end is movable. If the cross-section is 12 in. × 8 in., what is its safe load for a steady stress? Ans. 28,963 lb.

TRANSVERSE STRENGTH OF MATERIALS.

43. The **transverse strength** of any material is the resistance offered by its fibers to being broken by bending. As, for example, when a beam, bar, rod, etc. which is supported at its ends is broken by a force applied between the supports.

The transverse strength of any beam, bar, rod, etc. is proportional to the product of the square of its depth

multiplied by its width; consequently, it is more economical to increase the depth than the width.

44. The safe load that can be carried by beams, bars, rods, etc. of uniform cross-section depends on the material, the manner in which the beam is supported and loaded, and on the shape of the cross-section. A beam that is rigidly fixed at one end and free at the other, when subjected to a transverse stress, is called a **cantilever.**

45. The safe working loads on beams supported and loaded in different ways can be found by applying the corresponding formula given in Table IX. In the formulas given, W = the safe load in pounds, and l = length of beam in inches to be taken as shown in the illustrations of Table IX. To apply the formulas to a beam, the values of S must be taken from Table X; and the value of R is to be computed by the formulas given in Table XI. It may then be substituted. Attention is called to the fact that it is absolutely necessary to reduce the length of the beam to inches; furthermore, the load W on a uniformly loaded beam is the total load for the length l, and not the load per unit of length.

In the table giving the value of R, the letter A denotes the whole area of the section in square inches, that is, taking the hollow cylinder, for example, it denotes the area corresponding to its outside diameter. The letter a stands for the area of the hollow part of the section. In the formulas where A is applied to a solid section, it stands for the area of the section.

The values of the constant S have been determined from practical experience and are safe, conservative values for the conditions stated.

EXAMPLE 1.—A cast-iron, solid, rectangular beam 8 inches deep and 4 inches wide rests upon two supports 8 feet apart. What steady load will it safely support when the load is applied at the middle?

SOLUTION.—According to Table XI, the formula to be used is $W = \frac{4SR}{l}$. For a steady stress on cast iron, Table X gives 7,500 as

TABLE IX.

FORMULAS FOR STRENGTH OF BEAMS.

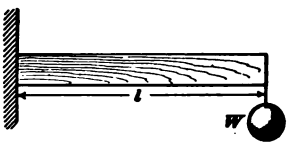
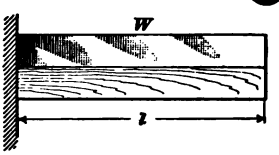
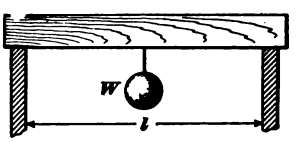
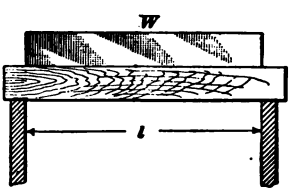
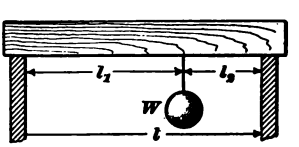
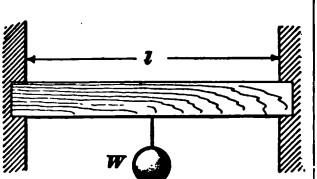
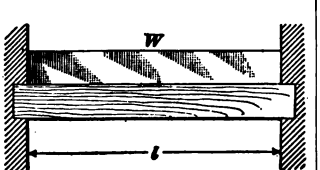
Manner of Supporting Beams.	Remarks.	Formula for Safe Load. Pounds.
	Cantilever load W at free end.	$W = \frac{S R}{l}$
	Cantilever, uniformly loaded. W = total load.	$W = \frac{2 S R}{l}$
	Simple beam resting on two supports, load W at middle.	$W = \frac{4 S R}{l}$
	Simple beam resting on two supports, uniformly loaded. W = total load on length l.	$W = \frac{8 S R}{l}$
	Simple beam resting on two supports, single load W not in the middle.	$W = \frac{S R}{\frac{l_1 l_2}{l}}$
	Beam rigidly fixed at both ends, load W in middle.	$W = \frac{8 S R}{l}$
	Beam rigidly fixed at both ends, uniformly loaded. W = total load on length l.	$W = \frac{12 S R}{l}$

TABLE X.

VALUES OF S.

Material.	Nature of Load.		
	Suddenly Applied.	Gradually Applied.	Steady.
Cast iron	2,250	3,000	7,500
Wrought iron	4,000	6,000	13,700
Structural steel	5,000	7,500	16,800
Brass	1,100	1,500	3,600
White pine	320	480	960
Yellow pine	500	730	1,460
Hemlock	240	360	720
Oak	400	600	1,200

the value of S. For a solid rectangular beam, Table XI gives $R = \frac{b h^2}{6}$. Substituting the width and depth of the beam in this last formula, we get $R = \frac{4 \times 8^2}{6} = 42.67$, nearly. The length of the beam in inches is $8 \times 12 = 96$ inches. Substituting all the values in the formula for the safe working load, we get

$$W = \frac{4 \times 7{,}500 \times 42.67}{96} = 13{,}334 \text{ lb., nearly.} \quad \text{Ans.}$$

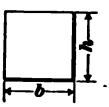
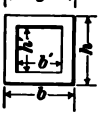
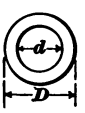
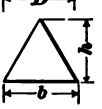
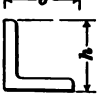
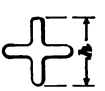
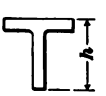
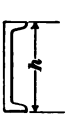
EXAMPLE 2.—An I beam having a depth of 10 inches and an area of section of 7.5 square inches is to be used as a cantilever to bear a varying load that is to be uniformly distributed. The beam is made of steel and is 100 inches long. What safe load will it carry?

SOLUTION.—From Table IX, it appears that the proper formula to be used is $W = \frac{2 S R}{l}$. According to Table X, the value of S for steel for a gradually applied load is 7,500. By Table XI for an I beam, $R = \frac{A h}{3.33}$. Substituting values in this last formula, we get $R = \frac{7.5 \times 10}{3.33} = 22.52$, nearly. Substituting values in the formula for the safe working load, we get

$$W = \frac{2 \times 7{,}500 \times 22.52}{100} = 3{,}378 \text{ lb.} \quad \text{Ans.}$$

TABLE XI.

VALUES OF *R*.

Section.	*R*.
	$\frac{b\,h^2}{6}$
	$\frac{b\,h^3 - b'\,h'^3}{6\,h}$
	$\frac{A\,D}{8}$
	$\frac{A\,D^2 - a\,d^2}{8\,D}$
	$\frac{b\,h^2}{24}$
	$\frac{A\,h}{7.2}$
	$\frac{A\,h}{6.5}$
	$\frac{A\,h}{9.5}$
	$\frac{A\,h}{8}$
	$\frac{A\,h}{3.33}$
	$\frac{A\,h}{3.67}$

46. While the formulas given in Table IX will allow the safe load on a given beam to be calculated, they cannot readily be transformed to give directly the size of beam required under given conditions to carry a given load.

In practice, when it is required to determine the size of beam, the length between supports is known. After the shape of cross-section of the beam has been chosen, dimensions for the beam may be assumed and its safe load for the given dimensions calculated. If the load thus found falls below the load the beam is to carry, larger dimensions must be chosen. In case of rolled sections, such as angle irons, **T** irons, channels, or **I** beams, catalogues of manufacturers should be consulted and standard sizes chosen. These catalogues usually contain the area of the section for different weights and dimensions of rolled sections.

47. The values of R given in Table XI *apply only when the dimension marked h is vertical.* If the beam is placed in any other position, R will have a different value, which can only be calculated by the aid of higher mathematics. Beams rigidly fixed at both ends are rarely met with in practice, and it is safer to assume that the beam merely rests on two supports.

EXAMPLES FOR PRACTICE.

1. What weight will a yellow-pine rectangular beam carry safely under a steady load when used as a cantilever and uniformly loaded? The beam is 9 feet 6 inches long, 16 inches deep, and 4 inches wide.
Ans. 4,371 lb.

2. What weight would the beam in example 1 carry safely if it rested on two supports? Ans. 17,484 lb.

3. What weight can safely be carried at the end of a wrought-iron round cantilever 3 inches in diameter when the load is suddenly applied 4 feet 2 inches from the support of the cantilever? Ans. 212 lb.

4. What steady load can be carried safely by the same beam given in the preceding example? Ans. 726 lb.

SHEARING, OR CUTTING, STRENGTH OF MATERIALS.

48. The **shearing strength** of any material is the resistance offered by its fibers to being cut in two. Thus, the pressure of the cutting edges of an ordinary shearing machine, Fig. 6, causes a shearing stress in the plane *a b*. The unit shearing force may be found by dividing the force *P* by the area of the plane *a b*.

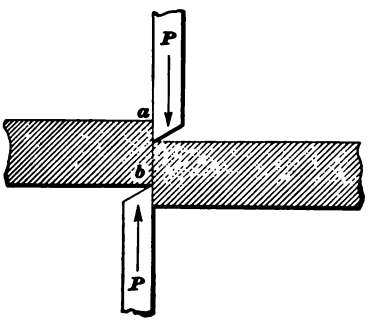


FIG. 6.

Fig. 7 shows a piece in double shear; here the central piece *c d* is forced out while the ends remain on their supports *M* and *N*.

The shearing strength of any body is directly proportional to its area.

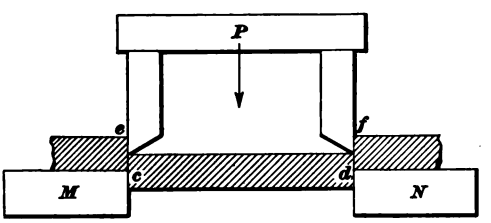


FIG. 7.

49. In Table XII. are given the greatest and the safe shearing strengths per square inch of different kinds of materials.

50. Rules for Shearing.—In general, the force required to shear a piece of material in double shear will be *twice* that required for single shear. This does not apply to all cases, however; a notable exception are rivets in double shear. Experiments have determined that the force required to shear both iron and steel rivets in double shear averages about 1.85 times that required for single shear.

TABLE XII.

SHEARING STRENGTH OF MATERIALS.

Material.	Average Ultimate Shearing Strength.		Safe Loads.					
			Sudden Load.		Gradual Load.		Steady Load.	
	Parallel to Grain.	Across Grain.	Parallel to Grain.	Across Grain.	Parallel to Grain.	Across Grain.	Parallel to Grain.	Across Grain.
Cast iron	25,000	....	2,500	...	3,700	...	5,000	...
Wrought-iron rivets	38,000	....	3,700	...	5,600	...	7,500	...
Soft-steel rivets	44,000	....	4,400	...	6,600	...	8,800	...
Ordinary wrought iron	44,000	....	4,400	...	6,600	...	8,800	...
Structural steel	52,000	....	5,000	...	7,500	...	10,000	...
White pine	300	2,500	30	250	45	375	60	500
Yellow pine	400	4,500	40	450	60	675	80	900
Hemlock	250	2,500	25	250	38	375	50	500
Spruce	300	3,000	30	300	45	450	60	600
Oak	600	5,000	60	500	90	750	120	1,000

Let a = area of cross-section in square inches;
S = shearing stress as given in table;
W = load in pounds.

Then, to find the safe load or the load that will shear the material:

Rule 9.—*Multiply the area of the section by the shearing stress.*

Or, $$W = a\,S.$$

In applying this rule, it should be remembered that when the safe load the material can bear is required, the value of S is to be taken from the column headed "Safe Loads," selecting the safe load corresponding to the nature of the load. When it is desired to find the load at which the material will fail, the value of S is to be taken from the column headed "Ultimate Shearing Strength."

EXAMPLE.—If the beam in double shear shown in Fig. 7 is rectangular and measures 4 in. × 2 in., what steady safe load would you allow if the beam were made of structural steel?

SOLUTION.—According to the table, a safe load of 10,000 pounds per square inch of section may be allowed. Applying rule **9** and multiplying by 2, since the beam is in double shear, we get

$$W = 4 \times 2 \times 10{,}000 \times 2 = 160{,}000 \text{ lb.} \quad \text{Ans.}$$

51. To find the area required for a material subjected to a shearing stress:

Rule 10.—*Divide the load by the shearing stress.*

Or, $$a = \frac{W}{S}.$$

In applying rule **10,** it should be remembered that for double shear the result is to be divided by 2 to obtain the area of the beam.

EXAMPLE.—A white pine beam is subjected to a suddenly applied shearing stress by a load of 4,000 pounds, the beam being in double shear and the load being applied across the grain. What should be the area of the beam to bear this stress safely?

Pulleys from which considerable power is to be taken should always be placed as close to a bearing as possible.

58. The diameters of the different lengths of shafts composing a line of shafting may be proportional to the quantity of power delivered by each respective length. In this connection it is to be observed that the positions of the various pulleys in reference to the bearings must be taken into consideration in deciding upon the size of a shaft. Suppose, for example, that a piece of shafting delivers a certain amount of power; then, it is obvious that the shaft will deflect or bend less if the pulley transmitting that power be placed close to a hanger or bearing than if it be placed midway between the two hangers or bearings. It is impossible to give any rule for the proper distance of bearings that could be used universally, as in some cases the requirements demand that the bearings be nearer than in others.

Wherever possible it is advisable to have the main line of shafting run through the center of the room, or at least far enough from either wall to allow countershafts to be placed on either side of it. When this is done, power may be taken off the main shaft from either side by alternate pulleys, and the deflection caused in the main shaft in one direction by one pulley will be counteracted by the deflection caused in the opposite direction by the next pulley.

If the work done by a line of shafting is distributed quite equally along its entire length and the power can be applied near the middle, the strength of the shaft need be only half as great as would be required if the power were applied at one end.

59. *To compute the horsepower that can be transmitted by a shaft of any given diameter:*

Let D = diameter of shaft;
R = revolutions per minute;
H = horsepower transmitted;
C = constant given in Table XIV.

TABLE XIV.

CONSTANTS FOR LINE SHAFTING.

Material of Shaft.	No. of Pulleys Between Bearings.	Pulleys Between Bearings.
Steel or cold-rolled iron.....	65	85
Wrought iron.....	70	95
Cast iron.....	90	120

In the above table the bearings are supposed to be spaced so as to relieve the shaft of excessive bending; also, in the third vertical column, an average number and weight of pulleys and power given off is assumed.

60. Rules and Formulas.—In determining the above constants, allowance has been made to insure the stiffness as well as strength of the shaft. Cold-rolled iron is considerably stronger than ordinary turned wrought iron; the increased strength is due to the process of rolling, which seems to compress the metal and so make it denser, not merely skin deep, but practically throughout the whole diameter. We have, then, the following

Rule 11.—*The horsepower that a shaft will transmit equals the product of the cube of the diameter and the number of revolutions, divided by the value of C for the given material.*

Or,
$$H = \frac{D^3 \times R}{C}.$$

EXAMPLE.—What horsepower will a 3-inch wrought-iron shaft transmit which makes 100 revolutions per minute, there being no pulleys between bearings?

SOLUTION.—Applying rule 11 and substituting, we have

$$H = \frac{3 \times 3 \times 3 \times 100}{70} = 38.57 \text{ H. P.} \quad \text{Ans.}$$

If there were the usual amount of power taken off, as mentioned above, we should take $C = 95$.

Then, $$H = \frac{27 \times 100}{95} = 28.42 \text{ H. P.} \quad \text{Ans.}$$

61. To compute the number of revolutions a shaft must make to transmit a given horsepower:

Rule 12.—*The number of revolutions necessary for a given horsepower equals the product of the value of C for the given material and the number of horsepower, divided by the cube of the diameter.*

Or, $$R = \frac{C \times H}{D^3}.$$

EXAMPLE.—How many revolutions must a 3-inch wrought-iron shaft make per minute to transmit 28.42 horsepower, power being taken off at intervals between the bearings?

SOLUTION.—Applying the rule just given and substituting, we have

$$R = \frac{95 \times 28.42}{3 \times 3 \times 3} = 100 \text{ rev.} \quad \text{Ans.}$$

62. To compute the diameter of a shaft that will transmit a given horsepower, the number of revolutions the shaft makes per minute being given:

Rule 13.—*The diameter of a shaft equals the cube root of the quotient obtained by dividing the product of the value of C for the given material and the number of horsepower by the number of revolutions.*

Or, $$D = \sqrt[3]{\frac{C \times H}{R}}.$$

EXAMPLE.—What must be the diameter of a wrought-iron shaft to transmit 38.57 horsepower, the shaft to make 100 revolutions per minute, no power being taken off between bearings?

SOLUTION.—By rule **13**, we have

$$D = \sqrt[3]{\frac{70 \times 38.57}{100}} = \sqrt[3]{27} = 3 \text{ in.} \quad \text{Ans.}$$

63. As the speed of shafting is used as a multiplier in the calculations of the horsepower of shafts, it is readily seen that a shaft having a given diameter will transmit more

power in proportion as its speed is increased. Thus, a shaft that is capable of transmitting 10 horsepower when making 100 revolutions per minute will transmit 20 horsepower when making 200 revolutions per minute. We may, therefore, say *the number of horsepower transmitted by a shaft is directly proportional to the number of revolutions.*

EXAMPLES FOR PRACTICE.

1. What horsepower will a 2½-inch wrought-iron shaft transmit when running at 110 revolutions per minute, it being used for transmission only? Ans. 24.55 H. P.

2. A 6-inch cast-iron shaft transmits 150 horsepower. How many revolutions per minute must it make, no power being taken off between bearings? Ans. 62½ R. P. M.

3. What should be the diameter of a wrought-iron shaft to transmit 100 horsepower at 150 revolutions per minute, power being taken off between bearings? Ans. 4 in., nearly.

4. The machines driven by a certain line of wrought-iron shafting take their power from various points between the bearings, and if all were working together at their full capacity, they would require 65 horsepower to drive them. What diameter should the shaft be if it runs at 150 revolutions per minute? Ans. 3½ in., nearly.

A SERIES

OF

QUESTIONS AND EXAMPLES

RELATING TO THE SUBJECTS
TREATED OF IN THIS VOLUME.

It will be noticed that the Examination Questions that follow have been divided into sections, which have been given the same numbers as the Instruction Papers to which they refer. No attempt should be made to answer any of the questions or to solve any of the examples until that portion of the text having the same section number as the section in which the questions or examples occur has been carefully studied.

ARITHMETIC.

(PART 1.)

EXAMINATION QUESTIONS.

(1) What is arithmetic?

(2) What is a number?

(3) What is the difference between a concrete number and an abstract number?

(4) Define notation and numeration.

(5) Write each of the following numbers in words: 980; 605; 28,284; 9,006,042; 850,317,002; 700,004.

(6) Represent in figures the following expressions:

Seven thousand six hundred. Eighty-one thousand four hundred two. Five million four thousand seven. One hundred eight million ten thousand one. Eighteen million six. Thirty thousand ten.

(7) What is the sum of 3,290, 504, 865,403, 2,074, 81, and 7? Ans. 871,359.

(8) 709 + 8,304,725 + 391 + 100,302 + 300 + 909 = what? Ans. 8,407,336.

(9) During a 12-hour test of a steam engine the counter showed the number of revolutions per hour to have been as follows: 12,600, 12,444, 12,467, 12,528, 12,468, 12,590, 12,610, 12,589, 12,576, 12,558, 12,546, 12,532. How many revolutions were made during the test? Ans. 150,508 rev.

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(10) Find the difference between the following: (*a*) 50,962 and 3,338; (*b*) 10,001 and 15,339.

Ans. { (*a*) 47,624. (*b*) 5,338.

(11) (*a*) 70,968 − 32,975 = ? (*b*) 100,000 − 98,735 = ?

Ans. { (*a*) 37,993. (*b*) 1,265.

(12) On a certain morning 7,240 gallons of water were drawn from an engine-room tank and 4,780 gallons were pumped in. In the afternoon 7,633 gallons were drawn out and 8,675 gallons pumped in. How many gallons remained in the tank at night, if it contained 3,040 gallons at the beginning of the day? Ans. 1,622 gal.

(13) Find the product of the following: (*a*) 526,387 × 7; (*b*) 700,298 × 17; (*c*) 217 × 103 × 67.

Ans. { (*a*) 3,684,709. (*b*) 11,905,066. (*c*) 1,497,517.

(14) If your watch ticks once every second, how many times will it tick in one week? Ans. 604,800.

(15) An engine and boiler in a manufactory are worth $3,246. The building is worth three times as much, plus $1,200, and the tools are worth twice as much as the building, plus $1,875. (*a*) What is the value of the building and tools? (*b*) What is the value of the whole plant?

Ans. { (*a*) $34,689. (*b*) $37,935.

(16) Divide the following:

(*a*) 962,842 by 84; (*b*) 39,728 by 63; (*c*) 29,714 by 108; (*d*) 406,089 by 135.

Ans. { (*a*) 11,462.4047. (*b*) 630.603. (*c*) 275.1296. (*d*) 3,008.0666.

(17) Suppose that in 1 hour 10 pounds of coal are burned per square foot of grate area in a certain boiler,

and that 9 pounds of water are evaporated per pound of coal burned. If the grate area is 30 square feet, how many pounds of water would be evaporated in a day of 10 hours?

Ans. 27,000 lb.

(18) If a mechanic receives $1,500 a year for his labor and his expenses are $968 per year, in what time can he save enough to buy 28 acres of land at $133 an acre?

Ans. 7 years.

(19) Solve the following by cancelation:

(*a*) $(72 \times 48 \times 28 \times 5) \div (84 \times 15 \times 7 \times 6)$.

(*b*) $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20)$.

Ans. (*a*) 9⅓. (*b*) 32.

(20) A freight train ran 365 miles in one week, and 3 times as far, lacking 246 miles, the next week; how far did it run the second week? Ans. 849 miles.

(21) If the driving wheel of a locomotive is 16 feet in circumference, how many revolutions will it make in going from Philadelphia to Pittsburg, the distance between them being 354 miles, and there being 5,280 feet in 1 mile?

Ans. 116,820 rev.

ARITHMETIC.

(PART 2.)

EXAMINATION QUESTIONS.

(1) What is a fraction?

(2) What are the terms of a fraction?

(3) What does the denominator show?

(4) What does the numerator show?

(5) Is $\frac{3}{8}$ a proper or an improper fraction, and why?

(6) Write three mixed numbers.

(7) Reduce the following fractions to their lowest terms: $\frac{4}{8}$, $\frac{4}{16}$, $\frac{8}{32}$, $\frac{32}{64}$. Ans. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{2}$.

(8) Reduce 6 to an improper fraction whose denominator is 4. Ans. $\frac{24}{4}$.

(9) Reduce $7\frac{7}{8}$, $13\frac{5}{16}$, and $10\frac{3}{4}$ to improper fractions.
Ans. $\frac{63}{8}$, $\frac{213}{16}$, $\frac{43}{4}$.

(10) How many feet does the piston of a steam engine pass over in a week of 6 days, running $8\frac{1}{2}$ hours a day, if the length of the stroke of the engine is $1\frac{1}{2}$ feet and the number of revolutions per minute is 160?
Ans. 1,468,800 ft.

(11) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = ?$ Ans. 1.

(12) $\frac{1}{4} + \frac{3}{8} + \frac{5}{16} = ?$ Ans. $\frac{15}{16}$.

(13) $42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Ans. $83\frac{1}{16}$.

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(14) An iron plate is divided into four sections; the first contains $29\frac{3}{4}$ square inches; the second, $50\frac{5}{8}$ square inches; the third, 41 square inches; and the fourth, $69\frac{3}{16}$ square inches. How many square inches are in the plate?

Ans. $190\frac{9}{16}$ sq. in.

(15) What is the difference between $\frac{7}{8}$ and $\frac{7}{16}$? 13 and $7\frac{7}{16}$? $312\frac{9}{16}$ and $229\frac{5}{32}$? Ans. $\frac{7}{16}$; $5\frac{9}{16}$; $83\frac{13}{32}$.

(16) Solve the following:

(*a*) $35 \div \frac{5}{16}$; (*b*) $\frac{9}{16} \div 3$; (*c*) $1\frac{7}{8} \div 9$; (*d*) $11\frac{13}{64} \div \frac{7}{16}$; (*e*) $15\frac{3}{4} \div 4\frac{3}{5}$.

Ans. (*a*) 112. (*b*) $\frac{3}{16}$. (*c*) $1\frac{7}{16}$. (*d*) $4\frac{1}{2}\frac{1}{8}$. (*e*) $3\frac{3}{5}$.

(17) The numerator of a fraction is 28 and the value of the fraction is $\frac{7}{8}$; what is the denominator? Ans. 32.

(18) Four bolts are required, $2\frac{1}{2}$, $6\frac{7}{8}$, $3\frac{1}{16}$, and 4 inches long. How long a piece of iron will be required from which to cut them, allowing $\frac{7}{16}$ of an inch to each bolt for cutting off and finishing? Ans. $18\frac{3}{16}$ in.

ARITHMETIC.

(PART 3.)

EXAMINATION QUESTIONS.

(1) State the difference between a common fraction and a decimal fraction.

(2) Reduce the following fractions to equivalent decimals: $\frac{1}{2}$, $\frac{7}{8}$, $\frac{5}{32}$, $\frac{65}{100}$, and $\frac{125}{1000}$.

Ans. .5. .875. .15625. .65. .125.

(3) Write out in words the following numbers: .08, .131, .0001, .000027, .0108, and 93.0101.

(4) What is the sum of .125, .7, .089, .4005, .9, and .000027? Ans. 2.214527.

(5) Add 17 thousandths, 2 tenths, and 47 millionths.

Ans. .217047.

(6) Work out the following examples:

(*a*) 709.63 − .8514; (*b*) 81.963 − 1.7; (*c*) 18 − .18; (*d*) 1 − .001; (*e*) 872.1 − (.8721 + .008); (*f*) (5.028 + .0073) − (6.704 − 2.38).

Ans. (*a*) 708.7786. (*b*) 80.263. (*c*) 17.82. (*d*) .999. (*e*) 871.2199. (*f*) .7113.

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(7) The cost of the coal consumed under a nest of steam boilers during a week's run was as follows: Monday, \$15.83; Tuesday, \$14.70; Wednesday, \$14.28; Thursday, \$13.87; Friday, \$14.98; Saturday, \$12.65. What was the cost of the week's supply of coal? Ans. \$86.31.

(8) It is desired to increase the capacity of an electric-light plant to 1,500 horsepower by adding a new engine. If the indicated horsepower of the engines already in use is $482\frac{4}{5}$, $316\frac{1}{3}$, and $390\frac{3}{4}$, what power must the new engine develop? Ans. $310.11\frac{2}{3}$ H. P.

(9) The inside diameter of a 6-inch steam pipe is 6.06 inches, and the outside diameter is 6.62 inches. How thick is the pipe? Ans. .28 in.

(10) Find the products of the following expressions: (*a*) $.013 \times .107$; (*b*) $203 \times 2.03 \times .203$; (*c*) $(2.7 \times 31.85) \times (3.16 - .316)$; (*d*) $(107.8 + 6.541 - 31.96) \times 1.742$.

Ans. (*a*) .001391. (*b*) 83.65427. (*c*) 244.56978. (*d*) 143.507702.

(11) How many square feet of heating surface are in the tubes of a boiler having sixty 3-inch tubes, each $15\frac{1}{2}$ feet long, if the heating surface of each tube per foot in length is .728 square foot? Ans. 677.04 sq. ft.

(12) Find the values of the following expressions:

(*a*) $\frac{7}{\frac{3}{16}}$; (*b*) $\frac{1\frac{5}{32}}{\frac{5}{8}}$; (*c*) $\frac{1.25 \times 20 \times 3}{\frac{87 + 88}{459 + 32}}$.

Ans. (*a*) $37\frac{1}{3}$. (*b*) .75. (*c*) $210\frac{3}{4}$.

(13) The distance around a cylindrical boiler is 166.85 inches. If there are 72 rivets in one of the circular seams, find what the pitch (distance between the centers of any two rivets) of the rivets is. Ans. 2.317+ in.

(14) A keg of $\frac{7}{8}'' \times 2\frac{3}{4}''$ boiler rivets weighs 100 pounds and contains 133 rivets. What is the weight of each rivet? Ans. .75+ lb.

(15) How many inches in .875 of a foot? Ans. $10\frac{1}{2}$ in.

(16) What decimal part of a foot is $\frac{3}{16}$ inch?

Ans. .015625 ft.

(17) In a steam-engine test of an hour's duration, the horsepower developed was found to be as follows at 10-minute intervals: 48.63, 45.7, 46.32, 47.9, 48.74, 48.38, 48.59. What was the average horsepower? Ans. 47.75+ H. P.

ARITHMETIC.

(PART 4.)

EXAMINATION QUESTIONS.

(1) What is 25% of 8,428 lb.? Ans. 2,107 lb.

(2) What is $\frac{1}{2}$% of $35,000? Ans. $175.

(3) What per cent. of 50 is 2? Ans. 4%.

(4) What per cent. of 10 is 10? Ans. 100%.

(5) The coal consumption of a steam plant is 5,500 lb. per day when the condenser is not running, or an increase of 15% over the consumption when the condenser is used. How many pounds are used per day when the condenser is running? Ans. 4,782.61 lb.

(6) An engineer receives a salary of $950. He pays 24% of it for board, 12$\frac{1}{2}$% of it for clothing, and 17% of it for other expenses. How much of it does he save a year?
Ans. $441.75.

(7) If 37$\frac{1}{2}$% of a number is 961.38, what is the number?
Ans. 2,563.68.

(8) The speed of an engine running unloaded was 1$\frac{1}{2}$% greater than when running loaded. If it made 298 revolutions per minute with the load, what was its speed running unloaded? Ans. 302.47 rev. per min.

(9) Reduce 4 yd. 2 ft. 10 in. to inches. Ans. 178 in.

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(10) Reduce 3,722 in. to higher denominations.
Ans. 103 yd. 1 ft. 2 in

(11) Reduce 764,325 cu. in. to cubic yards.
Ans. 16 cu. yd. 10 cu. ft. 549 cu. in.

(12) A carload of coal weighed 16 T. 8 cwt. 75 lb. How many pounds did this amount to? Ans. 32,875 lb.

(13) Reduce 25,396 lb. to higher denominations.
Ans. 12 T. 13 cwt. 96 lb.

(14) What is the sum of 2 yd. 2 ft. 3 in., 4 yd. 1 ft. 9 in., and 2 ft. 7 in.? Ans. 8 yd. 7 in.

(15) From a barrel of machine oil is sold at one time 10 gal. 2 qt. 1 pt. and at another time 16 gal. 3 qt. How much remained? Ans. 4 gal. 1 pt.

(16) If 1 iron rail is 17 ft. 3 in. long, how long would 51 such rails be if placed end to end? Ans. 879 ft. 9 in.

(17) Multiply 3 qt. 1 pt. by 4.7. Ans. 32.9 pt.

(18) A main line shaft is composed of four lengths each 15 ft. 5 in. long; one length 14 ft. 8 in. long; and one length 8 ft. 10 in. long. There are six hangers spaced equally distant apart, one being placed at each extremity of the shaft, 8 in. from the ends. What is the distance between the hangers? Ans. 16 ft. $9\frac{1}{5}$ in.

(19) If the length of a boiler shell is 18 ft. $11\frac{1}{4}$ in., how many rivets should there be in one of the longitudinal seams if it is a single-riveted seam, supposing the rivets to be $1\frac{1}{4}$ in. between centers and the two end rivets to be $1\frac{1}{8}$ in. from each end of the boiler? Ans. 181 rivets.

ARITHMETIC.

(PART 5.)

EXAMINATION QUESTIONS.

(1) What is the square of 108? Ans. 11,664.

(2) What is the cube of 181.25?

Ans. 5,954,345.703125.

(3) Find the values of the following: (*a*) $.0133^3$; (*b*) 301.011^3; (*c*) $(\frac{1}{8})^3$; (*d*) $(3\frac{3}{4})^3$.

Ans. (*a*) .000002352637.
(*b*) 27,273,890.942264331.
(*c*) $\frac{1}{512}$.
(*d*) $52\frac{47}{64}$, or 52.734375.

NOTE.—In the answers to the following examples requiring the extraction of a root, a minus sign after a number indicates that the last digit is not quite as large as the number printed. Thus, 12,497— indicates that the number really is 12.496+, and that the 6 has been made a 7 because the next succeeding figure was 5 or greater. The answers to the examples requiring the extraction of cube root are exact to the last digit given. If the student is unable to get the same answer by means of the method described in Arts. **30-34**, he should then use the method described in Arts. **35** and **36**.

(4) Find the square root of the following: (*a*) 3,486,784.401; (*b*) 9,000,099.4009.

Ans. (*a*) 1,867.29+.
(*b*) 3,000.017—.

(5) Extract the cube root of .32768. Ans. .6894+.

(6) Extract the cube root of 2 to four decimal places.

Ans. 1.2599+.

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(7) Solve the following: (*a*) $\sqrt{123.21}$; (*b*) $\sqrt{114.921}$.

Ans. { (*a*) 11.1. (*b*) 10.72+.

(8) Solve the following: (*a*) $\sqrt[3]{.0065}$; (*b*) $\sqrt[3]{.000021}$.

Ans. { (*a*) .18663—. (*b*) .02759—.

Find the value of x in the following:

(9) $11.7 : 13 :: 20 : x$. Ans. 22.22+.

(10) (*a*) $20 + 7 : 10 + 8 :: 3 : x$; (*b*) $12^2 : 100^2 :: 4 : x$.

Ans. { (*a*) 2. (*b*) 277.7+.

(11) (*a*) $\frac{4}{x} = \frac{7}{21}$; (*b*) $\frac{x}{24} = \frac{8}{16}$; (*c*) $\frac{2}{10} = \frac{x}{100}$; (*d*) $\frac{15}{45} = \frac{60}{x}$; (*e*) $\frac{10}{150} = \frac{x}{600}$.

Ans. { (*a*) 12. (*b*) 12. (*c*) 20. (*d*) 180. (*e*) 40.

(12) If a piece of 2-inch shafting $3\frac{1}{2}$ ft. long weighs 37.45 lb., how much would a piece $6\frac{3}{4}$ ft. long weigh?

Ans. 72.225 lb.

(13) If a railway train runs 444 miles in 8 hr. 40 min., in what time can it run 1,060 miles at the same rate of speed?

Ans. 20 hr. 41.44 min.

(14) If a pump discharging 135 gal. per min. fills a tank in 38 min., how long would it take a pump discharging 85 gal. per min. to fill it? Ans. $60\frac{6}{17}$ min.

(15) The distances around the driving wheels of two locomotives are 12.56 ft. and 15.7 ft., respectively. How many times will the larger turn while the smaller turns 520 times?

Ans. 416 times.

(16) If a cistern 28 ft. long, 12 ft. wide, 10 ft. deep holds 798 bbl. of water, how many barrels of water will a cistern hold that is 20 ft. long, 17 ft. wide, and 6 ft. deep?

Ans. $484\frac{1}{2}$ bbl.

MENSURATION AND USE OF LETTERS IN FORMULAS.

EXAMINATION QUESTIONS.

$A = 5$ $\quad h = 200$

$B = 10$ $\quad x = 12$

$i = 3.5$ $\quad D = 120$

Work out the solutions to the following formulas, using the above values for the letters:

(1) $C = \dfrac{D - x}{B + i}$. Ans. $C = 8$.

(2) $Q = \dfrac{A\,h + D}{2\,x + 6} + D$. Ans. $Q = 157\frac{1}{3}$.

(3) $v = \sqrt{\dfrac{A\,D}{i\,B + 1.5}}$. Ans. $v = 4.05+$.

(4) $g = \dfrac{(B - A)^2 - \sqrt{h + 2\,B + A}}{A^3 - (1 + D)}$. Ans. $g = 2\frac{1}{2}$.

(5) If one of the angles formed by one straight line meeting another straight line equals 152° 3′, what is the other angle equal to? Ans. 27° 57′.

(6) Draw an obtuse angle, a right angle, and an acute angle. State the name of each angle by using letters to designate them.

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(7) Draw a rhombus and then draw a rectangle having the same area.

(8) A sheet of zinc measures $11\frac{1}{2}$ inches by $2\frac{1}{2}$ feet. How many square inches does it contain? Ans. 345 sq. in.

(9) How many boards 16 feet long and 5 inches wide would be required to lay a floor measuring 15 ft. × 24 ft.?
Ans. 54 boards.

(10) The accompanying figure shows the floor plan of an electric-light station. From the dimensions given, calculate the number of square feet of unoccupied floor space.
Ans. 2,059.08 sq. ft.

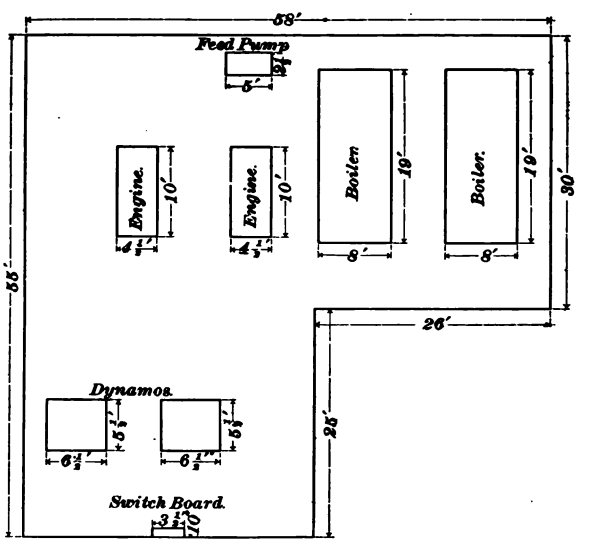

FIG. I.

(11) A triangle has three equal angles; what is it called?

(12) If a triangle has two equal angles, what kind of a triangle is it?

(13) In a triangle $A\ B\ C$, angle $A = 23°$ and $B = 32°\ 32'$; what does angle C equal? Ans. $C = 124°\ 28'$.

(14) In the figure, if $AD = 10$ inches, $AB = 24$ inches, and $BC = 13\frac{1}{2}$ inches, how long is DE, DE being parallel to BC? Ans. $DE = 5.625$ in.

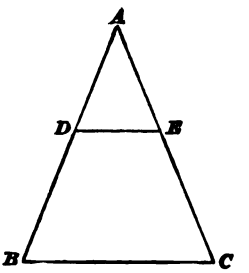

FIG. II.

(15) An engine room is 52 feet long and 39 feet wide. How many feet is it from one corner to a diagonally opposite one, measured in a straight line? Ans. 65 ft.

(16) It is required to make a miter-box in which to cut molding to fit around an octagon post. At what angle with the side of the box should the saw run? Ans. $67\frac{1}{2}°$.

(17) If the distance between two opposite corners of a hexagonal nut is 2 inches, what is the distance between two opposite sides? Ans. 1.732+ in.

(18) In the accompanying figure, if the distance BI is 6 inches and HK 18 inches, what is the diameter of the circle? Ans. 19.5 in.

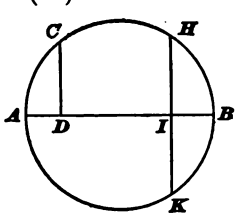

FIG. III.

(19) How many revolutions will a 72-inch locomotive driver make in going 1 mile? Ans. 280.112 revolutions.

(20) A pipe has an internal diameter of 6.06 inches; what is the area of a circle having this diameter? Ans. 28.8427 sq. in.

(21) How long must the arc of a circle be to contain 12°, supposing the radius of the circle to be 6 inches? Ans. 1.25664 in.

(22) What is the area of the sector of a circle 15 inches in diameter, the angle between the two radii forming the sector being $12\frac{1}{2}°$? Ans. 6.1359 sq. in.

(23) (*a*) What would be the length of the side of a square metal plate having an area of 103.8691 square inches?

(*b*) What would be the diameter of a round plate having this area? (*c*) How much shorter is the circumference of the round plate than the perimeter of the square plate?

Ans. { (*a*) 10.1916 in.
(*b*) $11\frac{1}{2}$ in.
(*c*) 4.638 in.

(24) Find the area in square feet of the entire surface of a hexagonal column 12 feet long, each edge of the ends of the column being 4 inches long. Ans. 24.5774 sq. ft.

(25) Find the cubical contents of the above column in cubic inches. Ans. 5,985.9648 cu. in.

(26) Compute the weight per foot of an iron boiler tube 4 inches outside diameter and 3.73 inches inside diameter, the weight of the iron being taken at .28 pound per cubic inch. Ans. $5\frac{1}{2}$ lb.

(27) The dimensions of a return-tubular boiler are as follows: Diameter, 60 inches; length between heads, 16 feet; outside diameter of tubes, $3\frac{1}{2}$ inches; number of tubes, 64; distance of mean water-line from top of boiler, 18 inches. (*a*) Compute the steam space of the boiler in cubic feet. (*b*) Determine the number of gallons of water that will be required to fill the boiler up to the mean water level.

Ans. { (*a*) 79.2 cu. ft.
(*b*) 1,246 gal., nearly.

(28) The length of the circumference of the base of a cone is 18.8496 inches and its slant height is 10 inches. Find the area of the entire surface of the cone.

Ans. 122.5224 sq. in.

(29) If the altitude of the above cone were 9 inches, what would be its volume? Ans. 84.8232 cu. in.

(30) A square vat is 11 feet deep, 15 feet square at the top, and 12 feet square at the bottom. How many gallons will it hold? Ans. 15,058.29 gal.

(31) How many pails of water would be required to fill the vat, the pail having the following dimensions: Depth,

11 inches; diameter at the top, 12 inches; diameter at the bottom, 9 inches? Ans. 3,627.28.

(32) Find (*a*) the area of the surface, and (*b*) the cubical contents of a ball 22½ inches in diameter.

Ans. { (*a*) 1,590.435 sq. in.
(*b*) 5,964.1313 cu. in.

(33) (*a*) What is the volume and area of a cylindrical ring whose outside diameter is 16 inches and inside diameter 13 inches? (*b*) If made of cast iron, what is its weight? Take the weight of 1 cubic inch of cast iron as .261 pound.

Ans. Weight = 21 lb.

PRINCIPLES OF MECHANICS.

EXAMINATION QUESTIONS.

(1) Define mechanics.

(2) Define (*a*) matter; (*b*) molecule; (*c*) atom.

(3) In what three states does matter exist?

(4) Explain clearly the distinction between general properties of matter and special properties of matter.

(5) Define (*a*) motion; (*b*) velocity; (*c*) uniform velocity; (*d*) variable velocity.

(6) Define (*a*) acceleration; (*b*) retardation; (*c*) average velocity.

(7) An ocean steamer made a run of 3,240 miles in 6 days and 16 hours. What was its average speed in miles per hour? Ans. 20¼ mi.

(8) How long would it take to make a tour around the world when traveling at an average speed of 3,000 feet per minute, assuming the distance around the world to be 25,000 miles? Ans. 30 da. 13 hr. 20 min.

(9) How do we recognize the existence of a force?

(10) What conditions are necessary to compare the relative effects of different forces on different bodies?

(11) State Newton's Laws of Motion.

(12) What do you understand by the term "inertia"?

(13) Define (*a*) dynamics; (*b*) statics.

(14) Why is not the weight of a given body the same at every point on the surface of the earth?

(15) Determine the mass of a body that weighs 346 pounds at a place where g is equal to 32.174. Ans. 10.75+.

(16) How does the position of a body above or below the surface of the earth affect its weight?

(17) A locomotive weighing 30 tons has to overcome a constant force of 15 pounds per ton when it is in motion. What total force must the locomotive exert so that its speed may increase at the rate of 3 feet per second?

Ans. 6,047 lb.

(18) What will be the momentum of the locomotive in the preceding example when it has attained a velocity of 1 mile per hour? Ans. 2,736.3 lb.

(19) Define (*a*) work; (*b*) power; (*c*) energy.

(20) What horsepower is required to raise a body weighing 66,000 pounds through a distance of 80 feet in $\frac{1}{2}$ hour?

Ans. $5\frac{1}{3}$ H. P.

(21) A body weighing 6,432 pounds is moving with a constant velocity of 60 feet per second. What horsepower will be required to bring the body to rest in 3 minutes?

Ans. $3\frac{7}{11}$ H. P.

(22) What is the tangential pressure on the crank of an engine when the crank is on either dead center?

(23) What is friction?

(24) A body weighing 5,000 pounds rests on a horizontal surface. In order to slide the body along the surface, a horizontal force of 300 pounds must be exerted. What is the coefficient of friction in this particular case? Ans. .06.

(25) A crosshead weighing 1,000 pounds and having bronze shoes slides on a slightly greased, horizontal wrought-iron guide. If the contact area of the bronze shoe is 100 square inches, what will be (*a*) the total friction and (*b*) the friction per square inch of contact surface?

Ans. { (*a*) 160 lb.
(*b*) 1.6 lb.

(26) What is the center of gravity of a body?

(27) Explain the method of finding the common center of gravity of several bodies whose weights and the distances between whose centers of gravity are known.

(28) Give a practical method of determining the center of gravity of a solid body.

(29) Define (*a*) centrifugal force; (*b*) centripetal force.

(30) What is the relation between the centrifugal and centripetal forces of a revolving body?

(31) A body weighing 10 pounds revolves at a speed of 60 revolutions per minute about a point 6 feet from its center of gravity. What is the centrifugal force tending to pull the body from the point about which it revolves?

Ans. 73.44 lb.

(32) Name the three states of equilibrium and give examples of each.

(33) In the case of a body at rest, what are the conditions of the forces acting on that body?

(34) How is the condition of equilibrium of the forces acting on a body affected by the position of its "line of direction" with respect to the base?

MACHINE ELEMENTS.

EXAMINATION QUESTIONS.

(1) Define (*a*) lever; (*b*) weight arm; (*c*) force arm; (*d*) fulcrum.

(2) What is the condition necessary for the equilibrium of the lever?

(3) State the general rule that expresses the relation existing between the weight, the force, and the distances through which they move.

(4) What must be the length of the weight arm in order that a force of 12 pounds at a distance of 20 inches from the fulcrum will raise a weight of 100 pounds at the end of the weight arm? Ans. 2.4 in.

(5) Into what two classes may pulleys be divided in reference to their construction?

(6) What advantages have split pulleys over solid pulleys?

(7) Explain how a crowned pulley tends to prevent the slipping off of the belt.

(8) What is meant by "balancing pulleys," and how is it accomplished?

(9) Define the terms "driver" and "driven" as applied to pulleys.

(10) Find what diameter driver will be required which, when running at 600 revolutions per minute, will cause the driven, whose diameter is 6 inches, to run at 1,800 revolutions per minute. Ans. 18 in.

(11) What must be the speed of a driver 12 inches in diameter in order that the driven, whose diameter is 5 inches, may make 1,600 revolutions per minute?

Ans. $666\frac{2}{3}$ rev. per min.

(12) The distance between the centers of two pulleys, whose diameters are 6 feet and 2 feet, respectively, is 40 feet. What is the required length of an open belt? Ans. 93 ft.

(13) A single belt running at 1,650 feet per minute is used to transmit 40 horsepower. If the arc of contact on the small pulley is 120°, how wide should the belt be?

Ans. 28 in.

(14) What horsepower will be transmitted by the belt in question 13 if the velocity is reduced to 1,200 feet per minute? Ans. 29.3 H. P.

(15) If, in question 13, a double belt were substituted for the single belt, how wide should it be? Ans. 20 in.

(16) Which side of a leather belt should be in contact with the pulley face, and why?

(17) Should rosin be used on a belt to prevent slipping?

(18) Give the causes of flapping belts and their remedies.

(19) State the different methods used in joining belts. Which of these makes the best joint?

(20) What precaution should be observed when using rubber belts?

(21) Define the terms "driver" and "follower" as applied to a train of gear-wheels.

(22) In Fig. 19, if the diameter of A is 90 inches, that of F 30 inches, and if B has 12 teeth, C 30 teeth, D 20 teeth, and E 36 teeth, find the weight that a force of 50 pounds at P can raise. Ans. 675 lb.

(23) What is (*a*) circular pitch? (*b*) diametral pitch?

(24) What are the most common forms of teeth used in ordinary practice?

(25) What advantages have involute teeth over epicycloidal teeth?

(26) Find the pitch diameter of a gear-wheel having 60 teeth and a circular pitch of 1.152 inches. Ans. 22 in.

(27) What is the circular pitch of a gear-wheel 30 inches in diameter having 60 teeth? Ans. 1.57 in.

(28) What is the over-all diameter of a gear-wheel having 80 teeth with a diametral pitch of 8? Ans. 10¼ in.

(29) Find the number of teeth in a gear-wheel whose outside diameter is 7½ inches and whose diametral pitch is 8. Ans. 58 teeth.

(30) What distinguishes a fixed pulley from a movable pulley?

(31) In a certain combination of pulleys there are seven movable ones. Neglecting losses due to friction, how heavy a weight can a force of 150 pounds raise when applied to the free end of the rope? Ans. 2,100 lb.

(32) How does friction affect the force required to raise a given weight by means of a rope or chain and pulleys?

(33) In an ordinary block and tackle having four movable pulleys, what is the probable actual force that must be applied to raise a weight of 1,500 pounds? Ans. 312.5 lb.

(34) In what respect is the Weston differential pulley block better than the ordinary block and tackle?

(35) An inclined plane is 70 feet long and 12 feet high. What force acting parallel to the plane will be required to sustain a weight of 600 pounds on the plane? Ans. 103 lb.

(36) What weight will a force of 36 pounds acting parallel to the base of the plane be able to sustain on an inclined plane having a base 50 feet long and a height of 15 feet? Ans. 120 lb.

(37) What is the probable actual weight that can be raised by means of a screw jack that has a screw $2\frac{1}{2}$ inches in diameter with 6 threads to the inch if a force of 50 pounds is applied at the end of a lever 20 inches from the shaft ?

Ans. 2,356.2 lb.

(38) Define (*a*) velocity ratio; (*b*) efficiency.

(39) What is the efficiency of the screw jack of question 37 ? Ans. $6\frac{1}{4}$ per cent.

MECHANICS OF FLUIDS.

EXAMINATION QUESTIONS.

(1) Define hydrostatics.

(2) How can it be proved that liquids transmit pressure in all directions and with the same intensity?

(3) State Pascal's law.

(4) In what direction does the pressure due to the weight of a body of water act in a vessel in which water is contained?

(5) What is the pressure on the bottom of a vessel if the base is a circle 3 inches in diameter, the height 8 inches, and the vessel is completely filled with water? Ans. 2.045 lb.

(6) State the law for the upward pressure of a liquid on a horizontal surface submerged in the liquid.

(7) Why is it that in several pipes that communicate with one another and that differ in shape and size, water will stand at the same height?

(8) Why is it that water issuing from a hose connected to a hydrant cannot be made to spout up to a level with the surface of the water in the reservoir that supplies the hydrant?

(9) Define specific gravity.

(10) Calculate the weight of a block of aluminum whose volume is 100 cubic feet. Ans. 15,605 lb.

(11) State the principle of Archimedes.

(12) Explain how, by the application of the principle of Archimedes, the volume of an irregularly shaped body may be accurately determined.

(13) (*a*) What are hydrometers? (*b*) Into what two classes may hydrometers be divided?

(14) A single-cylinder pump feeds a boiler through a delivery pipe 1 inch in actual diameter. The piston speed is such as to give a velocity of flow of 400 feet per minute. How many gallons of water can be pumped into the boiler in 1 hour? Ans. 979.2 gal.

(15) What should be the commercial size of a delivery pipe from a duplex pump to deliver 936 gallons of water per hour? Ans. 1 in.

(16) Why is it important to have as few bends as possible in the suction pipe leading to a pump?

(17) What simple experiment proves that gases tend to expand and increase their volume?

(18) How high a column of mercury will the pressure of the atmosphere support?

(19) How can the degree of the vacuum in a vessel be determined?

(20) How high a column of a liquid whose specific gravity is 2.5 will the atmospheric pressure support?

Ans. 163.2 in.

(21) How is the pressure of the atmosphere measured?

(22) Why does the pressure of the atmosphere decrease as the elevation above sea level increases?

(23) In what respect is the action of the pressure of the atmosphere similar to that of the pressure of a liquid?

(24) How does the tension of a gas change with the change in volume under constant temperature?

(25) Define (*a*) gauge pressure; (*b*) absolute pressure.

(26) A metallic tube closed at one end is fitted with an air-tight movable piston. When the piston is at the open end of the tube, the pressure of the air within the tube is equal to 14.7 pounds per square inch. What will be the pressure of the air between the piston and the closed end of the tube after the former has moved toward the latter a distance equal to $\frac{4}{5}$ the length of the tube, the temperature remaining constant? Ans. 73.5 lb. per sq. in.

(27) Suppose that the piston in the tube mentioned in question 26 had been moved toward the closed end until the pressure had reached 147 pounds per square inch. What fraction of the original volume would the compressed air have occupied? Ans. $\frac{1}{10}$.

(28) In pneumatics, what is an air pump?

(29) Can a perfect vacuum be produced with the air pump?

(30) Explain the operation of the dashpot of a Corliss engine.

(31) Explain the principle of operating the siphon.

(32) Suppose that two vessels, in one of which the water stands at a higher level than in the other, are connected by a siphon and water is siphoned from the vessel in which it stands at the higher level into the other vessel. What will happen when the water in each vessel reaches the same level?

(33) In practice, how far above the water level in the vessel from which water is siphoned can the highest point of the siphon be carried?

(34) (*a*) What is a pump? (*b*) Into how many types may pumps be divided in reference to their mode of action? (*c*) Name these types.

(35) Explain the principle on which the suction pump acts.

(36) What advantage has a lifting pump over a suction pump?

(37) In what respect does a force pump differ from a lifting pump?

(38) What is the difference between a single-acting and a double-acting force pump?

STRENGTH OF MATERIALS.

EXAMINATION QUESTIONS.

(1) (*a*) Define stress. (*b*) Name the various kinds of stresses to which a body can be subjected.

(2) A weight of 8,000 pounds rests on the top of a cubical block of wood the area of each face of which is 20 square inches. What is the unit stress, in pounds per square inch, to which the block is subjected? Ans. 400 lb. per sq. in.

(3) Define (*a*) strain; (*b*) elasticity; (*c*) elastic limit.

(4) Taking the ultimate tensile strength of wrought iron as 55,000 pounds per square inch, what tensile force will rupture a wrought-iron bar whose cross-sectional area is 4 square inches? Ans. 220,000 lb.

(5) How does annealing improve old chains?

(6) Give a practical method of determining the true condition of a given rope, supposing that its outer surface appears to be in good condition.

(7) (*a*) On what does the safe lifting load of a sling depend? (*b*) Explain how the style of attachment to the hook of the tackle block affects the amount of load to be raised.

(8) (*a*) For what is manila rope chiefly used? (*b*) What is the object of lubricating the fibers of a rope?

(9) In general, how should the size of the pulley compare with the size of manila rope used for transmitting power?

(10) What is the greatest load to which an iron-wire rope $1\frac{1}{2}$ inches in circumference should be subjected?

Ans. 1,350 lb.

(11) What should be the circumference of a steel-wire rope under a maximum working load of 16,000 pounds?

Ans. 4 in., nearly.

(12) How does the strength of a column having both ends flat compare with that of columns both of whose ends are not flat but which in every other respect are similar to the first column?

(13) Give practical examples of the three different classes into which columns are divided with respect to the condition of their ends.

(14) What is the safe steady working load that a cast-iron column, having fixed ends, 14 inches in diameter and 16 feet high can sustain? Ans. 1,291,480 lb.

(15) Would you consider a steel piston rod 6 inches in diameter of sufficient size for a 40-inch cylinder using steam at 110 pounds pressure? If so, why?

(16) A solid yellow-pine beam 14 inches square rests on two supports 12 feet apart. What steady safe load will the beam support at its middle point? Ans. 18,547 lb.

(17) An oak beam 4 inches wide and 6 inches deep is subjected to a sudden shearing stress of 10,000 pounds across the grain. Is this a safe load for the given conditions?

(18) What should be the area of a wrought-iron beam to safely support a sudden shearing stress of 400,000 pounds?

Ans. 91 sq. in., nearly.

(19) What do you understand by double shear?

(20) What is the use of countershafts?

(21) What is the distinction between cold-rolled shafting and bright shafting?

(22) How is bright shafting designated commercially with respect to size?

(23) Why is it good practice to place pulleys for transmitting or receiving power as near the bearings of the shaft as possible?

(24) What horsepower can be transmitted from a steel shaft 8 inches in diameter by means of pulleys between the bearings when running at a speed of 80 revolutions per minute? Ans. 482 H. P.

(25) What must be the speed of a 4-inch shaft of cold-rolled iron, having no pulleys between bearings, to transmit 75 horsepower? Ans. 76 rev.

(26) Find the diameter of a wrought-iron shaft running at 300 revolutions per minute to transmit 84 horsepower by means of pulleys between its bearings. Ans. 3 in.

(27) How does a change in the speed of a shaft affect the amount of power transmitted?

(28) Does a high tensile strength in metals necessarily imply an ability on the part of the metal to safely resist repeated applications of sudden stresses?

(29) What is the chief advantage of a steel rope over an iron rope?

A KEY

TO ALL THE

QUESTIONS AND EXAMPLES

INCLUDED IN THE

EXAMINATION QUESTIONS IN THIS VOLUME.

It will be noticed that the Key is divided into sections which correspond to the sections in the Examination Questions in this Volume. The answers and solutions are so numbered as to be similar to the numbers before the questions to which they refer.

ARITHMETIC.

(PART 1.)

---

(1) See Art. **1.**

(2) See Art. **3.**

(3) See Arts. **5** and **6.**

(4) See Arts. **10** and **11.**

(5) 980 = Nine hundred eighty.
605 = Six hundred five.
28,284 = Twenty-eight thousand two hundred eighty-four.
9,006,042 = Nine million six thousand forty-two.
850,317,002 = Eight hundred fifty million three hundred seventeen thousand two.
700,004 = Seven hundred thousand four.

(6) Seven thousand six hundred = 7,600.
Eighty-one thousand four hundred two = 81,402.
Five million four thousand seven = 5,004,007.
One hundred and eight million ten thousand one = 108,010,001.
Eighteen million six = 18,000,006.
Thirty thousand ten = 30,010.

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(7) In adding whole numbers, place the numbers to be added directly under each other, so that the extreme right-hand figures will stand in the same column, regardless of the position of those at the left. Add the first column of figures at the extreme right, which equals 19 units, or 1 ten and 9 units. We place 9 units under the units column and reserve 1 ten for the column of tens, $8 + 7 + 9 + 1 = 25$ tens, or 2 hundreds and 5 tens. Place 5 tens under the tens column and reserve 2 hundreds for the hundreds column. $4 + 5 + 2 + 2 = 13$ hundreds, or 1 thousand and 3 hundreds. Place 3 hundreds under the hundreds column and reserve the 1 thousand for the thousands column. $2 + 5 + 3 + 1 = 11$ thousands, or 1 ten-thousand and 1 thousand. Place the 1 thousand in the column of thousands and reserve the 1 ten-thousand for the column of ten-thousands. $6 + 1 = 7$ ten-thousands. Place this 7 ten-thousands in the ten-thousands column. There is but one figure, 8, in the hundreds of thousands place in the numbers to be added, so it is placed in the hundreds of thousands column of the sum.

```
  3290
   504
865403
  2074
    81
     7
------
871359  Ans.
```

A simpler (though less scientific) explanation of the same problem is the following: $7 + 1 + 4 + 3 + 4 + 0 = 19$; write the 9 and reserve the 1. $8 + 7 + 0 + 0 + 9 + 1$ reserved $= 25$; write the 5 and reserve the 2. $0 + 4 + 5 + 2 + 2$ reserved $= 13$; write the 3 and reserve the 1. $2 + 5 + 3 + 1$ reserved $= 11$; write the 1 and reserve 1. $6 + 1$ reserved $= 7$; write the 7. Bring down the 8 to its place in the sum.

(8)

```
      709
  8304725
      391
   100302
      300
      909
-------
8407336  Ans.
```

(9) The steam engine, during the 12-hour test, showed that the number of revolutions made were 150,508, since 12,600 + 12,444 + 12,467 + 12,528 + 12,468 + 12,590 + 12,610 + 12,589 + 12,576 + 12,558 + 12,546 + 12,532 = 150,508 rev.

$$
\begin{array}{rl}
12600 & \text{revolutions.} \\
12444 & \text{revolutions.} \\
12467 & \text{revolutions.} \\
12528 & \text{revolutions.} \\
12468 & \text{revolutions.} \\
12590 & \text{revolutions.} \\
12610 & \text{revolutions.} \\
12589 & \text{revolutions.} \\
12576 & \text{revolutions.} \\
12558 & \text{revolutions.} \\
12546 & \text{revolutions.} \\
12532 & \text{revolutions.} \\
\hline
150508 & \text{revolutions.} \quad \text{Ans.}
\end{array}
$$

(10) In subtracting whole numbers, place the subtrahend, or smaller number, under the minuend, or larger number, so that the right-hand figures stand directly under each other. Begin *at the right* to subtract. We cannot subtract 8 units from 2 units, so we take 1 ten from the 6 tens and add it to the 2 units. 1 *ten* = 10 *units*, so we have 10 units + 2 units = 12 units. Then, 8 units from 12 units leaves 4 units. We took 1 ten from 6 tens, so only 5 tens remain. 3 tens from 5 tens leaves 2 tens. In the hundreds column we have 3 hundreds from 9 hundreds leaves 6 hundreds. We cannot subtract 3 thousands from 0 thousands, so we take 1 ten-thousand from 5 ten-thousands and add it to the 0 thousands. 1 *ten-thousand* = 10 *thousands*, and 10 thousands + 0 thousands = 10 thousands. Subtracting, we have 3 thousands from 10 thousands leaves 7 thousands. We took 1 ten-thousand from 5 ten-thousands and have 4 ten-thousands remaining. Since there are no ten-thousands in the subtrahend, the 4 in the

(*a*)
$$
\begin{array}{r}
50962 \\
3338 \\
\hline
47624
\end{array}
\quad \text{Ans.}
$$

ten-thousands column in the minuend is brought down into the same column in the remainder, because 0 from 4 leaves 4.

(*b*) 1 5 3 3 9
1 0 0 0 1
5 3 3 8. Ans.

(11) (*a*) 7 0 9 6 8
3 2 9 7 5
3 7 9 9 3 Ans.

(*b*) 1 0 0 0 0 0
9 8 7 3 5
1 2 6 5 Ans.

(12)

3 0 4 0 = No. of gallons in the tank at the beginning of the day.
4 7 8 0 = No. of gallons pumped in during the morning.
7 8 2 0 = No. of gallons in the tank after 4,780 gallons were added.
7 2 4 0 = No. of gallons drawn out during the morning.
5 8 0 = No. of gallons in the tank at the beginning of the afternoon.
8 6 7 5 = No. of gallons pumped in during the afternoon.
9 2 5 5 = No. of gallons in the tank after 8,675 gallons were added.
7 6 3 3 = No. of gallons drawn out during the afternoon.
1 6 2 2 = No. of gallons remaining in the tank at night. Ans.

(13) In the multiplication of whole numbers, place the multiplier under the multiplicand and multiply each term of the multiplicand by each term of the multiplier, writing the right-hand figure of each product obtained under the term of the multiplier which produces it.

(*a*) 5 2 6 3 8 7
7
3 6 8 4 7 0 9 Ans.

7 times 7 units = 49 units, or 4 tens and 9 units. We write the 9 units and reserve the 4 tens. 7 times 8 tens = 56 tens; 56 + 4 tens reserved = 60 tens, or 6 hundreds and 0 tens. Write the 0 tens and reserve the 6 hundreds. 7 × 3 hundreds = 21 hundreds; 21 + 6 hundreds reserved = 27 hundreds, or 2 thousands and 7 hundreds. Write the 7 hundreds and reserve the 2 thousands. 7 × 6 thousands =

42 thousands; 42 + 2 thousands reserved = 44 thousands, or 4 ten-thousands and 4 thousands. Write the 4 thousands and reserve the 4 ten-thousands. 7 × 2 ten-thousands = 14 ten-thousands; 14 + 4 ten-thousands reserved = 18 ten-thousands, or 1 hundred-thousand and 8 ten-thousands. Write the 8 ten-thousands and reserve the 1 hundred-thousand. 7 × 5 hundred-thousands = 35 hundred-thousands; 35 + 1 hundred-thousand reserved = 36 hundred-thousands. Since there are no more figures in the multiplicand to be multiplied, we write the 36 hundred-thousands in the product. This completes the multiplication.

A simpler (though less scientific) explanation of the same problem is the following:

7 times 7 = 49; write the 9 and reserve the 4. 7 times 8 = 56; 56 + 4 reserved = 60; write the 0 and reserve the 6. 7 times 3 = 21; 21 + 6 reserved = 27; write the 7 and reserve the 2. 7 × 6 = 42; 42 + 2 reserved = 44; write the 4 and reserve 4. 7 × 2 = 14; 14 + 4 reserved = 18; write the 8 and reserve 1. 7 × 5 = 35; 35 + 1 reserved = 36; write the 36.

(*b*)

```
  700298
      17
 -------
 4902086
  700298
 -------
11905066   Ans.
```

In this case the multiplier is 17 *units*, or 1 *ten* and 7 *units*, so that the product is obtained by adding two partial products, namely, 7 × 700,298 and 10 × 700,298. The actual operation is performed as follows:

7 times 8 = 56; write the 6 and reserve the 5. 7 times 9 = 63; 63 + 5 reserved = 68; write the 8 and reserve the 6. 7 times 2 = 14; 14 + 6 reserved = 20; write the 0 and reserve the 2. 7 times 0 = 0; 0 + 2 reserved = 2; write the 2. 7 times 0 = 0; 0 + 0 reserved = 0; write the 0. 7 times 7 = 49; 49 + 0 reserved = 49; write the 49.

To multiply by the 1 ten we have 1 ten times 8 units = 8 tens and 0 units. We do not write the 0 units, but write the 8 tens under the 8 tens in the first partial product above. We next multiply 1 ten and 9 tens = 90 tens = 9 hundreds + 0 tens; write the 9 hundreds under the 0 hundreds

of the first partial product above. Again, 1 ten times 2 hundreds = 2 thousands; write the 2 thousands under the 2 thousands above. 1 ten times 0 thousands = 0 ten-thousands; write the 0 in the ten-thousands place. 1 ten times 0 ten-thousands = 0 hundred-thousands; write the 0 in the hundred-thousands place. 1 ten times 7 hundred-thousands = 7 millions; write the 7 in the millions place. This completes the second partial product. Add the two partial products; their sum equals the entire product.

(*c*)

```
      2 1 7
      1 0 3
      -----
      6 5 1
    0 0 0
  2 1 7
  -------
  2 2 3 5 1
        6 7
  ---------
  1 5 6 4 5 7
1 3 4 1 0 6
-----------
1 4 9 7 5 1 7   Ans.
```

Multiply any two of the numbers together, and multiply their product by the third number.

(14) If your watch ticks every second, to find how many times it ticks in 1 week, it is necessary to find the number of seconds in 1 week.

```
      6 0 seconds = 1 minute.
      6 0 minutes = 1 hour.
    -----
    3 6 0 0 seconds = 1 hour.
      2 4 hours = 1 day.
  -------
  1 4 4 0 0
  7 2 0 0
  -------
  8 6 4 0 0 seconds = 1 day.
          7 days = 1 week.
  -------
6 0 4 8 0 0 seconds in 1 week, or the number of times that
            your watch ticks in a week.  Ans.
```

(15) (*a*) If an engine and boiler are worth $3,246, and the building is worth three times as much, plus $1,200, then the building is worth

3246
3 times
9738
plus 1200
$10938 = value of building.

If the tools are worth twice as much as the building, plus $1,875, then the tools are worth

10938
2
21876
plus 1875
$23751 = value of tools.

Value of building = 10938
Value of tools = 23751
$34689 = value of the building and tools. Ans.

(*b*) Value of engine and boiler = 3246
Value of building and tools = 34689
$37935 = value of the whole plant. Ans.

(16) (*a*) 84)962842.0000(11462.40474+ Ans.

84
122
84
388
336
524
504
202
168
340
336
400
336
640
588
52

84 is contained once in 96. Place 1 as the first figure in the quotient and multiply the divisor 84 by it. Subtract the product, which is 84, from 96, leaving a remainder of 12. Bring down the next figure in the dividend, which is 2, and annex it to 12, making a new dividend of 122.

84 is contained in 122 once. Place 1 as the second figure in the quotient and multiply the divisor 84 by it. Subtract the product (84) from 122, leaving a remainder of 38. Bring down the next figure in the dividend, which is 8, and annex it to 38, making a new dividend of 388.

84 is contained in 388 4 times. Place 4 as the third figure in the quotient and multiply the divisor 84 by it. The product is 336. Subtract the product from 388, leaving a remainder of 52. Bring down the next figure, which is 4, and annex it to 52, making a new dividend of 524.

84 is contained in 524 6 times. Place 6 as the fourth figure in the quotient. Multiply the divisor 84 by it and subtract the product (504) from 524, leaving a remainder of 20. Bring down the next figure, which is 2, and annex it to 20, making a new dividend of 202.

84 is contained in 202 2 times. Place 2 as the fifth figure in the quotient. Multiply the divisor 84 by it and subtract the product (168) from 202, leaving a remainder of 34. If it is desired to carry the quotient to 4 decimal places, annex 4 ciphers to the dividend and continue in the same way. In the quotient point off as many decimal places as there are ciphers annexed, or, in this case, 4 decimal places.

(*b*)

```
63 ) 3 9 7 2 8.0 0 0 ( 6 3 0.6 0 3+   Ans.
     3 7 8
     -----
       1 9 2
       1 8 9
       -----
           3 8 0*
           3 7 8
           -----
               2 0 0
               1 8 9
               -----
                 1 1
```

*63 is not contained in 38, so we place a cipher in the quotient and bring down the next figure in the dividend, which is a cipher that has been annexed, making a new dividend of 380. 63 is contained in 380 6 times. $6 \times 63 = 378$. Subtracting 378 from 380 leaves 2. Bringing down the next figure in the dividend, we have 20 for a new dividend. 63 is not contained in 20, so we place a cipher in the quotient and bring down the next cipher in the dividend, making a new dividend of 200. 63 is contained 3 times in 200.

(*c*)
```
108)29714.0000(275.1296  Ans.
   216
   ---
   811
   756
   ---
    554
    540
    ---
     140
     108
     ---
      320
      216
      ---
      1040
       972
       ---
        680
        648
        ---
         32
```

(*d*)
```
135)406089.0000(3008.0666
405                      Ans.
---
 1089*
 1080
 ----
   900
   810
   ---
    900
    810
    ---
     900
     810
     ---
      90
```

*135 is not contained in 10, so we place a cipher as the second figure in the quotient. Bringing down the next figure, 8, and annexing it to 10, we have a new dividend of 108. 135 is not contained in 108, so we place a cipher as the third figure in the quotient and bring down the next figure in the dividend, or 9, and annex it to 108, making a new dividend of 1,089. 135 is contained in 1,089 8 times. Write 8 as the fourth figure in the quotient. Multiply 135 by 8 and subtract the product (1,080) from 1,089, which leaves a remainder of 9. Bring down the next figure in the dividend, which is a cipher that has been annexed, and annex

it to the remainder 9, making a new dividend of 90. As 135 is not contained in 90, we place a 0 in the quotient and bring down another cipher from the dividend, making a new dividend of 900. 135 is contained in 900 6 times. Write 6 as the next figure in the quotient, multiply 135 by 6, and subtract the product (810) from 900, which leaves a remainder of 90. Bring down the next figure (0) in the dividend and annex it to the remainder 90, making a new dividend of 900. 135 is contained in 900 6 times. Place 6 as the next figure in the quotient and multiply the divisor by it. It is plain that each succeeding figure of the quotient will be 6. Point off *four* decimal places in the quotient, since four ciphers were annexed.

(17) If in 1 hour 10 pounds of coal are burned for every square foot of grate area and 9 pounds of water are evaporated for every pound of coal burned, then in 1 hour there would be 9×10 or 90 pounds of water evaporated for 1 sq. ft. of grate area; and since the grate area is 30 sq. ft., the amount of water evaporated would be $30 \times 90 = 2{,}700$ lb. Since 2,700 lb. of water are evaporated in 1 hour, in a day of 10 hours there would be $10 \times 2{,}700$ lb., or 27,000 lb. of water evaporated.

(18) If a mechanic earns \$1,500 a year and his expenses are \$968 per year, then he would save \$1500 — \$968, or \$532 per year.

$$\begin{array}{r} 1\,5\,0\,0 \\ 9\,6\,8 \\ \hline \$\,5\,3\,2 \end{array}$$

If he saves \$532 in 1 year, to save \$3,724 it would take as many years as \$532 is contained times in \$3,724, or 7 years.

$$\begin{array}{l} 5\,3\,2\ )\ 3\,7\,2\,4\ (\ 7 \text{ years.} \quad \text{Ans.} \\ \quad\ \underline{3\,7\,2\,4} \end{array}$$

(19) (*a*) $(72 \times 48 \times 28 \times 5) \div (84 \times 15 \times 7 \times 6)$.

Placing the numerator over the denominator, the problem becomes

$$\frac{72 \times 48 \times 28 \times 5}{84 \times 15 \times 7 \times 6} = ?$$

The 5 in the numerator and 15 in the denominator are both *divisible* by 5, since 5 divided by 5 equals 1 and 15 divided by 5 equals 3. *Cross off* the 5; also *cross off* the 15 and write the 3 *under* it. Thus,

$$\frac{72 \times 48 \times 28 \times \cancel{5}}{84 \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

72 in the numerator and 84 in the denominator are *divisible* by 12, since 72 divided by 12 equals 6 and 84 divided by 12 equals 7. *Cross off* the 72 and write the 6 *over* it; also, *cross off* the 84 and write the 7 *under* it. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times 28 \times \cancel{5}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

Again, 28 in the numerator and 7 in the denominator are *divisible* by 7, since 28 divided by 7 equals 4 and 7 divided by 7 equals 1. *Cross off* the 28 and write the 4 *over* it; also, *cross off* the 7. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times \overset{4}{\cancel{28}} \times \cancel{5}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times \cancel{7} \times 6} =$$

Again, 48 in the numerator and 6 in the denominator are *divisible* by 6, since 48 divided by 6 equals 8 and 6 divided by 6 equals 1. *Cross off* the 48 and write the 8 *over* it; also, *cross off* the 6. Thus,

$$\frac{\overset{6}{\cancel{72}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \cancel{5}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times \cancel{7} \times \cancel{6}} =$$

Again, 6 in the numerator and 3 in the denominator are *divisible* by 3, since 6 divided by 3 equals 2 and 3 divided by 3 equals 1. *Cross off* the 6 and write the 2 *over* it; also, *cross off* the 3. Thus,

$$\frac{\overset{2}{\cancel{72}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \cancel{5}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times \cancel{7} \times \cancel{6}} =$$

Since there are *no two remaining numbers* (one in the numerator and one in the denominator) *divisible* by *any number* except 1, without a remainder, it is *impossible* to cancel further.

Multiply all the *uncanceled numbers* in the numerator together and divide their *product* by the *product* of all the *uncanceled* numbers in the denominator. The *result* will be the *quotient.* The *product* of all the *uncanceled numbers* in the numerator equals $2 \times 8 \times 4 = 64$; the product of all the *uncanceled* numbers in the denominator equals 7.

Hence, $\frac{\overset{2}{\cancel{72}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \cancel{5}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times \cancel{7} \times \cancel{6}} = \frac{2 \times 8 \times 4}{7} = \frac{64}{7} = 9\frac{1}{7}$. Ans.

(*b*) $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20)$.

Placing the numerator over the denominator, the problem becomes

$$\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$$

The 50 in the numerator and 70 in the denominator are both *divisible* by 10, since 50 divided by 10 equals 5 and 70 divided by 10 equals 7. *Cross off* the 50 and write the 5 *over* it; also, *cross off* the 70 and write the 7 *under* it. Thus,

$$\frac{80 \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times 20} =$$

Also, 80 in the numerator and 20 in the denominator are *divisible* by 20, since 80 divided by 20 equals 4 and 20 divided by 20 equals 1. *Cross off* the 80 and write the 4 *over* it; also, cross off the 20. Thus,

$$\frac{\overset{4}{\not 80} \times 60 \times \overset{5}{\not 50} \times 16 \times 14}{\underset{7}{\not 70} \times 50 \times 24 \times \not 20} =$$

Again, 16 in the numerator and 24 in the denominator are *divisible* by 8, since 16 divided by 8 equals 2 and 24 divided by 8 equals 3. *Cross off* the 16 and write the 2 *over* it; also, cross off the 24 and write the 3 *under* it. Thus,

$$\frac{\overset{4}{\not 80} \times 60 \times \overset{5}{\not 50} \times \overset{2}{\not 16} \times 14}{\underset{7}{\not 70} \times 50 \times \underset{3}{\not 24} \times \not 20} =$$

Again, 60 in the numerator and 50 in the denominator are *divisible* by 10, since 60 divided by 10 equals 6 and 50 divided by 10 equals 5. *Cross off* the 60 and write the 6 *over* it; also, cross off the 50 and write the 5 *under* it. Thus,

$$\frac{\overset{4}{\not 80} \times \overset{6}{\not 60} \times \overset{5}{\not 50} \times \overset{2}{\not 16} \times 14}{\underset{7}{\not 70} \times \underset{5}{\not 50} \times \underset{3}{\not 24} \times \not 20} =$$

The 14 in the numerator and 7 in the denominator are *divisible* by 7, since 14 divided by 7 equals 2 and 7 divided by 7 equals 1. *Cross off* the 14 and write the 2 *over* it; also, cross off the 7. Thus,

$$\frac{\overset{4}{\not 80} \times \overset{6}{\not 60} \times \overset{5}{\not 50} \times \overset{2}{\not 16} \times \overset{2}{\not 14}}{\underset{7}{\not 70} \times \underset{5}{\not 50} \times \underset{3}{\not 24} \times \not 20} =$$

The 5 in the numerator and 5 in the denominator are *divisible* by 5, since 5 divided by 5 equals 1. *Cross off* the 5 of the *dividend;* also, cross off the 5 of the *divisor.* Thus,

$$\frac{\overset{4}{\not 80} \times \overset{6}{\not 60} \times \not 50 \times \overset{2}{\not 16} \times \overset{2}{\not 14}}{\underset{7}{\not 70} \times \underset{5}{\not 50} \times \underset{3}{\not 24} \times \not 20} =$$

The 6 in the numerator and 3 in the denominator are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and place 2 *over* it; also, cross off the 3. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{\overset{2}{\cancel{6}}}{\cancel{60}} \times \overset{\cancel{5}}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\cancel{7}}{\cancel{70}} \times \underset{\cancel{5}}{\cancel{50}} \times \underset{\cancel{3}}{\cancel{24}} \times \cancel{20}} =$$

Hence, $$\frac{\overset{4}{\cancel{80}} \times \overset{\overset{2}{\cancel{6}}}{\cancel{60}} \times \overset{\cancel{5}}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\cancel{7}}{\cancel{70}} \times \underset{\cancel{5}}{\cancel{50}} \times \underset{\cancel{3}}{\cancel{24}} \times \cancel{20}} = 4 \times 2 \times 2 \times 2 = 32.$$

Ans.

(20) If the freight train ran 365 miles in one week, and 3 times as far, lacking 246 miles the next week, then it ran (3 × 365 miles) − 246 miles, or 849 miles the second week. Thus,

$$\begin{array}{r} 365 \\ 3 \\ \hline 1095 \\ -\ 246 \\ \hline 849 \end{array} \text{ miles. Ans.}$$

(21) The distance from Philadelphia to Pittsburg is 354 miles. Since there are 5,280 feet in 1 mile, in 354 miles there are 354 × 5,280 feet, or 1,869,120 feet. If the driving wheel of the locomotive is 16 feet in circumference, in going from Philadelphia to Pittsburg, a distance of 1,869,120 feet, it will make 1,869,120 ÷ 16, or 116,820 revolutions.

```
16 ) 1869120 ( 116820 rev.  Ans.
     16
     --
      26
      16
      --
      109
       96
      ---
       131
       128
       ---
         32
         32
         --
          0
```

ARITHMETIC.

(PART 2.)

---

(1) See Art. **1.**

(2) See Art. **6.**

(3) See Art. **3.**

(4) See Art. **3.**

(5) $\frac{13}{8}$ is an improper fraction, since its numerator 13 is greater than its denominator 8.

(6) $4\frac{1}{2}$; $14\frac{3}{10}$; $85\frac{4}{19}$.

(7) To reduce a fraction to its lowest terms means to change its form without changing its value. In order to do this, we must divide both numerator and denominator by the same number until we can no longer find any number (except 1) which will divide both of these terms without a remainder.

To reduce the fraction $\frac{4}{8}$ to its lowest terms, we divide both numerator and denominator by 4 and obtain as a result the fraction $\frac{1}{2}$. Thus, $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$; similarly, $\frac{4 \div 4}{16 \div 4} = \frac{1}{4}$; $\frac{8 \div 4}{32 \div 4} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$; $\frac{32 \div 8}{64 \div 8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$.

(8) When the denominator of any number is not expressed, it is understood to be 1, so that $\frac{6}{1}$ is the same as $6 \div 1$, or 6.

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To reduce $\frac{6}{1}$ to an improper fraction whose denominator is 4, we must multiply both numerator and denominator by some number which will make the denominator of 6 equal to 4. Since the denominator is 1, by multiplying both terms of $\frac{6}{1}$ by 4, we will have $\frac{6 \times 4}{1 \times 4} = \frac{24}{4}$, which has the *same value* as 6, but has a *different form.*

(9) In order to reduce a mixed number to an improper fraction, we must multiply the whole number by the denominator of the fraction and add the numerator of the fraction to the product. This result is the *numerator* of the improper fraction, of which the *denominator* is the denominator of the fractional part of the mixed number.

$7\frac{7}{8}$ means the same as $7 + \frac{7}{8}$. In 1 there are $\frac{8}{8}$; hence, in 7 there are $7 \times \frac{8}{8} = \frac{56}{8}$. Add the $\frac{7}{8}$ of the mixed number and we obtain $\frac{56}{8} + \frac{7}{8} = \frac{63}{8}$, which is the required improper fraction.

$$13\tfrac{5}{16} = \frac{(13 \times 16) + 5}{16} = \tfrac{213}{16}; \quad 10\tfrac{3}{4} = \frac{(10 \times 4) + 3}{4} = \tfrac{43}{4}.$$

(10) Each stroke of the engine is 18 inches in length. Since the piston makes 2 strokes for each revolution, it would pass over a distance of 2×18 inches $= 36$ inches, or 3 feet, in 1 minute, and in making 160 revolutions it would pass over 160×3, or 480 ft. Since 480 ft. are passed over in 1 minute, in 1 hour, or 60 minutes, the distance passed over equals $60 \times 480 = 28{,}800$ ft. Since the steam engine runs 6 days a week and $8\frac{1}{2}$ hours per day, the total number of hours it runs per week $= 6 \times 8\frac{1}{2}$, or 51 hours. If the piston passes over a distance of 28,800 ft. in 1 hour, in 51 hours it would pass over $51 \times 28{,}800$ ft., or 1,468,800 ft. Ans.

$$\begin{array}{r} 480 \text{ ft.} \\ \times \quad 60 \\ \hline 28800 \text{ ft.} \end{array}$$

$$\begin{array}{r} 28800 \\ \times \quad 51 \\ \hline 28800 \\ 144000 \\ \hline 1468800 \end{array}$$

(11) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = \frac{1 + 2 + 5}{8} = \frac{8}{8} = 1$. Ans.

When the denominators of the fractions to be added are alike, we know that the units are divided into the same number of parts (in this case eighths); we therefore add the numerators of the fractions to find the number of parts (eighths) taken or considered, thereby obtaining $\frac{8}{8}$, or 1, as the sum.

(12) When the denominators are not alike, we know that the units are divided into unequal parts, so before adding them we must find a common denominator for the denominators of all the fractions. Reduce the fractions to fractions having this common denominator, add the numerators, and write the sum over the common denominator.

In this case, the least common denominator, or the least number that will contain all the denominators, is 16; hence, we must reduce all these fractions to 16ths and then add their numerators.

$\frac{1}{4} + \frac{3}{8} + \frac{5}{16} = ?$ To reduce the fraction $\frac{1}{4}$ to a fraction having 16 for a denominator, we must multiply both terms of the fraction by some number which will make the denominator 16. This number evidently is 4, hence, $\frac{1 \times 4}{4 \times 4} = \frac{4}{16}$.

Similarly, both terms of the fraction $\frac{3}{8}$ must be multiplied by 2 to make the denominator 16, and we have $\frac{3 \times 2}{8 \times 2} = \frac{6}{16}$. The fractions now have a common denominator 16; hence, we find their sum by adding the numerators and placing their sum over the common denominator, thus: $\frac{4}{16} + \frac{6}{16} + \frac{5}{16}$ $= \frac{4 + 6 + 5}{16} = \frac{15}{16}$. Ans.

(13) When mixed numbers and whole numbers are to be added, add the fractional parts of the mixed numbers separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction.

$42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Reducing $\frac{5}{8}$ to a fraction having a denominator of 16, we have $\frac{5 \times 2}{8 \times 2} = \frac{10}{16}$. Adding the two fractional parts of the mixed numbers, we have $\frac{10}{16} + \frac{7}{16} = \frac{10 + 7}{16} = \frac{17}{16} = 1\frac{1}{16}$.

The problem now becomes $42 + 31 + 9 + 1\frac{1}{16} = ?$

Adding all the whole numbers and the number obtained from adding the fractional parts of the mixed numbers, we obtain $83\frac{1}{16}$ as their sum.

$$\begin{array}{r} 42 \\ 31 \\ 9 \\ 1\frac{1}{16} \\ \hline 83\frac{1}{16} \end{array}$$ Ans.

(14) $29\frac{3}{4} + 50\frac{5}{8} + 41 + 69\frac{3}{16} = ?$ $\frac{3}{4} = \frac{3 \times 4}{4 \times 4} = \frac{12}{16}$;

$\frac{5}{8} = \frac{5 \times 2}{8 \times 2} = \frac{10}{16}$. $\frac{12}{16} + \frac{10}{16} + \frac{3}{16} = \frac{12 + 10 + 3}{16} = \frac{25}{16} = 1\frac{9}{16}$.

The problem now becomes $29 + 50 + 41 + 69 + 1\frac{9}{16} = ?$

$$\begin{array}{rl} 29 & \text{square inches.} \\ 50 & \text{square inches.} \\ 41 & \text{square inches.} \\ 69 & \text{square inches.} \\ 1\frac{9}{16} & \text{square inches.} \\ \hline 190\frac{9}{16} & \text{square inches.} \end{array}$$ Ans.

(15) $\frac{7}{8} - \frac{7}{16} = ?$ When the denominators of fractions are not alike, it is evident that the units are divided into unequal parts; therefore, before subtracting, reduce the fractions to fractions having a common denominator. Then, subtract the numerators and place the remainder over the common denominator.

$\frac{7 \times 2}{8 \times 2} = \frac{14}{16}$. $\frac{14}{16} - \frac{7}{16} = \frac{14 - 7}{16} = \frac{7}{16}$. Ans.

$13 - 7\frac{7}{16} = ?$ This problem may be solved in two ways:

First: $13 = 12\frac{16}{16}$, since $\frac{16}{16} = 1$, and $12\frac{16}{16} = 12 + \frac{16}{16} = 12 + 1 = 13$.

$12\frac{16}{16}$
$7\frac{7}{16}$
$5\frac{9}{16}$

We can now subtract the whole numbers separately and the fractions separately, and obtain $12 - 7 = 5$ and $\frac{16}{16} - \frac{7}{16} = \frac{16 - 7}{16} = \frac{9}{16}$. $5 + \frac{9}{16} = 5\frac{9}{16}$. Ans.

Second: By reducing both numbers to improper fractions having a denominator of 16.

$$13 = \frac{13}{1} = \frac{13 \times 16}{1 \times 16} = \frac{208}{16}. \quad 7\frac{7}{16} = \frac{(7 \times 16) + 7}{16} = \frac{112 + 7}{16} = \frac{119}{16}.$$

Subtracting, we have $\frac{208}{16} - \frac{119}{16} = \frac{208 - 119}{16} = \frac{89}{16}$, and

$\frac{89}{16} = 16)89(5\frac{9}{16}$
80
9

the same result that was obtained by the first method.

$312\frac{9}{16} - 229\frac{5}{32} = ?$ We first reduce the fractions of the two mixed numbers to fractions having a common denominator. Doing this, we have $\frac{9}{16} = \frac{9 \times 2}{16 \times 2} = \frac{18}{32}$. We can now subtract the whole numbers and fractions separately, and have $312 - 229 = 83$ and $\frac{18}{32} - \frac{5}{32} = \frac{18 - 5}{32} = \frac{13}{32}$. $83 + \frac{13}{32} = 83\frac{13}{32}$. Ans.

$312\frac{18}{32}$
$229\frac{5}{32}$
$83\frac{13}{32}$

(16) In division of fractions, invert the divisor (or, in other words, turn it upside down) and proceed as in multiplication.

(*a*) $35 \div \frac{5}{16} = \frac{35}{1} \times \frac{16}{5} = \frac{35 \times 16}{1 \times 5} = \frac{560}{5} = 112$. Ans.

(*b*) $\frac{9}{16} \div 3 = \frac{9}{16} \div \frac{3}{1} = \frac{9}{16} \times \frac{1}{3} = \frac{9 \times 1}{16 \times 3} = \frac{9}{48} = \frac{3}{16}$. Ans.

(*c*) $\frac{17}{2} \div 9 = \frac{17}{2} \div \frac{9}{1} = \frac{17}{2} \times \frac{1}{9} = \frac{17 \times 1}{2 \times 9} = \frac{17}{18}$. Ans.

(*d*) $\frac{113}{64} \div \frac{7}{16} = \frac{113}{64} \times \frac{16}{7} = \frac{113 \times 16}{64 \times 7} = \frac{1808}{448} = 4\frac{1}{28}$. Ans.

$$\begin{array}{r} 448\,)\,1808\,(\,4\frac{1}{28} \\ 1792 \\ \hline 16 \\ \hline 448 \end{array} = \frac{1}{28}$$

(*e*) $15\frac{3}{4} \div 4\frac{3}{8} = ?$ Before proceeding with the division, reduce both mixed numbers to improper fractions. Thus, $15\frac{3}{4} = \frac{(15 \times 4) + 3}{4} = \frac{60 + 3}{4} = \frac{63}{4}$, and $4\frac{3}{8} = \frac{(4 \times 8) + 3}{8} = \frac{32 + 3}{8} = \frac{35}{8}$. The problem is now $\frac{63}{4} \div \frac{35}{8} = ?$ As before, invert the divisor, multiply, and cancel; $\frac{63}{4} \div \frac{35}{8} = \frac{63}{4} \times \frac{8}{35} = \frac{\overset{9}{\cancel{63}} \times \overset{2}{\cancel{8}}}{\cancel{4} \times \underset{5}{\cancel{35}}} = \frac{18}{5} = 3\frac{3}{5}$. Ans.

(17) $\frac{7}{8}$ = value of the fraction, and 28 the numerator. We find that 4 multiplied by 7 = 28, so multiplying 8, the denominator of the fraction, by 4, we have 32 for the required denominator, and $\frac{28}{32} = \frac{7}{8}$. Hence, 32 is the required denominator.

(18) Since these four bolts measure $2\frac{1}{2}$, $6\frac{7}{8}$, $3\frac{1}{16}$, and 4 inches, respectively, together they will measure $16\frac{7}{16}$ inches, since $2\frac{1}{2} + 6\frac{7}{8} + 3\frac{1}{16} + 4 = 16\frac{7}{16}$. Reducing the fractions of the mixed numbers to a common denominator, we have $\frac{1}{2} = \frac{1}{2} \times \frac{8}{8} = \frac{8}{16}$; $\frac{7}{8} = \frac{7}{8} \times \frac{2}{2} = \frac{14}{16}$; $\frac{8}{16} + \frac{14}{16} + \frac{1}{16} = \frac{8 + 14 + 1}{16} = \frac{23}{16} = 1\frac{7}{16}$. $2 + 6 + 3 + 4 + 1\frac{7}{16} = 16\frac{7}{16}$. If $\frac{7}{16}$ of an inch is allowed for cutting and finishing each bolt, then the allowance for the 4 bolts would equal $4 \times \frac{7}{16} = \frac{28}{16} = 1\frac{12}{16}$ inches, which added to $16\frac{7}{16}$ inches equals $18\frac{3}{16}$ inches, the length of the piece of iron required. Ans.

$$\begin{array}{r} 16\frac{7}{16} \\ 1\frac{12}{16} \\ \hline 17\frac{19}{16} \end{array}$$

or $18\frac{3}{16}$, since $\frac{19}{16} = 1\frac{3}{16}$.

ARITHMETIC.

(PART 3.)

(1) A fraction is one or more of the equal parts of a unit and is expressed by a numerator and a denominator, while a decimal fraction is a number of *tenths*, *hundredths*, *thousandths*, etc. of a unit and is expressed by placing a period (.), called a decimal point, to the left of the figures of the numerator and omitting the denominator.

(2) To reduce the fraction $\frac{1}{2}$ to a decimal, we annex one cipher to the numerator, which makes it 1.0. Dividing 1.0, the numerator, by 2, the denominator, gives a quotient of .5, the decimal point being placed before the *one* figure of the quotient, or .5, since only *one* cipher was annexed to the numerator.

$$\begin{array}{r} 7 \\ \hline 8\,)\,7.000 \\ \underline{.875} \end{array} \qquad \begin{array}{r} 5 \\ \hline 32\,)\,5.000000\,(.15625 \\ \underline{32} \\ 180 \\ \underline{160} \\ 200 \\ \underline{192} \\ 80 \\ \underline{64} \\ 160 \\ \underline{160} \end{array}$$

To express $\frac{65}{100}$ as a decimal, write the numerator and, beginning with the right-hand figure, point off towards the left as many decimal places as there are ciphers in the

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denominator, prefixing ciphers to the numerator if necessary, and then insert the decimal point. Proceeding thus, $\frac{65}{100} = .65$ and $\frac{125}{1000} = .125$. Similarly, $\frac{24}{100000} = .00024$.

(3) . 0 8 = *Eight* **hundredths.**

(tenths. hundredths.)

. 1 3 1 = *One hundred thirty-one* **thousandths.**

(tenths. hundredths. thousandths.)

. 0 0 0 1 = *One* **ten-thousandths.**

(tenths. hundredths. thousandths. ten-thousandths.)

. 0 0 0 0 2 7 = *Twenty-seven* **millionths.**

(tenths. hundredths. thousandths. ten-thousandths. hundred-thousandths. millionths.)

.0 1 0 8 = *One hundred eight* **ten-thousandths.**

(tenths. hundredths. thousandths. ten-thousandths.)

9 3 .0 1 0 1 = Ninety-three and *one hundred one* **ten-thousandths.**

In reading decimals, read the number just as you would if there were no ciphers before it. Then count from the

decimal point towards the right, beginning with tenths, to as many places as there are figures, and the name of the last figure must be annexed to the previous reading of the figures to give the decimal reading. Thus, in the first example given, the simple reading of the figure is eight and the name of its position in the decimal scale is hundredths, so that the decimal reading is eight hundredths. Similarly, the figures in the fourth example are ordinarily read twenty-seven; the name of the position of the figure 7 in the decimal scale is millionths, giving, therefore, the decimal reading as twenty-seven millionths.

If there should be a whole number before the decimal point, read it as you would read any whole number and read the decimal as you would if the whole number were not there; or, read the whole number and then say "and" so many hundredths, thousandths, or whatever it may be, as "ninety-three and *one* hundred one ten-thousandths."

(4) In addition of decimals, the decimal points must be placed directly under each other, so that tenths will come under tenths, hundredths under hundredths, thousandths under thousandths, etc. The addition is then performed as in whole numbers, the decimal point of the sum being placed directly under the decimal points above.

```
 .1 2 5
 .7
 .0 8 9
 .4 0 0 5
 .9
 .0 0 0 0 2 7
-------------
2.2 1 4 5 2 7   Ans.
```

(5)

```
tenths.
hundredths.
thousandths.
ten-thousandths.
hundred-thousandths.
millionths.
.0 1 7
.2
.0 0 0 0 4 7
-------------
.2 1 7 0 4 7 = Two hundred and seventeen thousand
                and forty-seven millionths.  Ans.
```

(6) (*a*) In subtraction of decimals, place the decimal points directly under each other and proceed as in the subtraction of whole numbers, placing the decimal point in the remainder directly under the decimal points above.

$$\begin{array}{r} 709.6300 \\ .8514 \\ \hline 708.7786 \end{array}$$ Ans.

In the above example, we proceed as follows: We cannot subtract 4 ten-thousandths from 0 ten-thousandths, and as there are no thousandths, we take 1 hundredth from the 3 hundredths. 1 hundredth = 10 thousandths = 100 ten-thousandths. 4 ten-thousandths from 100 ten-thousandths leaves 96 ten-thousandths. 96 ten-thousandths = 9 thousandths + 6 ten-thousandths. Write the 6 ten-thousandths in the ten-thousandths place in the remainder. The next figure in the subtrahend is 1 thousandth. This must be subtracted from the 9 thousandths which is a part of the 1 hundredth taken previously from the 3 hundredths. Subtracting, we have 1 thousandth from 9 thousandths leaves 8 thousandths, the 8 being written in its place in the remainder. Next we have to subtract 5 hundredths from 2 hundredths (1 hundredth having been taken from the 3 hundredths leaves but 2 hundredths now). Since we cannot do this, we take 1 tenth from 6 tenths. 1 tenth = 10 hundredths, and 10 hundredths + 2 hundredths = 12 hundredths. 5 hundredths from 12 hundredths leaves 7 hundredths. Write the 7 in the hundredths place in the remainder. Next we have to subtract 8 tenths from 5 tenths (5 tenths now, because 1 tenth was taken from the 6 tenths). Since this cannot be done, we take 1 unit from the 9 units. 1 unit = 10 tenths. 10 tenths + 5 tenths = 15 tenths, and 8 tenths from 15 tenths leaves 7 tenths. Write the 7 in the tenths place in the remainder. In the minuend we now have 708 units (1 unit having been taken away) and 0 units in the subtrahend. 0 units from 708 units leaves 708 units; hence, we write 708 in the remainder.

(*b*)
$$\begin{array}{r} 81.963 \\ 1.700 \\ \hline 80.263 \end{array}$$ Ans.

(*c*)
$$\begin{array}{r} 18.00 \\ .18 \\ \hline 17.82 \end{array}$$ Ans.

(*d*)
$$\begin{array}{r} 1.000 \\ .001 \\ \hline .999 \end{array}$$ Ans.

(*e*) $872.1 - (.8721 + .008) = ?$ In this problem we are to subtract $(.8721 + .008)$ from 872.1. First perform the operation as indicated by the sign between the decimals enclosed by the parenthesis.

```
 .8 7 2 1
 .0 0 8 0
 ---------
 .8 8 0 1  sum.
```

Subtracting the sum (obtained by adding the decimals enclosed within the parenthesis) from the number 872.1 (as required by the minus sign before the parenthesis), we obtain the required remainder.

```
8 7 2.1 0 0 0
      .8 8 0 1
---------------
8 7 1.2 1 9 9  Ans.
```

(*f*) $(5.028 + .0073) - (6.704 - 2.38) = ?$ First perform the operations as indicated by the signs between the numbers enclosed by the parentheses. The first parenthesis shows that 5.028 and .0073 are to be added. This gives 5.0353 as their sum.

```
 5.0 2 8 0
 .0 0 7 3
 ---------
 5.0 3 5 3  sum.
```

```
 6.7 0 4
 2.3 8 0
 -------
 4.3 2 4  diff.
```

The second parenthesis shows that 2.38 is to be subtracted from 6.704. The difference is found to be 4.324.

The sign between the parentheses indicates that the quantities obtained by performing the above operations are to be subtracted—namely, that 4.324 is to be subtracted from 5.0353. Performing this operation, we obtain .7113 as the final result.

```
 5.0 3 5 3
 4.3 2 4 0
 ---------
  .7 1 1 3  Ans.
```

(7) If the cost of the coal consumed by a nest of steam boilers amounts to \$15.83 on Monday, to \$14.70 on Tuesday, to \$14.28 on Wednesday, to \$13.87 on Thursday, to \$14.98 on Friday, and to \$12.65 on Saturday, then we find the total cost of the week's supply by adding the different amounts together; hence, \$15.83 + \$14.70 + \$14.28 + \$13.87 + \$14.98 + \$12.65 = \$86.31.

```
\$ 1 5.8 3
  1 4.7 0
  1 4.2 8
  1 3.8 7
  1 4.9 8
  1 2.6 5
---------
\$ 8 6.3 1  Ans.
```

(8) $482\frac{4}{5} + 316\frac{1}{3} + 390\frac{3}{4} =$ what ?

When mixed numbers are to be added, add the fractional parts of the mixed numbers separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction. First, we will reduce the fractional parts $\frac{4}{5}$, $\frac{1}{3}$, and $\frac{3}{4}$ to equivalent fractions having the least common denominator. In this case the least common denominator equals the product of the denominators 5, 3, and 4, since we cannot divide any two of them by any number (except 1) without having a remainder, as can be done in the examples in Art. **26,** Part 2. Hence, the least common denominator $= 5 \times 3 \times 4 = 60$. Reducing $\frac{4}{5}$, $\frac{1}{3}$, and $\frac{3}{4}$ to fractions having this least common denominator, we have 60 divided by the first denominator, 5, equals 12. Then, $\frac{4}{5} \times \frac{12}{12} = \frac{48}{60}$. 60 divided by the second denominator, 3, equals 20. Then, $\frac{1}{3} \times \frac{20}{20} = \frac{20}{60}$. 60 divided by the third denominator, 4, equals 15. Then, $\frac{3}{4} \times \frac{15}{15} = \frac{45}{60}$. The sum of these fractions equals

$$\frac{48}{60} + \frac{20}{60} + \frac{45}{60} = \frac{48 + 20 + 45}{60} = \frac{113}{60}, \text{ or } 1\frac{53}{60}.$$

The problem now becomes $482 + 316 + 390 + 1\frac{53}{60}$, the sum of which equals $1,189\frac{53}{60}$.

$$\begin{array}{r} 482 \\ 316 \\ 390 \\ 1\frac{53}{60} \\ \hline 1189\frac{53}{60} \end{array}$$

1500 represents the actual horsepower required.
1189 $\frac{53}{60}$ represents the indicated horsepower of the engines in use.

---

310 $\frac{7}{60}$, or $310.11\frac{2}{3}$ = the H. P. to be developed by the new engine. Ans.

$\frac{7}{60}$ reduced to its equivalent decimal $= \frac{7}{60}$

```
60 ) 7.00 (.11 2/3
     60
     ---
     100
      60
     ---
      40
      ---
      60 = 2/3.
```

(9) Since the inside diameter of the steam pipe is 6.06 inches and the outside diameter is 6.62 inches, there is a difference of 6.62 − 6.06, or .56 of an inch, in both diameters. But .56 of an inch is just twice the thickness of the pipe; hence, the pipe is $\frac{1}{2}$ of .56, or .28 of an inch thick.

(10) (*a*) There are 3 decimal places in the multiplicand and 3 in the multiplier; hence, there are 3 + 3, or 6, decimal places in the product. Since the product contains but four figures, we prefix two ciphers in order to obtain the necessary 6 decimal places.

```
     .107
     .013
     ----
      321
     107
  -------
  .001391   Ans.
```

(*b*)

```
      203
      2.03
      ----
      609
     000
    406
   -------
    412.09
      .203
   -------
   123627
   00000
  82418
  -------
  83.65427   Ans.
```

There are 2 decimal places in the multiplier and none in the multiplicand; hence, there are 2 + 0, or 2, decimal places in the first product.

Since in the second multiplication there are 2 decimal places in the multiplicand and 3 decimal places in the multiplier, there are 3 + 2, or 5, decimal places in the second product.

When there are one or more ciphers in the multiplier, multiply just the same as with the other figures.

(*c*) First perform the operations indicated by the signs between the numbers enclosed by the parentheses, and then whatever may be required by the sign between the parentheses.

```
  3 1.8 5
      2.7
 ---------
 2 2 2 9 5
 6 3 7 0
 ---------
 8 5.9 9 5
```

The first parenthesis shows that the numbers 2.7 and 31.85 are to be multiplied together.

The second parenthesis shows that .316 is to be taken from 3.16.

```
 3.1 6 0
  .3 1 6
 -------
 2.8 4 4
```

The product obtained by performing the operation indicated by the signs within the first parenthesis is now multiplied by the remainder obtained by performing the operation indicated by the signs within the second parenthesis.

```
      8 5.9 9 5
        2.8 4 4
    -----------
    3 4 3 9 8 0
  3 4 3 9 8 0
6 8 7 9 6 0
1 7 1 9 9 0
-------------
2 4 4.5 6 9 7 8 0  Ans.
```

(*d*) $(107.8 + 6.541 - 31.96) \times 1.742 = ?$

```
    1 0 7.8                    8 2.3 8 1
+     6.5 4 1              ×   1.7 4 2
  -----------              -------------
    1 1 4.3 4 1                1 6 4 7 6 2
-     3 1.9 6                3 2 9 5 2 4
  -----------              5 7 6 6 6 7
      8 2.3 8 1            8 2 3 8 1
                           -------------
                           1 4 3.5 0 7 7 0 2  Ans.
```

(11) If one 3-inch tube measures $15\frac{1}{2}$ ft. in length, 60 of these tubes would measure $60 \times 15\frac{1}{2}$ ft., or 930 ft. in length. If 1 foot of tubing heats a surface of .728 sq. ft., then it is evident that 930 ft. of tubing would heat a surface of $930 \times .728$ ft., or 677.04 sq. ft.

```
1 5 ½
  6 0
-----
9 0 0
  3 0
-----
9 3 0 ft.
```

```
    .7 2 8
      9 3 0
  ---------
  2 1 8 4 0
6 5 5 2
-----------
6 7 7.0 4 0 sq. ft.
```

(12) (*a*) $\dfrac{7}{\frac{3}{16}} = 7 \div \frac{3}{16} = 7 \times \frac{16}{3} = \dfrac{7 \times 16}{3} = 11\frac{2}{3} = 37\frac{1}{3}$. Ans.

The heavy line indicates that 7 is to be divided by $\frac{3}{16}$.

(*b*) $\dfrac{\frac{15}{32}}{\frac{5}{8}} = \frac{15}{32} \div \frac{5}{8} = \frac{15}{32} \times \frac{8}{5} = \dfrac{\cancel{15} \times \cancel{8}}{\cancel{32} \times \cancel{5}} = \frac{3}{4} = .75$. Ans.

(*c*) $\dfrac{1.25 \times 20 \times 3}{\frac{87 + 88}{459 + 32}} = ?$ In this problem $1.25 \times 20 \times 3$ constitutes the numerator of the complex fraction.

$$\begin{array}{r} 1.2\ 5 \\ \times \quad 2\ 0 \\ \hline 2\ 5.0\ 0 \\ \times \quad 3 \\ \hline 7\ 5 \end{array}$$

Multiplying the factors of the numerator together, we find their product to be 75.

The fraction $\dfrac{87 + 88}{459 + 32}$ constitutes the denominator of the complex fraction. The value of the numerator of this fraction equals $87 + 88 = 175$.

The value of the denominator of this fraction is equal to $459 + 32 = 491$. The problem then becomes

$$\frac{75}{\frac{175}{491}} = \frac{\frac{75}{1}}{\frac{175}{491}} = \frac{75}{1} \div \frac{175}{491} = \frac{75}{1} \times \frac{491}{175} = \frac{\cancel{75} \times 491}{\cancel{175}} = \frac{1{,}473}{7} = 210\frac{3}{7}.$$

Ans.

(13) The pitch of the rivets is the distance between the centers of the rivets. Hence, since the distance around the cylindrical boiler is 166.85 in., and there are 72 rivets in one of the seams, the pitch of the rivets equals $166.85 \div 72 = 2.317+$ in. Ans.

$$\begin{array}{r} 7\ 2\ )\ 1\ 6\ 6.8\ 5\ 0\ (\ 2.3\ 1\ 7+ \\ \underline{1\ 4\ 4} \\ 2\ 2\ 8 \\ \underline{2\ 1\ 6} \\ 1\ 2\ 5 \\ \underline{7\ 2} \\ 5\ 3\ 0 \\ \underline{5\ 0\ 4} \\ 2\ 6 \end{array}$$

(14) If a keg containing 133 boiler rivets weighs 100 pounds, then each rivet must weigh as much as 133 is contained times in 100, or .75 of a pound.

```
1 3 3 ) 1 0 0.0 0 (.7 5+   Ans.
        9 3 1
        -----
          6 9 0
          6 6 5
          -----
            2 5
```

Since there are 2 decimal places in the dividend and 0 decimal places in the divisor, we must point off $2-0 = 2$ decimal places in the quotient, or answer.

(15) $.875 = \frac{875}{1,000} = \frac{175}{200} = \frac{7}{8}$ of a foot.

1 foot = 12 inches.

$$\tfrac{7}{8} \text{ of 1 foot} = \frac{7}{\underset{2}{\cancel{8}}} \times \frac{\overset{3}{\cancel{12}}}{1} = \frac{21}{2} = 10\tfrac{1}{2} \text{ inches.} \quad \text{Ans.}$$

(16) 12 inches = 1 foot.

$$\tfrac{3}{16} \text{ of an inch} = \tfrac{3}{16} \div 12 = \frac{\cancel{3}}{16} \times \frac{1}{\underset{4}{\cancel{12}}} = \tfrac{1}{64} \text{ of a foot.}$$

```
1
---
6 4 ) 1.0 0 0 0 0 0 (.0 1 5 6 2 5   Ans.
      6 4
      -----
      3 6 0
      3 2 0
      -------
        4 0 0
        3 8 4
        -------
          1 6 0
          1 2 8
          -------
            3 2 0
            3 2 0
            -------
                0
```

Point off 6 decimal places in the quotient, since we annexed six ciphers to the dividend, the divisor containing no decimal places; hence, $6 - 0 = 6$ places to be pointed off.

(17) The total horsepower developed equals $48.63 + 45.7 + 46.32 + 47.9 + 48.74 + 48.38 + 48.59 = 334.26$.

Since the horsepower developed equals 334.26, then the average horsepower developed must equal $334.26 \div 7$, or 47.75+ H. P.

$$
\begin{array}{r}
48.63 \\
45.7 \\
46.32 \\
47.9 \\
48.74 \\
48.38 \\
48.59 \\
\hline
7\,)\,334.26 \\
\hline
47.75+
\end{array}
$$

Ans.

ARITHMETIC.

(PART 4.)

---

(1) A certain per cent. of a number means so many hundredths of that number.

25% of 8,428 lb. means 25 hundredths of 8,428 lb. $\frac{25}{100}$ = .25. Hence, 8,428 lb. × .25 = 2,107 lb. Ans.

(2) ½% means one-half of 1 per cent. Since 1% is .01, ½% is .005, for $\begin{array}{r} 2\,)\,.0\,1\,0 \\ \hline 0\,0\,5 \end{array}$. And \$35,000 × .005 = \$175. Ans.

$$\begin{array}{r} \$\,3\,5\,0\,0\,0 \\ .0\,0\,5 \\ \hline \$\,1\,7\,5.0\,0\,0 \end{array}$$

(3) If 2 is a certain per cent. of 50, then 50 multiplied by a certain rate gives a product of 2, and that rate is equal to 2 divided by 50. Dividing 2 by 50, the quotient is .04, which means that 2 is 4% of 50, or, since percentage = base × rate,

$$\begin{array}{l} 5\,0\,)\,2.0\,0\,(\,.0\,4 \quad \text{Ans.} \\ \quad\;\; \underline{2\,0\,0} \end{array}$$

rate = percentage ÷ base
= 2 ÷ 50 = .04, or 4%. Ans.

(4) Since percentage = base × rate, rate = percentage ÷ base.

As percentage = 10 and base = 10, we have rate = 10 ÷ 10 = 1.

§ 2

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But $1 = \frac{100}{100}$ and $\frac{100}{100} = 100\%$; hence, the rate (1) means that 10 is 100% of 10.

(5) Since 5,500 lb. represent an increase of 15% over the consumption when the condenser is used, 5,500 lb. must be the amount, .15 the rate, and the number of pounds consumed when the condenser is running (to be found) the base.

$$\text{Base} = \text{amount} \div (1 + \text{rate}) = 5{,}500 \div (1 + .15)$$
$$= 5{,}500 \div 1.15 = 4{,}782.61 \text{ lb., nearly.} \quad \text{Ans.}$$

```
1.1 5 ) 5 5 0 0.0 0 0 0 ( 4 7 8 2.6 1
        4 6 0
        -----
          9 0 0
          8 0 5
          -----
            9 5 0
            9 2 0
            -----
              3 0 0
              2 3 0
              -----
                7 0 0
                6 9 0
                -----
                  1 0 0
                  1 1 5
```

Or, this problem could also have been solved as follows:

100% = the number of pounds consumed when the condenser is running. If there is a gain of 15%, then 100% + 15%, or 115% = 5,500 lb., the amount used when the condenser is not running. If 115% = 5,500 lb., $1\% = \frac{1}{115}$ of 5,500 = 47.8261 lb., and 100% = 100 × 47.8261 = 4,782.61 lb. Ans.

(6)
$$24\% \text{ of } \$950 = 950 \times .24 = \$228$$
$$12\tfrac{1}{2}\% \text{ of } \$950 = 950 \times .125 = 118.75$$
$$17\% \text{ of } \$950 = 950 \times .17 = 161.50$$
$$53\tfrac{1}{2}\% \text{ of } \$950 = \$508.25$$

The total amount of his yearly expenses, then, is \$508.25; hence, his savings are \$950 − \$508.25 = \$441.75. Ans.

Or, as above, $24\% + 12\frac{1}{2}\% + 17\% = 53\frac{1}{2}\%$, the total percentage of expenditures; hence,

$\$950 \times .535 = \$508.25$, and
$\$950 - \$508.25 = \$441.75$ = his yearly savings. Ans.

(7) The percentage is 961.38 and the rate is $.37\frac{1}{2}$.

Base = percentage ÷ rate
= 961.38 ÷ .375 = 2,563.68, the number. Ans.

```
.3 7 5 ) 9 6 1.3 8 0 0 0 ( 2 5 6 3.6 8
         7 5 0
         ------
         2 1 1 3
         1 8 7 5
         -------
           2 3 8 8
           2 2 5 0
           -------
             1 3 8 0
             1 1 2 5
             -------
               2 5 5 0
               2 2 5 0
               -------
                 3 0 0 0
                 3 0 0 0
                 -------
```

Another method of solving is the following:

If $37\frac{1}{2}\%$ of a number is 961.38, then $.37\frac{1}{2}$ times the number = 961.38 and the number = 961.38 ÷ $.37\frac{1}{2}$, which, as above = 2,563.68. Ans.

(8) 298 revolutions per minute with the load = base, $.01\frac{1}{2}$ = rate, and the amount (to be found) will equal the speed of the engine when running unloaded.

Amount = base × (1 + rate)
= 298 × (1 + .015) = 302.47 rev. per min. Ans.

```
      2 9 8
  ×   1.0 1 5
      -------
      1 4 9 0
      2 9 8
    0 0 0
  2 9 8
  -------
  3 0 2.4 7 0
```

(9) 4 yd. 2 ft. 10 in. to inches.

$\times$ 3

12

$+$ 2

14 feet

$\times$ 12

28

14

168

$+$ 10

178 inches. Ans.

Since there are 3 feet in 1 yard, in 4 yards there are 4×3 feet, or 12 feet. 12 feet plus 2 feet = 14 feet.

There are 12 inches in 1 foot; therefore, in 14 feet there are 12×14, or 168 inches. 168 inches plus 10 inches = 178 inches.

(10) 12) 3722 inches.

3) 310 + 2 inches.

103 + 1 foot.

Ans. = 103 yd. 1 ft. 2 in.

EXPLANATION.—There are 12 inches in 1 foot; hence, in 3,722 inches there are as many feet as 12 is contained times in 3,722, or 310 feet and 2 inches remaining. Write 2 inches as a remainder. There are 3 feet in 1 yard; hence, in 310 yards there are as many feet as 3 is contained times in 310, or 103 yards and 1 foot remaining. Hence, in 3,722 inches there are 103 yd. 1 ft. 2 in.

(11) 1728) 764325 cu. in.

27) 442 + 549 cu. in.

16 cu. yd. + 10 cu. ft.

Ans. = 16 cu. yd. 10 cu. ft. 549 cu. in.

EXPLANATION.—There are 1,728 cubic inches in 1 cubic foot; hence, in 764,325 cu. in. there are as many cubic feet as 1,728 is contained times in 764,325, or 442 cubic feet and 549 cubic inches remaining. Write the 549 cubic inches as a remainder. There are 27 cubic feet in 1 cubic yard; hence, in 442 cubic feet there are as many cubic yards as 27

is contained times in 442 cubic feet, or 16 cubic yards and 10 cubic feet remaining. Then, in 764,325 cubic inches there are 16 cu. yd. 10 cu. ft. 549 cu. in.

(12)

	T.	cwt.	lb.
	16	8	75
×		20	
		320	
+		8	
		328	cwt.
×		100	
		32800	
+		75	
		32875	lb. Ans.

Since in 1 ton there are 20 cwt., in 16 tons there are $16 \times 20 = 320$ cwt. 320 cwt. $+ 8$ cwt. $= 328$ cwt. There are 100 lb. in 1 cwt.; hence, in 328 cwt. there are $328 \times 100 = 32{,}800$ lb. $32{,}800$ lb. $+ 75$ lb. $= 32{,}875$ lb. Ans.

(13)

$$100\overline{)25396}\text{ lb.}$$
$$20\overline{)253}\text{ cwt.} + 96\text{ lb.}$$
$$12\text{ T.} + 13\text{ cwt.}$$

There are 100 lb. in 1 cwt.; hence, in 25,396 lb. there are as many cwt. as 100 is contained times in 25,396, or 253 cwt. and 96 lb. remaining.

There are 20 cwt. in 1 ton, and in 253 cwt. there are as many tons as 20 is contained times in 253, or 12 tons and 13 cwt. remaining. Hence, 25,396 lb. = 12 T. 13 cwt. 96 lb. Ans.

(14) Arrange the different terms in columns, taking care to have like denominations in the same column. We begin to add at the right-hand column. $7 + 9 + 3 = 19$ in.; since 12 in. = 1 ft., 19 in. = 1 ft. and 7 in. Place the 7 in. in the inches column and reserve the 1 ft. to add to the sum of the feet. $2 + 1 + 2 + 1$ (reserved) $= 6$ ft. Since 3 ft. = 1 yd., 6 ft. = 2 yd. and 0 ft. remaining. Place the 0 in the feet column and reserve the 2 yd. to add to the sum of the yards. $4 + 2 + 2$ (reserved) $= 8$ yd., which we place in yards column. Ans. = 8 yd. 7 in.

yd.	ft.	in.	
2	2	3	
4	1	9	
	2	7	
8	0	7	Ans.

(15) Since 10 gal. 2 qt. 1 pt. of machine oil is sold at one time and 16 gal. 3 qt. at another time, together there was sold 27 gal. 1 qt. 1 pt. 0 + 1 = 1 pt. We cannot reduce 1 pt. to any higher denomination, so place it under pints column. 3 qt. + 2 qt. = 5 qt. Since 4 qt. = 1 gal., 5 qt. = 1 gal. and 1 qt. remaining. Place 1 qt. under quarts column and reserve the 1 gal. to add to the gallons. 16 gal. + 10 gal. + 1 gal. (reserved) = 27 gal.

gal.	qt.	pt.
10	2	1
16	3	0
27	1	1

Since the barrel contained 31½, or 31.5 gal., and 27 gal. 1 qt. 1 pt. were sold, there remained the difference, or 4 gal. 1 pt. 31.5 gal. = 31 gal. 2 qt., since .5 = ½, and ½ of 1 gal. = ½ of 4 qt. = 2 qt.

gal.	qt.	pt.	
31	2	0	
27	1	1	
4	0	1	Ans.

1 pt. cannot be taken from 0 pt., so we take 1 qt. from the 2 qt. The 1 qt. taken = 2 pt. 1 pt. from 2 pt. = 1 pt. Place 1 pt. under pints column. Since we took 1 qt. from the quarts column, there remains 2 − 1, or 1 qt. 1 qt. from 1 qt. leaves 0 qt. Place 0 qt. under the quarts column. 27 gal. from 31 gal. leaves 4 gal. Place 4 gal. under the gallons column. We therefore find that 4 gal. 1 pt. of machine oil remained in the barrel.

(16) In multiplication of denominate numbers, we place the multiplier under the lowest denomination of the multiplicand, as

17 ft. 3 in.
51
879 ft. 9 in.

and begin at the right to multiply. 51 × 3 = 153 in. Since there are 12 in. in 1 ft., in 153 in. there are as many feet as 12 is contained times in 153, or 12 ft. and 9 in. remaining. Place the 9 in. under the inches and reserve the 12 ft. 51 × 17 ft. = 867 ft.; 867 ft. + 12 ft. (reserved) = 879 ft. 879 ft.

can be reduced to higher denominations by dividing by 3 ft. to find the number of yards, and by $5\frac{1}{2}$ to find the number of rods.

$$\begin{array}{r} 3\,)\,8\,7\,9 \text{ ft. } 9 \text{ in.} \\ 5.5\,)\,2\,9\,3 \text{ yd.} \\ 5\,3 \text{ rd. } 1\frac{1}{2} \text{ yd.} \end{array}$$

Then, 879 ft. 9 in. = 53 rd. $1\frac{1}{2}$ yd. 0 ft. 9 in., or 53 rd. 1 yd. 2 ft. 3 in.

(17) Since 2 pt. = 1 qt., 3 qt. = 3 × 2, or 6 pt. 6 pt. + 1 pt. = 7 pt. 4.7 × 7 = 32.9 pt. Ans.

qt.	pt.
3	1
× 2	
6	
+ 1	
7 pt.	

7 pt.
4.7
4 9
2 8
3 2.9 pt.

(18) If there are four lengths, each 15 ft. 5 in., 15 ft. 5 in. × 4 = 60 ft. 20 in., or the length of the four pieces.

ft.	in.
6 0	2 0
1 4	8
8	1 0
8 2	3 8, or 85 ft. 2 in. = the length of the shaft.

1 5 ft.	5 in.
	4
6 0 ft.	2 0 in.

From the length of the shaft we must subtract 8 in. × 2 = 16 in. to get the distance between the end hangers.

ft.	in.
8 2	3 8
	1 6
8 2	2 2, or 83 ft. 10 in.

Since there are six hangers, there are *five* spaces. The length of one space is 83 ft. 10 in ÷ 5 = 16 ft. $9\frac{1}{5}$ in. Ans.

(19) Reducing 18 ft. $11\frac{1}{4}$ in. to inches, we have $227\frac{1}{4}$ in., or 227.25 in.

```
  ft.      in.
  1 8      1 1 ¼
  1 2
 ------
  3 6
1 8
 ------
2 1 6
  1 1 ¼
 ------
2 2 7 ¼ in.
```

$1\frac{1}{8} \times 2$, or $2\frac{1}{4}$ in., for the two end rivets is deducted from the length, leaving 225 in., which is divided into equal spaces by the rivets.

```
  2 2 7.2 5 in.
—     2.2 5 in.
 ----------
  2 2 5     in.
```

The pitch of the rivets (or the distance between their centers) is $1\frac{1}{4}$ in., or 1.25 in.; hence, $225 \div 1.25 = 180$ spaces between the rivets. But since there will be one more rivet than the number of spaces, the number of rivets required for this boiler shell will be $180 + 1 = 181$. Ans.

```
1.2 5 ) 2 2 5.0 0 ( 1 8 0
        1 2 5
        -------
        1 0 0 0
        1 0 0 0
        -------
              0
```

ARITHMETIC.

(PART 5.)

(1) To find the second power of a number, we must multiply the number by itself once; that is, use the number twice as a factor. Thus, the second power of 108 is $108 \times 108 = 11{,}664$.

$$\begin{array}{r} 108 \\ 108 \\ \hline 864 \\ 108 \\ \hline 11664 \end{array}$$ Ans.

(2) The third power of 181.25 equals the number obtained by using 181.25 as a factor three times. Thus, the third power of 181.25 is $181.25 \times 181.25 \times 181.25 = 5{,}954{,}345.703125$.

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2 + 2 = 4$ decimal places in the first product.

Since there are 4 decimal places in the multiplicand and 2 in the multiplier, there are $4 + 2 = 6$ decimal places in the final product.

$$\begin{array}{r} 181.25 \\ 181.25 \\ \hline 90625 \\ 36250 \\ 18125 \\ 145000 \\ 18125 \\ \hline 32851.5625 \\ 181.25 \\ \hline 164257 8125 \\ 65703125 0 \\ 3285156 25 \\ 26281250 00 \\ 3285156 25 \\ \hline 5954345.703125 \end{array}$$ Ans.

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(3) (*a*) $.0133^3 = .0133 \times .0133 \times .0133 = .000002352637.$

```
          .0133
          .0133
          -----
            399
           399
          133
      ---------
      .00017689
          .0133
      ---------
          53067
         53067
        17689
      ---------
      .000002352637  Ans.
```

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, we should point off $4 + 4 = 8$ decimal places in the product; but as there are only 5 figures in the product, we prefix 3 ciphers to form the 8 necessary decimal places in the first product.

Since there are 8 decimal places in the multiplicand and 4 in the multiplier, we should point off $8 + 4 = 12$ decimal places in the product; but as there are only 7 figures in the product, we prefix 5 ciphers to make the necessary 12 decimal places in the final product.

(*b*) $301.011^3 = 301.011 \times 301.011 \times 301.011$
$= 27{,}273{,}890.942264331.$

```
                  301.011
                  301.011
                  -------
                  301011
                 301011
               301011
             903033
             ----------
             90607.622121
                  301.011
             ----------
             90607622121
            90607622121
          90607622121
        271822866363
        ------------------
        27273890.942264331  Ans.
```

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, we should point off $3 + 3 = 6$ decimal places in the first product.

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, we should point off $6 + 3 = 9$ decimal places in the final product.

(*c*) $(\frac{1}{8})^3 = \frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} = \frac{1 \times 1 \times 1}{8 \times 8 \times 8} = \frac{1}{512}$. Ans.

$$\begin{array}{r} 8 \\ \times\ 8 \\ \hline 64 \\ \times\ 8 \\ \hline 512 \end{array}$$

(*d*) To find any power of a mixed number, first reduce it to an improper fraction, and then multiply the numerators together for the numerator of the answer and multiply the denominators together for the denominator of the answer.

$$(3\tfrac{3}{4})^3 = \tfrac{15}{4} \times \tfrac{15}{4} \times \tfrac{15}{4} = \frac{15 \times 15 \times 15}{4 \times 4 \times 4} = \frac{3,375}{64} = 52.734375.$$

Ans.

$$3\tfrac{3}{4} = \frac{(3 \times 4) + 3}{4} = \frac{12 + 3}{4} = \tfrac{15}{4}.$$

$$\begin{array}{r} 15 \\ 15 \\ \hline 75 \\ 15 \\ \hline 225 \\ 15 \\ \hline 1125 \\ 225 \\ \hline 3375 \end{array}$$

$$\begin{array}{l} 64)3375.000000(52.734375 \\ \quad 320 \\ \quad \hline \quad 175 \\ \quad 128 \\ \quad \hline \quad 470 \\ \quad 448 \\ \quad \hline \quad 220 \\ \quad 192 \\ \quad \hline \quad 280 \\ \quad 256 \\ \quad \hline \quad 240 \\ \quad 192 \\ \quad \hline \quad 480 \\ \quad 448 \\ \quad \hline \quad 320 \\ \quad 320 \\ \quad \hline \end{array}$$

Since 6 ciphers were annexed to the dividend, 6 decimal places must be pointed off in the quotient.

(4) (*a*)

(*a*) 1 $\sqrt{3'48'67'84.40'10} = 1867.29+$ Ans.

1 (*b*) 1

(*d*) 20 (*c*) 248

8 224

28 (*e*) 2467

8 2196

360 27184

6 26089

366 109540

6 74684

3720 3485610

7 3361041

3727 124569

7

37340

2

37342

2

373440

9

373449

EXPLANATION.—(I) Divide the number into periods. In the above case, where the number consists of a whole number and a decimal, we begin at the decimal point and proceed to the *left* in pointing off the *whole* number and towards the *right* in pointing off the *decimal*, annexing a 0 to complete the last decimal period. In *square root two* figures constitute a period.

Find the greatest single number whose square is *less* than or *equal* to 3, the first period. This is evidently 1, since $2^2 = 4$, which is greater than 3. Write 1 as the first figure of the root; also write it to the left, as shown at (*a*). Now, multiply the 1 at (*a*) by the 1 in the root and write the result under the first period, or 3, as shown at (*b*). Subtract, bring down the next period, 48, and annex it to the remainder 2, thus making 248 the *dividend*, as shown at (*c*). Add the root already found to the 1 at (*a*), thereby

obtaining 2, and annex a cipher to this 2, thus making it 20, which we call the *trial divisor.*

Divide the dividend 248 at (*c*) by the trial divisor 20 at (*d*) and obtain 8, which is probably the next figure of the root. Write 8 in the root, as shown, and also add it to 20, the trial divisor, making it 28. This is called the *complete divisor.*

(II) Multiply the complete divisor 28 by 8, the second figure of the root, and subtract the result from the dividend (*c*). The remainder is 24, to which annex the next period, making 2,467, as shown at (*e*), which call the *new dividend.*

Add the second figure of the root, or 8, to the divisor 28 and annex a cipher, thus obtaining 360. Dividing 2467 by 360, we find 6 to be the next figure of the root. Adding this last figure of the root, or 6, to 360, we get 366, and multiplying by this last figure of the root, or 6, gives 2196, which we write under 2467 and subtract.

(III) Annexing the next period, 84, to the remainder 271 gives 27184 as the next new dividend. Now, adding the third figure of the root to 366 and annexing a cipher, as before, we have 3720. Dividing 27184 by 3720, the result is 7, which write as the next figure of the root. Adding the fourth figure of the root, or 7, to 3720, we get 3727, and multiplying by 7 of the root gives 26089, which write under 27184 and subtract, obtaining 1095 as a remainder.

(IV) Annexing the next period, or 40, to the remainder 1095 gives 109540 as the next new dividend. Adding the last figure of the root to 3727 and annexing a cipher, as before, the result is 37340. Dividing 109540 by 37340, the result is 2, which write as the next figure of the root. Adding the fifth figure of the root, or 2, to 37340, we obtain 37342, and multiplying by 2 of the root gives 74684, which write under 109540 and subtract, obtaining 34856 as a remainder.

(V) Annexing the next period, or 10, to the remainder 34856 gives 3485610 as the next new dividend. Now, adding

the last figure of the root to 37342 and annexing a cipher, as before, the result is 373440. Dividing 3485610 by 373440 gives 9 as a result, which write as the next, or last figure, of the root. Adding the last figure of the root, or 9, to 373440, we get 373449, and multiplying by 9 of the root gives 3361041, which write under 3485610 and subtract. Since there is a remainder, we know the given power is not a perfect square, so we place + after the root.

In this problem there are *six* periods—four in the whole number and two in the decimal—hence, there will be *six* figures in the root (since we obtain one figure of the root for each period), four figures constituting the whole number and two figures the decimal of the root. Hence,

$$\sqrt{3{,}486{,}784.401} = 1{,}867.29+.$$

```
(b) (a)  3        √9'0 0'0 0'9 9.4 0'0 9'0 0 = 3 0 0 0.0 1 6 +
         3   (b)  9
         ─          ─────────────
    (d)  6 0 (c)    0 0 0 0 9 9 4 0 0 9
           0              6 0 0 0 0 1
         ─────          ─────────────
         6 0 0          3 9 4 0 0 8 0 0
             0          3 6 0 0 0 1 5 6
         ─────          ─────────────
         6 0 0 0          3 4 0 0 6 4 4
             0
         ───────
         6 0 0 0 0
               0
         ─────────
         6 0 0 0 0 0
               1
         ─────────
         6 0 0 0 0 1
               1
         ─────────
         6 0 0 0 0 2 0
                   6
         ─────────────
         6 0 0 0 0 2 6
```

EXPLANATION.—Beginning at the decimal point, we point off the whole number into periods of two figures each, proceeding from right to left; also point off the decimals into periods of two figures each, proceeding from left to right. The largest number whose square is contained in the first period, 9, is 3; hence, 3 is the first figure of the root. Place 3 at the left, as shown at (*a*), and multiply it by the first figure in the root, or 3. The result is 9. Write 9 under the

first period, 9, as at (*b*), subtract, and there is no remainder. Bring down the next period, which is 00, as shown at (*c*). Add the root already found to the 3 at (*a*), obtaining 6, and annex a cipher to this 6, thus making it 60, which is the *trial divisor*, as shown at (*d*). Divide the dividend (*c*) by the trial divisor and obtain 0 as the next figure in the root. Write 0 in the root as shown, and also add it to the trial divisor 60, and annex a cipher, thereby making the next trial divisor 600. Bring down the next period, 00, annex it to the dividend already obtained, and divide it by the trial divisor. 600 is contained in 0000 no times, so we place another cipher in the root. Write 0 in the root, as shown, and also add it to the trial divisor 600, and annex a cipher, thereby making the next trial divisor 6000. Bring down the next period, 99. The trial divisor 6000 is contained in 000099 no times, so we place 0 as the next figure in the root, as shown, and also add it to the trial divisor 6000 and annex a cipher, thereby making the next trial divisor 60000. Bring down the next period, 40, and annex it to the dividend already obtained to form the new dividend, 00009940, and divide it by the trial divisor 60000. 60000 is contained in 00009940 no times, so we place another cipher in the root, as shown, and also add it to the trial divisor 60000 and annex one cipher, thereby making the next trial divisor 600000. Bring down the next period, 09, and annex it to the dividend already obtained to form the new dividend, 0000994009, and divide it by the trial divisor 600000. 600000 is contained in 0000994009 once, so we place 1 as the next figure in the root and also add it to the trial divisor 600000, thereby making the complete divisor 600001. Multiply the complete divisor 600001 by 1, the sixth figure in the root, and subtract the result obtained from the dividend. The remainder is 394008, to which we annex the next period, 00, to form the next new dividend, or 39400800. Add the sixth figure of the root, or 1, to the divisor 600001 and annex a cipher, thus obtaining 6000020 as the next trial divisor. Dividing 39400800 by 6000020, we find 6 to be the next figure of the root. Adding this last figure, 6, to the trial divisor, we obtain 6000026 for our next

complete divisor, and multiplying by the last figure of the root, or 6, gives 36000156, which write under 39400800 and subtract. Since there is a remainder, it is clearly evident that the given power is not a perfect square; and since the next figure of the root is 5, we increase the last figure just found by 1 and write — after the root.

In this problem there are *seven* periods—four in the whole number and three in the decimal; hence, there will be *seven* figures in the root, *four* figures constituting the whole number and three figures the decimal of the root. Hence, $\sqrt{9,000,099.4009} = 3,000.017-$. Ans.

(5) Beginning at the decimal point and pointing off into periods of three figures each, we have .327'680, annexing a cipher to complete the right-hand period. Since the number is entirely decimal, the root is entirely decimal. Neglecting the decimal point for the present, we find the cube root of 327,680. Referring to the table in Art. **30**, 327,680 lies between 314,432, the cube of 68, and 328,509, the cube of 69; the first two figures of the root are, therefore, 68. The first difference is 328,509 — 314,432 = 14,077; the second difference is 327,680 — 314,432 = 13,248; and 13,248 ÷ 14,077 =.94+. Therefore, the first four figures of the root are 6894. Locating the decimal point according to Art. **32**, we have the $\sqrt[3]{.32768} = .6894+$. Ans.

		14077) 13248.00 (.94+
		12669
328509	327680	5870
314432	314432	5630
14077	13248	1562

(6) Instead of finding the cube root of 2, we annex a cipher period and find the cube root of 2,000, in order to obtain two figures of the root from the table (see Art. **32**). Referring to the table, 2,000 lies between 1,728, the cube of 12, and 2,197, the cube of 13; the first two figures of the root are, therefore, 12. The first difference is 2,197 — 1,728 = 469; the second difference is 2,000 — 1,728 = 272. and 272 ÷ 469 = .58—. Therefore, the first four figures of the

root are 1258. Since there is only one period in the number, the whole-number part of the root will contain only one figure; hence, the root is 1.258—. We need one more figure, however; hence, proceeding as in Art. **35**, $2 \div 1.258 = 1.58983$; $1.58983 \div 1.258 = 1.26377$; $1.258 + 1.258 + 1.26377 = 3.77977$; $3.77977 \div 3 = 1.25992$. Therefore, $\sqrt[3]{2} = 1.2599$ to four decimal places.

```
2197    2000      469)272.000(.579+, or .58—
1728    1728          2345
 469     272          3750
                      3283
                       4670
                       4221

1.258)2.000000000(1.589825+, or 1.589983—
      1258
       7420
       6290
       11300      1.258)1.589830000(1.26377
       10064              1258
        12360              3318
        11322              2516
         10380              8023
         10064              7548
           3160              4750
           2516              3774
            6440              9760
            6290              8806
                               9540
                               8806
  1.258
  1.258
  1.26377
3)3.77977
  1.25992, or 1.2599  Ans.
```

(7) (*a*)

1	$\sqrt{1'23.21}$ = 11.1 Ans.	
1	1	
20	23	
1	21	
21	221	
1	221	
220		
1		
221		

(*b*)

1	$\sqrt{1'14.92'10}$ = 10.72+ Ans.	
1	1	
200	1492	
7	1449	
207	4310	
7	4284	
2140	26	
2		
2142		

(8) (*a*) Since the number is entirely decimal, the root is entirely decimal. Adding two ciphers to complete the second period, and neglecting the decimal point, we find the cube root of 6,500. Referring to the table, 6,500 lies between 5,832, the cube of 18, and 6,859, the cube of 19; the first two figures of the root are 18. The first difference is 6,859 − 5,832 = 1,027; the second difference is 6,500 − 5,832 = 668; and 668 ÷ 1,027 = .65+. Therefore, the first four figures of the root are .1865+. Since five figures are required, we proceed as in Art. **35**; .0065 ÷ .1865 = .03485254; .03485254 ÷ .1865 = .186876; .1865 + .1865 + .186876 = .559876; .559876 ÷ 3 = .186625, or .18663−, correct to five figures.

6859	6500	1027)	668.00 (.65+
5832	5832		6162
1027	668		5180
			5135

.1865).	00650000000000 (.03485254+
	5595
	9050
	7460
	15900
	14920
	9800
	9325
	4750
	3730
	10200
	9325
	8750
	7460
	1290

```
.1865).034852540 0 (.186876+
      1865
      -----
      16202
      14920
      -----
       12825
       11190
       -----
        16354
        14920
        -----
         14340
         13055
         -----
          12850
          11190
          -----
           1660

 1865
.1865
.186876
-------
3).559876
-------
```

.186625+, or .18663—. Ans.

(*b*) Since the number is entirely decimal, the root is entirely decimal. Pointing off into periods, we obtain .000'021. The figures of the root will be the same as for the cube root of 21. Hence, consider the given number as 21 and annex a cipher period so that two figures of the root may be obtained from the table; in other words, we extract the cube root of 21,000. Referring to the table, 21,000 lies between 19,683, the cube of 27, and 21,952, the cube of 28; the first two figures of the root are, therefore, 27. The first difference is 21,952 — 19,683 = 2,269; the second difference is 21,000 — 19,683 = 1,317; and 1,317 ÷ 2,269 = .58+. Hence, the first four figures of the root are 2758. Locating the decimal point according to the principle given in Art. **32,** the root is .02758+. The result does not agree with the printed answer. Hence, proceeding as directed in the note following example 3 (see Examination Questions), we apply the method described in Arts. **35** and **36.** .000021 ÷ .02758 = .000761421; .000761421 ÷ .02758 = .027607; .02758 + .02758 + .027607 = .082767; .082767 ÷ 3 = .027589, or .02759—, correct to four significant figures.

```
21952     21000      2269)1317.00(.58+
19683     19683          11345
-----     -----          -----
 2269      1317           18250
                          18152
                          -----
                             98
```

```
.02758).0000210000000000(.000761421+
              19306
              -----
               16940
               16548
               -----
                 3920
                 2758
                 ----
                11620
                11032
                -----
                  .5880
                   5516
                   ----
                    3640
                    2758
                    ----
                     882
```

```
02758).000761421000(.027607+
            5516
            ----
            20982
            19306
            -----
             16761          .02758
             16548          .02758
             -----          .027607
                21300       -------
                19306    3).082767
                -----       -------
                 1994       .027589, or
                                .02759— Ans.
```

(9) $11.7 : 13 :: 20 : x$ The product of the means equals the product of the extremes.

$11.7\,x = 13 \times 20$

$11.7\,x = 260$

$$x = \frac{260}{11.7}$$

```
11.7 ) 260.000 ( 22.22+   Ans.
       234
       ---
        260
        234
        ---
         260
         234
         ---
          260
          234
          ---
           26
```

(10) (*a*) $20 + 7 : 10 + 8 :: 3 : x.$

$27 : 18 :: 3 : x$

$27\,x = 18 \times 3$

$27\,x = 54$

$$x = \frac{54}{27} = 2. \quad \text{Ans.}$$

(*b*) $12^2 : 100^2 :: 4 : x$

$144 : 10{,}000 :: 4 : x$

$144\,x = 10{,}000 \times 4$

$144\,x = 40{,}000$

```
  12      100
  12        100
 ---     -----
 144     10000
```

$$x = \frac{40{,}000}{144}$$

```
144 ) 40000.0 ( 277.7+   Ans.
      288
      ---
      1120
      1008
      ----
       1120
       1008
       ----
        1120
        1008
        ----
```

(11) (*a*) $\frac{4}{x} = \frac{7}{21}$ is equivalent to $4 : x :: 7 : 21$. The product of the means equals the product of the extremes Hence,

$$7x = 4 \times 21$$
$$7x = 84$$
$$x = \frac{84}{7}, \text{ or } 12. \quad \text{Ans}$$

In like manner,

(*b*) $\frac{x}{24} = \frac{8}{16}$ is equivalent to $x : 24 :: 8 : 16$.

$$16x = 24 \times 8$$
$$16x = 192$$
$$x = \frac{192}{16} = 12. \quad \text{Ans.}$$

(*c*) $\frac{2}{10} = \frac{x}{100}$ is equivalent to $2 : 10 :: x = 100$.

$$10x = 2 \times 100$$
$$10x = 200$$
$$x = \frac{200}{10} = 20. \quad \text{Ans.}$$

(*d*) $\frac{15}{45} = \frac{60}{x}$ is equivalent to $15 : 45 :: 60 : x$

$$15x = 45 \times 60$$
$$15x = 2{,}700$$
$$x = \frac{2{,}700}{15} = 180. \quad \text{Ans.}$$

(*e*) $\frac{10}{150} = \frac{x}{600}$ is equivalent to $10 : 150 :: x : 600$.

$$150x = 10 \times 600$$
$$150x = 6{,}000$$
$$x = \frac{6{,}000}{150} = 40. \quad \text{Ans.}$$

(12) $3\frac{1}{2}$ ft. = 3.5 ft., since $\frac{1}{2} = .5$.
$6\frac{3}{4}$ ft. = 6.75 ft., since $\frac{3}{4} = .75$.

Consider the question "What do we wish to find?" In this case it is "pounds." We know that a piece of shafting 3.5 ft. long weighs 37.45 lb., and we wish to know the weight of a piece of shafting $6\frac{3}{4}$ ft. long. It is evident that the weight of a piece of shafting 6.75 ft. long bears the same relation to the weight of a piece 3.5 ft. long that 6.75 ft. bears to 3.5 ft. Letting x occupy any place in the proportion, we have the following, the value of x being the same in each. Thus,

(a) 3.5 ft. : 6.75 ft. :: 37.45 lb. : x lb.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb.} \quad \text{Ans.}$$

(b) 6.75 ft. : 3.5 ft. :: x lb. : 37.45 lb.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb.} \quad \text{Ans.}$$

(c) x lb. : 37.45 lb. :: 6.75 ft. : 3.5 ft.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb.} \quad \text{Ans.}$$

(d) 37.45 lb. : x lb. :: 3.5 ft. : 6.75 ft.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb.} \quad \text{Ans.}$$

(13) We will first reduce 8 hr. 40 min. to minutes. 8 hr. + 40 min. = (8 × 60 min.) + 40 min. = 520 min. In this problem we are required to find "time." We know that a railway train runs 444 miles in 520 minutes, and we want to know how long it will take it to run 1,060 miles at the same rate of speed. It is evident that the time it requires to run 1,060 miles bears the same relation to the time it takes to run 444 miles that 1,060 miles bears to 444 miles. Letting x occupy any place in the proportion, we have the following, the value of x being the same in each. Thus,

(*a*) 1,060 miles : 444 miles :: x min. : 520 min.,

$$\text{or } x = \frac{1{,}060 \times 520}{444} = \frac{551{,}200}{444} =$$

```
  1 0 6 0          4 4 4 ) 5 5 1 2 0 0.0 0 ( 1 2 4 1.4 4+ min.
      5 2 0                4 4 4
 -----------               -----
  2 1 2 0 0                1 0 7 2
5 3 0 0                      8 8 8
 -----------               -------
5 5 1 2 0 0                  1 8 4 0
                             1 7 7 6
                             -------
                                6 4 0
                                4 4 4
                                -----
                                1 9 6 0
                                1 7 7 6
                                -------
                                  1 8 4 0
                                  1 7 7 6
                                  -------
                                      6 4
```

Reducing 1,241.44 min. to hours by dividing by 60, we have

```
6 0 ) 1 2 4 1.4 4 ( 20 hr. 41.44 min.   Ans.
      1 2 0
      -----
        4 1
```

(*b*) 444 miles : 1,060 miles :: 520 min. : x min.

$$x = \frac{1{,}060 \times 520}{444} = 1{,}241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min.} \quad \text{Ans.}$$

(*c*) x min. : 520 min. :: 1,060 miles : 444 miles.

$$x = \frac{1{,}060 \times 520}{444} = 1{,}241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min.} \quad \text{Ans.}$$

(*d*) 520 min. : x min. :: 444 miles : 1,060 miles.

$$x = \frac{1{,}060 \times 520}{444} = 1{,}241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min.} \quad \text{Ans.}$$

(14) A pump discharging 135 gal. per min. fills the tank in 38 min. Therefore, a pump discharging 1 gal. per

min. fills it in 135×38 min. Hence, a pump discharging 85 gal. per min. fills it in $\frac{135 \times 38}{85} = 60\frac{6}{17}$ min. Ans.

$$
\begin{array}{r}
135 \\
\times\ 38 \\
\hline
1080 \\
405 \\
\hline
85\,)\,5130\,(\,60\,\tfrac{6}{17} \\
510 \\
\hline
30 \\
\end{array}
\qquad \frac{30}{85} = \tfrac{6}{17}
$$

(15) If a wheel measuring 12.56 ft. around it turns 520 times, a wheel measuring 1 ft. around it turns 520×12.56 times. Hence, a wheel measuring 15.7 ft. around it turns $\frac{520 \times 12.56}{15.7} = 416$ times. Ans.

$$
\begin{array}{r}
520 \\
12.56 \\
\hline
3120 \\
2600 \\
1040 \\
520 \\
\hline
15.7\,)\,6531.20\,(\,416\text{ times.} \\
628 \\
\hline
251 \\
157 \\
\hline
942 \\
942 \\
\hline
\end{array}
$$

(16) If a cistern 28 ft. by 12 ft. by 10 ft. holds 798 bbl. of water,

a cistern 1 ft. by 12 ft. by 10 ft. holds $\frac{798}{28}$ bbl.;

a cistern 1 ft. by 1 ft. by 10 ft. holds $\frac{798}{28 \times 12}$ bbl.;

a cistern 1 ft. by 1 ft. by 1 ft. holds $\frac{798}{28 \times 12 \times 10}$ bbl.

Therefore, by a similar course of reasoning, a cistern 20 ft. by 17 ft. by 6 ft. holds

$$\frac{798 \times 20 \times 17 \times 6}{28 \times 12 \times 10} = \frac{\overset{57}{\cancel{798}} \times \overset{2}{\cancel{20}} \times 17 \times \cancel{6}}{\underset{4}{\cancel{28}} \times \underset{2}{\cancel{12}} \times \cancel{10}} = \frac{969}{2} = 484\tfrac{1}{2} \text{ bbl.}$$

Ans.

$$\begin{array}{r} 57 \\ 17 \\ \hline 399 \\ 57 \\ \hline 2\,)\,969 \\ \hline 484\tfrac{1}{2} \end{array}$$

MENSURATION AND USE OF LETTERS IN FORMULAS.

(1) Substituting for D, x, B, and i their values,

$$C = \frac{D - x}{B + i} = \frac{120 - 12}{10 + 3.5} = \frac{108}{13.5} = 8. \quad \text{Ans.}$$

A line between two numbers signifies that the one above the line is to be divided by the one below the line.

(2) Substituting for A, h, D, and x their values,

$$\frac{A\,h + D}{2\,x + 6} = \frac{(5 \times 200) + 120}{(2 \times 12) + 6} = \frac{1{,}000 + 120}{24 + 6} = \frac{1{,}120}{30} = 37\tfrac{1}{3}$$

$$37\tfrac{1}{3} + D = 37\tfrac{1}{3} + 120 = 157\tfrac{1}{3}. \quad \text{Ans.}$$

When there is no sign between the letters, multiplication is understood.

(3) Substituting for A, D, i, and B their values,

$$v = \sqrt{\frac{A\,D}{i\,B + 1.5}} = \sqrt{\frac{5 \times 120}{(3.5 \times 10) + 1.5}} = \sqrt{\frac{600}{36.5}}$$

$$= \sqrt{16.4383} = 4.05+. \quad \text{Ans.}$$

The square root sign extends over both numerator and denominator, thus indicating that the square root of the entire fraction is to be extracted.

(4) Substituting for A, B, D, and h their values,

$$g = \frac{(B - A)^2 - \sqrt{h + 2\,B + A}}{A^2 - (1 + D)} = \frac{(10 - 5)^2 - \sqrt{200 + 2 \times 10 + 5}}{5^2 - (1 + 120)}$$

$$= \frac{5^2 - \sqrt{225}}{125 - 121} = \frac{25 - 15}{4} = \tfrac{10}{4} = 2\tfrac{1}{2}. \quad \text{Ans.}$$

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(5) When one straight line meets another straight line, two angles are formed which together equal 180°. Hence, if one of the angles = 152° 3′, the other angle = 180° − 152° 3′, or

$$\begin{array}{rr} 180^\circ = & 179^\circ\ 60' \\ \text{subtracting,} & 152^\circ\ \ 3' \\ \hline & 27^\circ\ 57' \end{array}$$ Ans.

(6) See Arts. **26-28.**

(7) See Art. **41.** A rectangle with the same area would have the same base and altitude.

(8) Since the area is to be found in square inches, the $2\frac{1}{2}$ feet must be reduced to inches. $2\frac{1}{2}$ ft. = 30 in. Area = $30 \times 11\frac{1}{2} = 345$ sq. in. Ans.

(9) It will take $1\frac{1}{2}$ boards to reach lengthways of the room. Since the room is 15 feet wide and each board is 5 *inches* wide, it will take $15 \div \frac{5}{12} = 36$ boards, laid side by side, to extend across the width of the room. Hence, number of boards required = $36 \times 1\frac{1}{2} = 54$. Ans.

(10) The total area of the floor of the station = 55×58 ft. = 3,190 sq. ft. − 25×26 ft. = 650 sq. ft., the area represented by the lower right-hand corner of the figure. Hence, total area of floor = 3,190 − 650 = 2,540 sq. ft.

From this we have to deduct the following areas:

$$\begin{array}{lrrl} 2 \text{ boilers} & = 2 \times 8 \times 19 & = 304 & \text{sq. ft.} \\ \text{Feed-pump} & = 2\frac{1}{2} \times 5 & = 12.5 & \text{sq. ft.} \\ 2 \text{ engines} & = 2 \times 4\frac{1}{2} \times 10 & = 90 & \text{sq. ft.} \\ 2 \text{ dynamos} & = 2 \times 5\frac{1}{2} \times 6\frac{1}{2} & = 71.5 & \text{sq. ft.} \\ \text{Switchboard} & = \frac{10 \times 3.5}{12} & = 2.92 & \text{sq. ft.} \\ & & \overline{480.92} & \text{sq. ft.} \end{array}$$

The unoccupied floor space, therefore, equals

$$2{,}540 - 480.92 = 2{,}059.08 \text{ sq. ft.} \quad \text{Ans.}$$

(11) A triangle with three equal angles has three equal sides, and is therefore an equilateral triangle.

(12) A triangle with two equal angles has two equal sides, and is therefore an isosceles triangle.

(13) The sum of the three angles in any triangle = 2 right angles, or 180°. In the given triangle, the sum of two angles = 23° + 32° 32′ = 55° 32′, and the third angle = 180° − 55° 32′, or

$$\begin{array}{rr} 180^\circ = & 179^\circ\ 60' \\ \text{subtracting,} & 55^\circ\ 32' \\ \hline & 124^\circ\ 28' \end{array} \quad \text{Ans.}$$

(14) In Fig. I we have the proportion $AD : DE :: AB : BC$, in which $AD = 10$ in., $AB = 24$ in., and $BC = 13\frac{1}{2}$ in., to find DE.

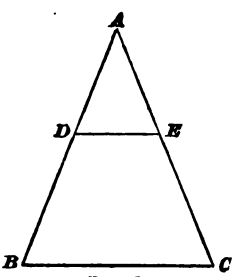

FIG. I.

Substituting the given values,

$$10 : DE :: 24 : 13\tfrac{1}{2}, \text{ or}$$

$$DE = \frac{10 \times 13.5}{24} = 5.625 \text{ in.} \quad \text{Ans.}$$

(15) A line drawn diagonally from one corner to the opposite one would form the hypotenuse of a right triangle, whose two sides are 39 and 52 feet. By rule **6**, Art. **58**, the length of the diagonal = $\sqrt{52^2 + 39^2} = 65$ ft. Ans.

(16) See example, Art. **64.** The process is simply to find one of the angles of the polygon, and then to divide it by 2. By rule **10**, Art. **64**, one of the interior angles $= \frac{180 \times (8-2)}{8} = 135^\circ$. This divided by $2 = 67\frac{1}{2}^\circ$. Ans.

(17) Since this is a regular hexagon, it may be inscribed in a circle (Fig. II), and the radius of the inscribing circle will be equal to one side of the hexagon. Since the diameter $EF = 2$ inches, the radii AB and AC, and the side BC each = 1 inch, and the triangle ABC is equilateral. Draw the line AD perpendicular to the side BC; it will bisect BC. Then, in the right-angled triangle ADB, $AB = 1''$ and $BD = \frac{1}{2}''$,

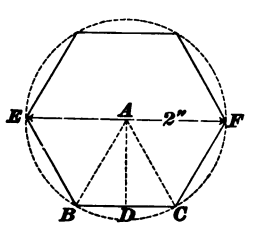

FIG. II.

to find AD. According to rule **7**, Art. **59**, $AD = \sqrt{1^2 - .5^2} = \sqrt{.75} = .866''$. Hence, the distance between two opposite sides of the hexagon $= AD \times 2 = .866 \times 2 = 1.732''$. Ans.

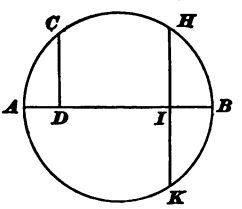

FIG. III.

(18) In Fig. III, we have the proportion $BI : HI :: HI : IA$, in which $BI = 6$ and $HI = \frac{1}{2}$ of $HK = \frac{18}{2} = 9$. Substituting, $6 : 9 :: 9 : IA$, or $IA = \frac{81}{6} = 13.5$ in. Hence, the diameter $AB = IA + BI = 13.5 + 6 = 19.5$ in. Ans.

(19) One mile $= 5{,}280$ feet. The circumference of the wheel in feet $= \frac{72 \times 3.1416}{12} = 18.8496$. (See rule **12**, Art. **77**.) Number of revolutions in going 1 mile $= 5{,}280 \div 18.8496 = 280.112$. Ans.

(20) Using rule **15**, Art. **80**, area = diameter squared $\times .7854$. $6.06^2 = 36.7236$; $36.7236 \times .7854 = 28.8427$ sq. in. Ans.

(21) Since the radius of the circle $= 6$ in., its diameter $= 12$ in., and its circumference $= 12 \times 3.1416 = 37.6992$ in. There are 360° in the circumference, and the length of an arc of 12° $= 37.6992 \times \frac{12}{360} = 1.25664$ in. Ans.

(22) The area of a circle 15 in. in diameter $= 15^2 \times .7854 = 176.715$ sq. in. Hence, the area of a sector of this circle whose angle is $12\frac{1}{2}° = 176.715 \times \frac{12\frac{1}{2}}{360} = \frac{2{,}208.937}{360} = 6.1359$ sq. in. Ans. (See rule **17**, Art. **82**.)

(23) (*a*) The side of a square whose area $= 103.8691$ sq. in. $= \sqrt{103.8691} = 10.1916$ in. Ans.

(*b*) By rule **16**, Art. **81**, the diameter of a circle having the same area $= \sqrt{\frac{103.8691}{.7854}} = 11\frac{1}{2}$ in. Ans.

(*c*) Perimeter of the square = 10.1916 × 4 = 40.7664 in.; circumference of the circle = 11.5 × 3.1416 = 36.1284 in.; difference = 40.7664 − 36.1284 = 4.638 in. Ans.

(24) The perimeter of the base = 4 × 6 = 24 in. = 2 ft. Convex area = 2 × 12 = 24 sq. ft. The area of the bases is found as follows: In Fig. IV, AB = 4 in. and AC = 2 in.; since this is a regular hexagon, $AO = AB = 4$ in. By rule **7**, Art. **59**, $OC = \sqrt{4^2 - 2^2} = \sqrt{12}$ = 3.4641 in.; area of triangle AOB = $\frac{4 \times 3.4641}{2}$ = 6.9282 sq. in.; area of base = 6.9282 × 6 = 41.5692; and the area of both bases = 41.5692 × 2 = 83.1384 sq. in. This reduced to square feet = $\frac{83.1384}{144}$ = .5774. Hence, the area of the entire surface of the column is 24 + .5774 = 24.5774 sq. ft. Ans.

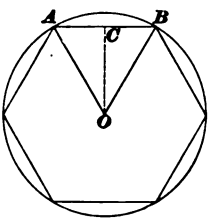

FIG. IV.

(25) The cubical contents in cubic inches = area of base in square inches × altitude in inches. The area of the base in the last example was found to be 41.5692 sq. in.; altitude = 12 × 12 = 144 in. Hence, the cubical contents = 41.5692 × 144 = 5,985.9648 cu. in. Ans.

(26) This example is solved by combining the rules for the circular ring (see example, Art. **81**) and for the cylinder. To obtain the area of one end of the tube, we have $4^2 \times .7854$ = 12.5664 = area of a circle 4 inches in diameter; 3.73^2 × .7854 = 10.9272 = area of a circle 3.73 inches in diameter; difference = 12.5664 − 10.9272 = 1.6392 = area of one end of the tube. The cubical contents = 1.6392 × 12 = 19.6704 cu. in.; the weight = 19.6704 × .28 = 5.5, or 5½ lb. Ans.

(27) This example is done exactly like the one in Art. **92**, and the solution is given here without explanation.

(*a*) In the formula of rule **18**, Art. **83**, $\frac{4h^2}{3}\sqrt{\frac{D}{h} - .608}$, h in this case = 18, and D = 60.

Substituting, area =

$\frac{4 \times 18^2}{3}\sqrt{\frac{60}{18} - .608} = \frac{4 \times 324}{3}\sqrt{3.333 - .608} = 432 \times \sqrt{2.725}$

$= 432 \times 1.65 = 712.8$ sq. in. This reduced to square feet $= 712.8 \div 144 = 4.95$. Hence, the steam space $= 4.95 \times 16 = 79.2$ cu. ft. Ans.

(*b*) Total area of one end of boiler in square inches $= 60^2 \times .7854 = 2{,}827.44$. From this is to be subtracted the area of the tube ends and of the segment found above.

Area of ends of tubes $= 3.5^2 \times .7854 \times 64 = 615.75$ sq. in.
Area of segment = 712.8 sq. in.
1,328.55 sq. in.

Area of water space $= 2{,}827.44 - 1{,}328.55 = 1{,}498.89$ sq. in.

Contents of water space $= 1{,}498.89 \times 16 \times 12 = 287{,}786.88$ cu. in., and $287{,}786.88 \div 231 = 1{,}245.83$, number of gallons, or say 1,246 gal. Ans.

(28) The area of the convex surface = circumference of base $\times$ $\frac{1}{2}$ slant height $= 18.8496 \times \frac{10}{2} = 94.248$ sq. in. (See rule **21**, Art. **97.**) The area of the entire surface $= 94.248$ sq. in. + the area of the base. The diameter of the base $= \frac{18.8496}{3.1416} = 6$ in.; hence, the area of the base $= 6^2 \times .7854 = 28.2744$ (rules **13** and **15**, Arts. **78** and **80**); therefore, the area of the entire surface $= 94.248 + 28.2744 = 122.5224$ sq. in. Ans.

(29) Using rule **22**, Art. **98**, volume = area of base $\times$ $\frac{1}{3}$ altitude $= 28.2744 \times \frac{9}{3} = 84.8232$ cu. in. Ans.

(30) The vat has the form of an inverted frustum of a pyramid. Area of larger base $= 15^2 = 225$ sq. ft.; area of smaller base $= 12^2 = 144$ sq. ft. Hence, by rule **24**, Art. **102**, the contents of the vat in cubic feet =

$(225 + 144 + \sqrt{225 \times 144})\frac{11}{3} = (369 + 180) \times \frac{11}{3} = 549 \times \frac{11}{3}$

$= 2{,}013$ cu. ft. This should be reduced to cubic inches by

multiplying by 1,728, the number of cubic inches in a cubic foot. $2{,}013 \times 1{,}728 = 3{,}478{,}464$ cu. in. Since there are 231 cubic inches in a gallon, the number of gallons that the vat will hold $= \frac{3{,}478{,}464}{231} = 15{,}058.29$. Ans.

(31) The pail is in the form of a frustum of a cone. Area of larger base $= 12^2 \times .7854 = 113.0976$ cu. in. Area of smaller base $= 63.6174$ cu. in. Hence, the contents in cubic inches $=$

$$(113.0976 + 63.6174 + \sqrt{113.0976 \times 63.6174}) \times \frac{11}{3}$$

$$= (176.715 + \sqrt{7{,}194.9753})\frac{11}{3} = (176.715 + 84.8232) \times \frac{11}{3}$$

$$= 261.5382 \times \frac{11}{3} = 958.9734.$$

The contents of the vat in cubic inches were found in the last example to be 3,478,464. Hence, the number of pails of water required to fill the vat $= 3{,}478{,}464 \div 958.9734 = 3{,}627.28$. Ans.

(32) (*a*) By rule **25**, Art. **104**, area of the surface $= 22.5^2 \times 3.1416 = 506.25 \times 3.1416 = 1{,}590.435$ sq. in. Ans.

(*b*) Using rule **26**, Art. **105**, the cubical contents $=$ the cube of the diameter $\times .5236 = 11{,}390.625 \times .5236 = 5{,}964.1313$ cu. in. Ans.

(33) (*a*) Given $OB = \frac{16}{2}$, or 8 inches, and $OA = \frac{13}{2}$, or $6\frac{1}{2}$ inches, to find the volume, area, and weight (see Fig. V):

Radius of center circle equals $\frac{8 + 6.5}{2}$, or $7\frac{1}{4}$ inches. Length of center line $= 2 \times 3.1416 \times 7\frac{1}{4} = 45.5532$ inches.

The radius of the inner circle is $6\frac{1}{2}$ inches and of the outer circle 8 inches; therefore, the diameter of the cross-section on the line AB is $1\frac{1}{2}$ inches. Then, according to rule **27**, Art. **106**,

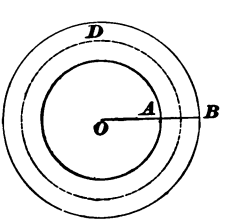

FIG. V.

the area of the ring is $1\frac{1}{2} \times 3.1416 \times 45.553 = 214.665$ square inches. Ans.

Diameter of cross-section of ring $= 1\frac{1}{2}$ inches.

Area of cross-section of ring $= (1\frac{1}{2})^2 \times .7854 = 1.76715$ sq. in. Ans.

By rule **28**, Art. **107**, volume of ring $= 1.76715 \times 45.553 = 80.499$ cu. in. Ans.

(*b*) Weight of ring $= 80.499 \times .261 = 21$ lb. Ans.

PRINCIPLES OF MECHANICS.

(1) Broadly speaking, mechanics is the science that treats of forces and their effects on material bodies. See Art. **1.**

(2) See Arts. **2, 3,** and **4.**

(3) Solid, liquid, and gaseous. See Arts. **6, 7,** and **8.**

(4) General properties are such as are common to all substances; special properties are such as are possessed by certain substances only. See Arts. **12** and **24.**

(5) (*a*) Motion is that condition of a body that causes it to change its position in relation to some other body. See Art. **30.**

(*b*) Velocity is the rate of motion, that is, the distance passed through in a unit of time. See Art. **32.**

(*c*) Uniform velocity means passing over equal distances in equal intervals of time. See Art. **32.**

(*d*) Variable velocity means passing over equal distances in unequal intervals of time, or passing over unequal distances in equal intervals of time. See Art. **33.**

(6) (*a*) Acceleration is the rate of change of velocity. See Arts. **35** and **57.**

(*b*) Retardation denotes the rate of decrease of velocity. See Arts. **36** and **58.**

(*c*) The average velocity is that uniform velocity that carries the body over the same distance in the same time as the variable velocity. See Art. **37.**

(28) Suspend the body successively from two different points and find the point of intersection of two plumb-lines from the points of suspension. The center of gravity will be on the line passing through the point of intersection and perpendicular to the plumb-lines. See Art. **103.**

(29) (*a*) Centrifugal force is that force that tends to pull a revolving body away from the point about which the body revolves. See Art. **104.**

(*b*) Centripetal force is that force that tends to pull a revolving body toward the point about which it revolves. See Art. **105.**

(30) The centrifugal and centripetal forces of a given revolving body are always equal and opposite. See Art. **104.**

(31) Applying rule **14**, given in Art. **106,**

$$F = .00034 \times 10 \times 6 \times 60^2 = 73.44 \text{ lb.} \quad \text{Ans.}$$

(32) See Arts. **110, 111,** and **112.**

(33) The forces acting on a body at rest are in equilibrium. See Art. **109.**

(34) As long as the line of direction falls within the base, the body will stand; if it falls outside the base, the body will fall. In the former case the forces will be in equilibrium; in the latter they will not be in equilibrium. See Art. **113.**

MACHINE ELEMENTS.

(1) (*a*) A lever is a rigid bar or rod capable of being turned about a pivot.

(*b*) The weight arm is that part of the lever between the fulcrum and the weight.

(*c*) The force arm is that part of the lever between the fulcrum and the force.

(*d*) The fulcrum is the point or pivot about which the lever turns. See Arts. **1** and **2.**

(2) The product of the weight and the weight arm must be equal to the product of the force and the force arm. See Art. **2.**

(3) See rule 1, Art. **3.**

(4) $\frac{12 \times 20}{100} = 2.4$ in. Ans. See Art. **3.**

(5) Solid pulleys and split pulleys. See Art. **9.**

(6) Split pulleys can be more easily put on and removed from shafts than solid pulleys. See Art. **9.**

(7) As explained in Art. **11,** the belt climbs toward the highest part of the pulley; and since in a crowned pulley the highest portion is in the center, the belt will tend to stay on the pulley.

(8) "Balancing pulleys" is the operation of making opposite sides of pulleys equal in weight, so that the centrifugal

forces of the opposite sides of a revolving pulley will be equal. A pulley is balanced by first finding which side is the lighter and then adding weights to the lighter side until it is as heavy as the other. See Art. **12.**

(9) The "driver" is the pulley that imparts motion to the belt. The "driven" is the pulley that receives motion from the belt. See Art. **14.**

(10) Applying rule **3,** Art. **15,**

$$D = \frac{6 \times 1{,}800}{600} = 18 \text{ in.} \quad \text{Ans.}$$

(11) Applying rule **6,** Art. **18,**

$$N = \frac{5 \times 1{,}600}{12} = 666\tfrac{2}{3} \text{ rev. per min.} \quad \text{Ans.}$$

(12) Applying rule **7,** Art. **23,**

$$B = 3\tfrac{1}{4}\left(\frac{6+2}{2}\right) + 2 \times 40 = 93 \text{ ft.} \quad \text{Ans.}$$

(13) From Table I we find that the allowable effective pull C for an arc of contact of 120° is 28.8 pounds. Applying rule **8,** Art. **27,**

$$W = \frac{33{,}000 \times 40}{1{,}650 \times 28.8} = 27.7, \text{ say } 28 \text{ in.} \quad \text{Ans.}$$

(14) Applying rule **9,** Art. **28,**

$$H = \frac{28.8 \times 28 \times 1{,}200}{33{,}000} = 29.3+ \text{ H. P.} \quad \text{Ans.}$$

(15) Applying rule **10,** Art. **31,**

$$W = \frac{23{,}100 \times 40}{1{,}650 \times 28.8} = 19.44, \text{ say } 20 \text{ in.} \quad \text{Ans.}$$

(16) The hair or grain side is commonly considered to be the proper side to be in contact with the pulley face. For reasons see Art. **32.**

(17) No; rosin makes the belt gummy and causes it to crack. See Art. **32.**

(18) Pulleys run out of true; they should be turned true. Pulleys out of line; they should be lined up. Belts sometimes flap if they are run at speeds over 4,000 feet per minute; perforating the belt with a series of small holes is said to remedy the defect in this case. Lack of steadiness in running; take steps to insure steady running. A defective joint; unlace the joint and relace it properly. Too great a distance between pulleys; reduce the distance if possible or substitute a wider belt. See Art. **33.**

(19) Lacing, sewing, riveting, and cementing. The cemented joint is considered the best.

(20) See Art. **38.**

(21) A wheel that imparts motion to another is called a "driver." A "follower" is a wheel that receives motion from another.

(22) Letting W represent the weight we have, by rule **12**, Art. **39**,

$$50 \times 90 \times 30 \times 36 = W \times 30 \times 12 \times 20,$$

or

$$4{,}860{,}000 = 7{,}200\ W.$$

Hence, $$W = \frac{4{,}860{,}000}{7{,}200} = 675 \text{ lb.} \quad \text{Ans.}$$

(23) (*a*) Circular pitch is the distance measured along the pitch circle from a point on one tooth to the corresponding point on the next tooth. See Art. **47.**

(*b*) Diametral pitch is the number of teeth per inch of pitch diameter. See Art. **47.**

(24) The epicycloidal and the involute. See Arts. **51** and **52.**

(25) Involute teeth are stronger and their action is more satisfactory. See Art. **52.**

(26) Applying rule **13**, Art. **54**,

$$D = \frac{1.152 \times 60}{3.1416} = 22 \text{ in.} \quad \text{Ans.}$$

(27) Applying rule **15**, Art. **56**,

$$P = \frac{3.1416 \times 30}{60} = 1.57 \text{ in.} \quad \text{Ans.}$$

(28) Applying rule **17**, Art. **59**,

$$D_o = \frac{80 + 2}{8} = 10\tfrac{1}{4} \text{ in.} \quad \text{Ans.}$$

(29) Applying rule **21**, Art. **63**,

$$N = 7.5 \times 8 - 2 = 58 \text{ teeth.} \quad \text{Ans.}$$

(30) A fixed pulley has a stationary block. A movable pulley has a movable block. See Arts. **71** and **72.**

(31) Calling W the weight that can be raised, we have, by applying rule **27**, Art. **74**,

$$W = 150 \times 14 = 2{,}100 \text{ lb.} \quad \text{Ans.}$$

(32) The force that must be applied is greater than if there were no frictional losses. See Art. **75.**

(33) Applying rule **28**, Art. **75**,

$$f_a = \frac{100 \times 1{,}500}{2 \times 4 \times 60} = 312.5 \text{ lb.} \quad \text{Ans.}$$

(34) With the Weston differential pulley block a much greater weight can be raised with a given force than with the ordinary pulley; and the load can be stopped anywhere by ceasing to pull on the chain. See Art. **78.**

(35) Applying rule **30**, Art. **82**,

$$P = \frac{600 \times 12}{70} = 102.8+, \text{ say } 103 \text{ lb.} \quad \text{Ans.}$$

(36) Applying rule **31**, Art. **83**,

$$W = \frac{36 \times 50}{15} = 120 \text{ lb.} \quad \text{Ans.}$$

(37) First applying rule **34**, Art. **89**, we find the factor by which the theoretical weight is to be multiplied.

Thus, $$E = \frac{\frac{1}{6}}{\frac{1}{6} + 2\frac{1}{2}} = \frac{1}{16}.$$

Then applying rule **35**, Art. **90**, to obtain the theoretical weight, we find

$$W = \frac{6.2832 \times 50 \times 20}{\frac{1}{6}} = 37{,}699.2 \text{ pounds.}$$

Finally applying the principle of Art. **92**, we find the probable actual weight to be

$$37{,}699.2 \times \tfrac{1}{16} = 2{,}356.2 \text{ lb.} \quad \text{Ans.}$$

(38) (*a*) The velocity ratio is the ratio between the distance through which the force acts and that through which the weight moves. See Art. **103.**

(*b*) The efficiency is the ratio of the actual work to the theoretical work. See Art. **105.**

(39) By Art. **105**, the efficiency would be $\frac{2{,}356.2}{37{,}699.2}$ $= .0625$, or $6\frac{1}{4}$ per cent. Ans.

MECHANICS OF FLUIDS.

(1) Hydrostatics is that branch of mechanics that treats of liquids at rest. See Art. **1.**

(2) See Art. **4.**

(3) See Art. **5.**

(4) The pressure due to the weight of water acts, like any other pressure to which the water may be subjected, in *all* directions. See Art. **5.**

(5) First find the area of the base. This equals $.7854 \times 3^2 = 7.0686$ square inches. Multiplying this by the height of the water, we get $7.0686 \times 8 = 56.5488$ cubic inches as the volume of a column of water whose weight is equal to the pressure on the base. Therefore, the pressure on the base $= 56.5488 \times .03617 = 2.045+$ lb. Ans. See Arts. **7, 8,** and **9.**

(6) See Art. **12.**

(7) The water stands at the same level in each tube, because it is then in equilibrium. See Art. **16.**

(8) Because of the resistance of the air and the friction in the pipe. See Art. **17.**

(9) Specific gravity is the ratio between weights of equal volumes of any substance and water. Thus, the specific gravity of iron is the ratio between the weight of a given volume of iron and the weight of an equal volume of water. See Art. **23.**

(10) Using the principle of Art. **25** and taking the specific gravity from Table I, we have for the weight of the aluminum $62.42 \times 2.50 \times 100 = 15{,}605$ lb. Ans.

(11) A body immersed in a liquid will be buoyed up with a force equal to the weight of the liquid displaced. See Art. **26.**

(12) See Art. **29.**

(13) (*a*) Hydrometers are instruments for determining the specific gravity of liquids and of some forms of solids.

(*b*) Hydrometers of constant weight and hydrometers of constant volume. See Art. **30.**

(14) Applying rule **5**, we have

$$n = \frac{.7854 \times 1^2 \times 400}{19.25} = 16.32 \text{ gallons.}$$

As there are 60 minutes in 1 hour, the amount of water that can be pumped in 1 hour will be $16.32 \times 60 = 979.2$ gal. Ans.

(15) The amount of water to be delivered per minute will be $\frac{9{,}360}{60} = 15.6$ gallons. The velocity of flow for this case should not exceed 500 feet per minute. Applying rule **4**, we have

$$A = \frac{19.25 \times 15.6}{500} = .601 \text{ square inch, nearly.}$$

From Table VI we see that the nearest larger size pipe is the 1-inch pipe, which has an actual area of .863 square inch. Hence, a 1-inch pipe should be used.

(16) Bends in a pipe greatly increase the resistance to the flow of water, and, hence, also the power required to move the water through the pipe. As the power available for the suction pipe of a pump is quite small, any increase in the resistance to the flow will materially interfere with the satisfactory working of the pump. See Art. **39.**

(17) See Art. **41.**

(18) About 30 in., as explained in Art. **43.**

(19) By means of a vacuum gauge. See Arts. **45** to **47**, inclusive.

(20) Applying the principle of Art. **48**, we have $\frac{30 \times 13.6}{2.5}$ = 163.2 in. Ans.

(21) The pressure of the atmosphere is measured with an instrument called a barometer. See Arts. **49** to **52**, inclusive.

(22) The higher the elevation the shorter is the column of air and the lower the density of the air. Consequently, the pressure exerted by the air becomes less the higher we go above sea level. See Art. **53**.

(23) Like the pressure exerted by a liquid, the pressure of the atmosphere acts in all directions with the same intensity. See Art. **54**.

(24) The pressure of a gas varies inversely as the volume, the temperature remaining constant. Thus, if the volume is reduced one-half, the pressure is doubled; if the volume is reduced to one-third the original volume, the pressure is increased to three times the original pressure. See Art. **55**.

(25) (*a*) Gauge pressure is pressure measured above the atmospheric pressure. See Art. **56**.

(*b*) Absolute pressure is pressure measured from vacuum and is equal to gauge pressure plus atmospheric pressure. See Art. **56**.

(26) Call the original volume, or volume of the tube, 1; then the volume after the piston has moved $\frac{4}{5}$ the length of the tube will be $1 - \frac{4}{5} = \frac{1}{5}$. Applying rule **7**, we have

$$p_1 = \frac{14.7 \times 1}{\frac{1}{5}} = 73.5 \text{ lb. per sq. in. Ans}$$

(27) Applying rule **8**,

$$v_1 = \frac{14.7 \times 1}{147} = .1 = \tfrac{1}{10} \text{ the original volume. Ans.}$$

(28) An air pump is an apparatus for removing air from any vessel; in other words, it is an apparatus for producing a vacuum in any vessel. See Art. **60.**

(29) No. See Art. **61.**

(30) See Art. **62.**

(31) See Arts. **63** and **64.**

(32) When the water reaches the same level in both vessels, the flow of water from one vessel into the other will cease. This follows from Arts. **63** and **64.**

(33) The highest point of the siphon should not be more than 28 feet above the level of the water that is being siphoned, or the siphon will not work. See Art. **65.**

(34) (*a*) A pump is an apparatus for raising water. See Art. **68.**

(*b*) Pumps may be divided into three classes, depending on their mode of operation. See Art. **69.**

(*c*) (1) Suction pumps; (2) lifting pumps; and (3) force pumps. See Art. **69.**

(35) A suction pump acts by producing a vacuum in the suction pipe, which causes the water into which the lower end of the suction pipe is immersed to rise in the suction pipe. See Art. **70.**

(36) A lifting pump will raise water to a higher point than a suction pump. See Art. **72.**

(37) A force pump piston does not have any valve like the piston of a lifting pump. See Art. **73.**

(38) A single-acting force pump discharges water at every second stroke; a double-acting force pump discharges water during each stroke. See Art. **76.**

STRENGTH OF MATERIALS.

(1) (*a*) A stress is a force that acts in the interior of a body and tends to produce a deformation in the body. See Art. **1.**

(*b*) Tensile, compressive, shearing, transverse, and torsional. See Art. **2.**

(2) By Art. **3,** we have

$$\text{unit stress} = \tfrac{8000}{20} = 400 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(3) (*a*) Strain is the amount of deformation produced in a body by the action of a stress. See Art. **4.**

(*b*) Elasticity is that property of a body by virtue of which it will tend to return to its original shape after the force producing the deformation is removed. See Art. **5.**

(*c*) The elastic limit is that unit stress at which permanent set begins. See Art. **7.**

(4) Since the ultimate tensile strength per square inch is 55,000 pounds, the force at which a bar having an area of 4 square inches will rupture is $4 \times 55{,}000 = 220{,}000$ lb. Ans. See Art. **10.**

(5) Annealing makes the chain tougher, more ductile, and less liable to break from sudden jerks. See Art. **16.**

(6) Untwist a portion of the rope and examine the condition of the inner surfaces. See Art. **20.**

(7) (*a*) The safe load to be lifted depends on the size of the rope and on its manner of attachment to the hook of the tackle block or to the work. See Art. **23.**

(*b*) See Art. **23.**

(8) (*a*) Manila rope is chiefly used for hoisting and for power transmission. See Art. **26.**

(*b*) The fibers are oiled to reduce internal wear. See Art. **26.**

(9) The diameter of the pulley should be at least 40 times the diameter of the rope. See Art. **28.**

(10) Applying rule **4**, Art. **30,**

$$W = 600 \times (1\tfrac{1}{2})^2 = 1{,}350 \text{ lb.} \quad \text{Ans.}$$

(11) Applying rule **6**, Art. **30,**

$$C = \sqrt{16{,}000} \times .0316 = 4 \text{ in., nearly.} \quad \text{Ans.}$$

(12) A column both of whose ends are fixed is 4 times as strong as one that has both ends movable and $1\frac{7}{9}$ times as strong as one that has one fixed end and one movable end. See Art. **34.**

(13) See Art. **34.**

(14) Applying rule **7**, Art. **37,** and taking the proper values from Tables V and VII, we have

$$W = \frac{14{,}000 \times .7854 \times 14^2}{1 + \dfrac{(16 \times 12)^2}{281.25 \times 14^2}} = 1{,}291{,}480 \text{ lb.} \quad \text{Ans.}$$

(15) By rule **8**, Art. **42,** the area of the piston rod should be

$$\frac{.7854 \times 40^2 \times 110}{3{,}600} = 38.397 \text{ square inches.}$$

The corresponding diameter is $\sqrt{\dfrac{38.397}{.7854}} = 7$ inches, nearly. Hence a 6″ piston rod is too small. Ans.

(16) From Table X, S for this case is found to be 1,460. From Table XI, $R = \frac{14 \times 14^2}{6} = 457.33$. Now, applying the proper formula from Table IX, we have

$$W = \frac{4 \times 1{,}460 \times 457.33}{12 \times 12} = 18{,}547 \text{ lb.} \quad \text{Ans.}$$

(17) By rule **9**, Art. **50**, and Table XII, the safe load would be $4 \times 6 \times 500 = 12{,}000$ pounds. Hence, a load of 10,000 pounds is safe. Ans.

(18) From Table XII, $S = 4{,}400$. Applying rule **10**, Art. **51**, we have

$$a = \frac{400{,}000}{4{,}400} = 91 \text{ sq. in., nearly.} \quad \text{Ans.}$$

(19) Double shear means that the body subjected to the shearing stress resists this stress at two planes, at both of which shear must occur to produce failure. See Art. **48.**

(20) Countershafts serve to effect changes in speed and to stop and start the machinery. See Art. **54.**

(21) Cold-rolled shafting is made cylindrically true by a special rolling process. Bright shafting is turned up in a lathe. See Art. **55.**

(22) The diameter by which bright shafting is designated is that of the bar from which it is turned. See Art. **56.**

(23) Pulleys should be placed as near the bearings as possible so that the deflection of the shaft may be as small as possible. See Art. **58.**

(24) Applying rule **11**, Art. **60**, after finding from Table XIV that C for this case is 85, we have

$$H = \frac{8^3 \times 80}{85} = 481.8, \text{ nearly, say 482 H. P.} \quad \text{Ans.}$$

(25) From Table XIV we find C for this case to be 65. Applying rule **12**, Art. **61**,

$$R = \frac{65 \times 75}{4^3} = 76 \text{ rev.} \quad \text{Ans.}$$

(26) From Table XIV, $C = 95$. Applying rule **13**, Art. **62**,

$$D = \sqrt[3]{\frac{95 \times 84}{300}} = \sqrt[3]{26.6}, \text{ say } 3 \text{ in. Ans.}$$

(27) The horsepower of a shaft will vary directly with the speed. See Art. **63.**

(28) See Art. **11.**

(29) A steel rope wears better than an iron rope. See Art. **29.**

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